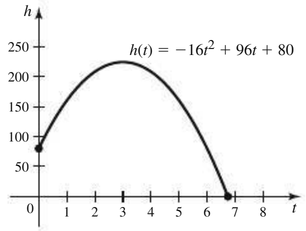
Answers

CHAPTER 1

Section 1.1 Exercises, pp. 9–13

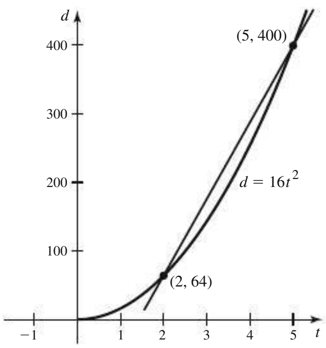
1. A function is a rule that assigns to each value of the independent variable in the domain a unique value of the dependent variable in the range. **3.** B **5.** The first statement **7.** $D = \mathbb{R}, R = [-10, \infty)$ **9.** The independent variable is h; the dependent variable is V; $D = [0, 50]$. **11.** -3; $1/8$; $1/(2x)$ **13.** The domain of $f \circ g$ consists of all x in the domain of g such that $g(x)$ is in the domain of f. **15. a.** 4 **b.** 1 **c.** 3 **d.** 3 **e.** 8 **f.** 1 **17.** 15.4 ft/s; radiosonde rises at an average rate of 15.4 ft/s during the first 5 seconds of its flight. **19.** 2; 2; 2; -2 **21.** A is even, B is odd, and C is even. **23.** $D = \{x: x \neq 2\}$; $R = \{y: y \neq -1\}$ **25.** $D = [-\sqrt{7}, \sqrt{7}]$; $R = [0, \sqrt{7}]$ **27.** $D = \mathbb{R}$ **29.** $D = [-3, 3]$ **31. a.** $[0, 3 + \sqrt{14}]$

b.



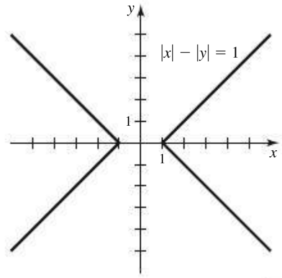
At time $t = 3$, the maximum height is 224 ft.

33. $1/z^3$ **35.** $1/(y^3 - 3)$ **37.** $(u^2 - 4)^3$ **39.** $\dfrac{x - 3}{10 - 3x}$ **41.** x **43.** $g(x) = x^3 - 5, f(x) = x^{10}$ **45.** $g(x) = x^4 + 2, f(x) = \sqrt{x}$ **47.** $|x^2 - 4|$; $D = \mathbb{R}$ **49.** $\dfrac{1}{|x - 2|}$; $D = \{x: x \neq 2\}$ **51.** $\dfrac{1}{x^2 - 6}$; $D = \{x: x \neq \sqrt{6}, -\sqrt{6}\}$ **53.** $x^4 - 8x^2 + 12$; $D = \mathbb{R}$ **55.** $f(x) = x - 3$ **57.** $f(x) = x^2$ **59.** $f(x) = x^2$ **61. a.** True **b.** False **c.** True **d.** False **e.** False **f.** True **g.** True **h.** False **i.** True **63.** 3 **65.** $2x + h$ **67.** $-\dfrac{2}{x(x + h)}$ **69.** $x + a + 1$ **71.** $x^2 + ax + a^2 - 2$ **73.** $\dfrac{4(x + a)}{a^2x^2}$ **75. a.** 864 ft/hr; the hiker's elevation increases at an average rate of 864 ft/hr. **b.** -487 ft/hr; the hiker's elevation decreases at an average rate of 487 ft/hr. **c.** The hiker might have stopped to rest during this interval of time and/or the trail was level during this portion of the hike. **77. a.**



b. $m_{\text{sec}} = 112$ ft/s; the object falls at an average rate of 112 ft/s.

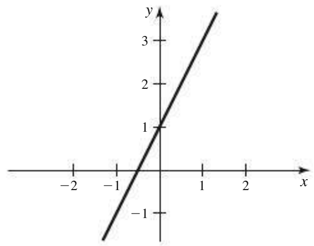
79. y-axis **81.** No symmetry **83.** x-axis, y-axis, origin **85.** Origin **87. a.** 4 **b.** 1 **c.** 3 **d.** -2 **e.** -1 **f.** 7 **89.**



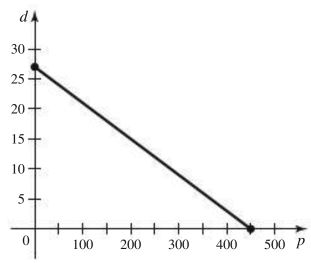
91. The equation $y = 2 - \sqrt{-x^2 + 6x + 16}$ can be rewritten as $(x - 3)^2 + (y - 2)^2 = 5^2$. Because $y \leq 2$, the function is the lower half of a circle of radius 5 centered at $(3, 2)$. $D = [-2, 8]$; $R = [-3, 2]$ **93.** $f(x) = 3x - 2$ or $f(x) = -3x + 4$ **95.** $f(x) = x^2 - 6$ **97.** $\dfrac{1}{\sqrt{x + h} + \sqrt{x}}$; $\dfrac{1}{\sqrt{x} + \sqrt{a}}$ **99.** $\dfrac{3}{\sqrt{x}(x + h) + x\sqrt{x + h}}$; $\dfrac{3}{x\sqrt{a} + a\sqrt{x}}$ **101.** None **103.** y-axis

Section 1.2 Exercises, pp. 22–27

1. A formula, a graph, a table, words **3.** $y = -\frac{2}{3}x - 1$ **5.** The set of all real numbers for which the denominator does not equal 0 **7.** $y = \begin{cases} x + 3 & \text{if } x < 0 \\ -\frac{1}{2}x + 3 & \text{if } x \geq 0 \end{cases}$ **9.** Shift the graph to the left 2 units. **11.** Compress the graph horizontally by a factor of $\frac{1}{3}$. **13.** $f(x) = |x - 2| + 3$; $g(x) = -|x + 2| - 1$ **15.** $f(x) = 2x + 1$

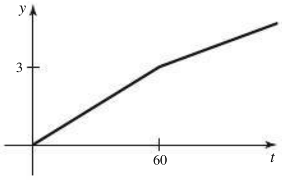


17. $f(x) = 3x - 7$ **19.** $C_s = 5.71$; 856.5 million **21.** $d = -3p/50 + 27$; $D = [0, 450]$

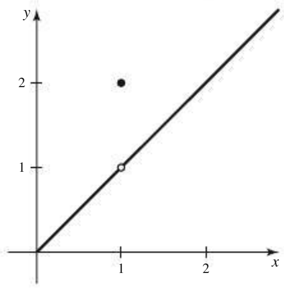


23. a. $p(t) = 328.3t + 1875$ **b.** 4830 **25.** $f(x) = \begin{cases} 3 & \text{if } x \leq 3 \\ 2x - 3 & \text{if } x > 3 \end{cases}$

27. $c(t) = \begin{cases} 0.05t & \text{if } 0 \le t \le 60 \\ 1.2 + 0.03t & \text{if } 60 < t \le 120 \end{cases}$



29.



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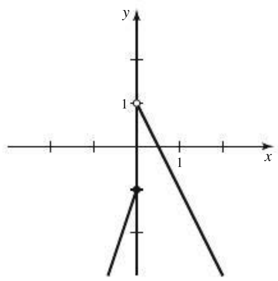
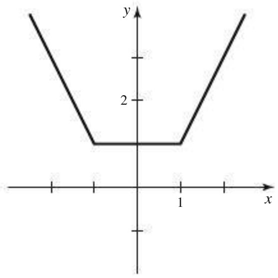
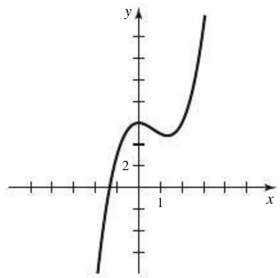


33.

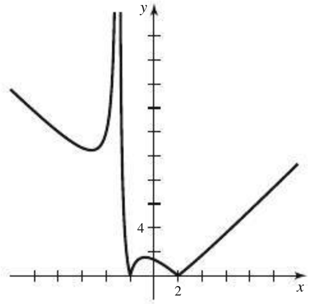


35. a.



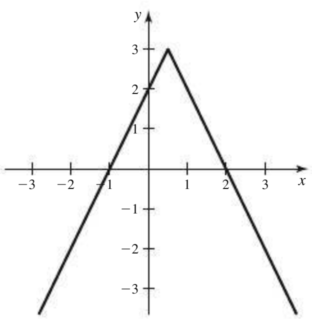
b. $D = \mathbb{R}$ **c.** One peak near $x = 0$; one valley near $x = 4/3$; x-intercept approx. $(-1.3, 0)$, y-intercept $(0, 6)$

37. a.



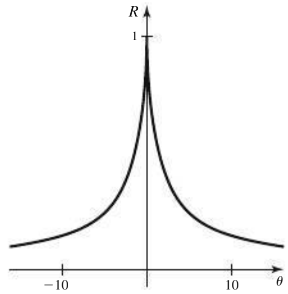
b. $D = \{x \colon x \neq -3\}$ **c.** Undefined at $x = -3$; a valley near $x = -5.2$; x-intercepts (and valleys) at $(-2, 0)$ and $(2, 0)$; a peak near $x = -0.8$; y-intercept $(0, \frac{4}{3})$

39. a.



b. $D = \mathbb{R}$ **c.** One peak at $x = \frac{1}{2}$; x-intercepts $(-1, 0)$ and $(2, 0)$; y-intercept $(0, 2)$ **41. a.** A, D, F, I **b.** E **c.** B, H **d.** I **e.** A

43. a.



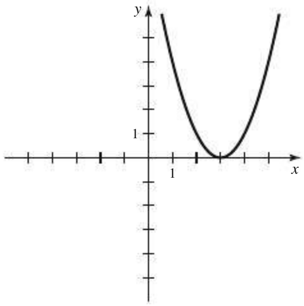
b. $\theta = 0$; vision is sharpest when we look straight ahead.
c. $|\theta| \le 0.19°$ (less than $\frac{1}{5}$ of a degree) **45.** $S(x) = 2$

47. $S(x) = \begin{cases} 1 & \text{if } x < 0 \\ -\frac{1}{2} & \text{if } x > 0 \end{cases}$

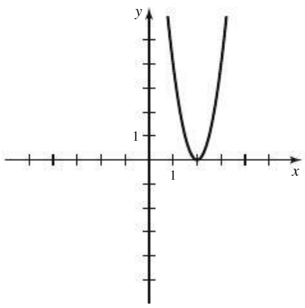
49. a. 12 **b.** 36 **c.** $A(x) = 6x$ **51. a.** 12 **b.** 21
c. $A(x) = \begin{cases} 8x - x^2 & \text{if } 0 \le x \le 3 \\ 2x + 9 & \text{if } x > 3 \end{cases}$

53. a. True **b.** False **c.** True **d.** False

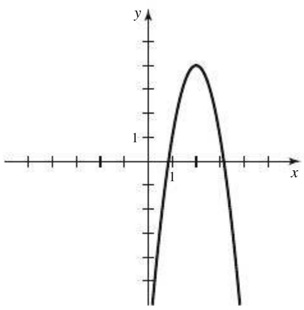
55. a. Shift 3 units to the right.



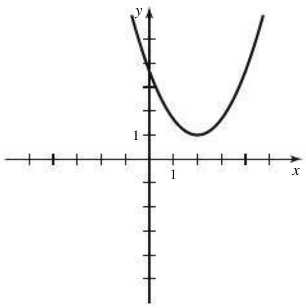
b. Horizontal compression by a factor of $\frac{1}{2}$, then shift 2 units to the right.



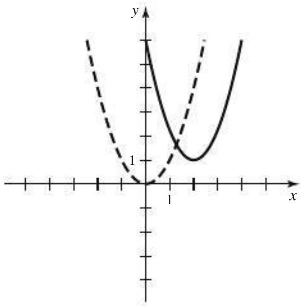
c. Shift to the right 2 units, vertically stretch by a factor of 3, reflect across the x-axis, and shift up 4 units.



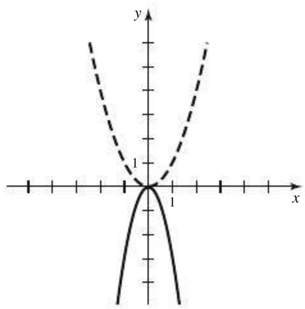
d. Horizontal stretch by a factor of 3, horizontal shift right 2 units, vertical stretch by a factor of 6, and vertical shift up 1 unit.



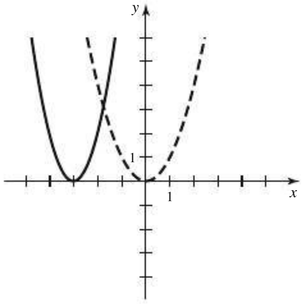
57. Shift the graph of $y = x^2$ right 2 units and up 1 unit.



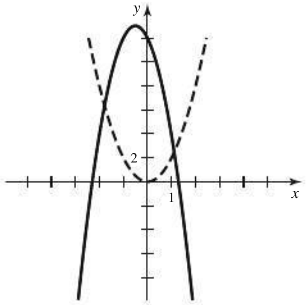
59. Stretch the graph of $y = x^2$ vertically by a factor of 3 and reflect across the x-axis.



61. Shift the graph of $y = x^2$ left 3 units and stretch vertically by a factor of 2.

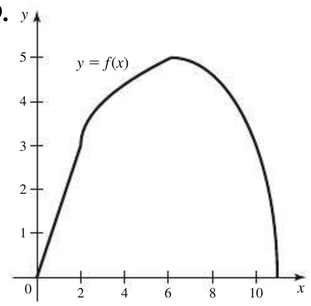


63. Shift the graph of $y = x^2$ to the left $\frac{1}{2}$ unit, stretch vertically by a factor of 4, reflect across the x-axis, and then shift up 13 units.

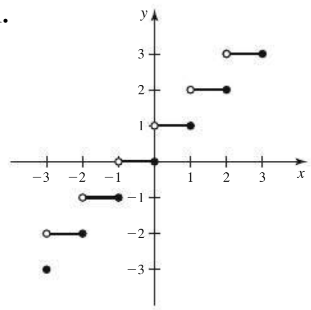


65. (0, 0); (2, 8) **67.** (0, 0); (4, 16)

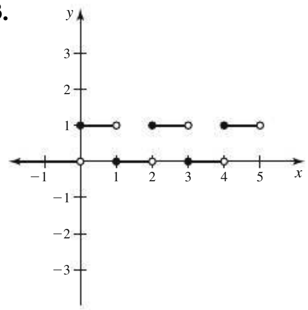
69.



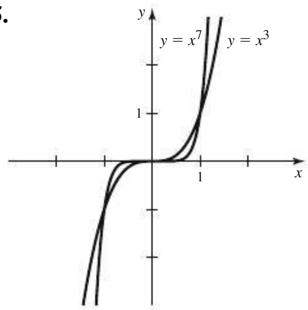
71.



73.

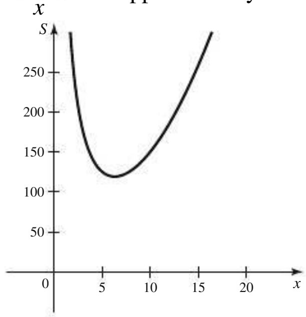


75.



77. a. 0.9; 90% chance that server will win from deuce given that such servers win 75% of their service points **b.** 0.1; 10% chance that server will win from deuce given that such servers win 25% of their service points **79. a.** $f(m) = 350m + 1200$ **b.** Buy

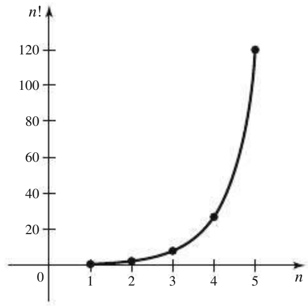
81. a. $S(x) = x^2 + \dfrac{500}{x}$ **b.** Approximately 6.3 ft



85. a.

n	1	2	3	4	5
$n!$	1	2	6	24	120

b.

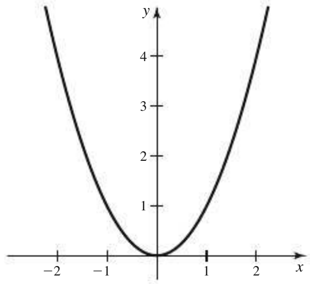


c. 10

Section 1.3 Exercises, pp. 35–39

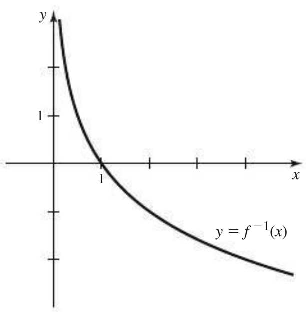
1. $D = \mathbb{R}$; $R = (0, \infty)$

3.



5. $(-\infty, -1], [-1, 1], [1, \infty)$ **7.** If a function f is not one-to-one, then there are domain values, x_1 and x_2, such that $x_1 \neq x_2$ but $f(x_1) = f(x_2)$. If f^{-1} exists, then by definition, $f^{-1}(f(x_1)) = x_1$ and $f^{-1}(f(x_2)) = x_2$, so f^{-1} assigns two different range values to the single domain value of $f(x_1)$.

9. $f^{-1}(x) = \dfrac{1}{2}x$ **11.**



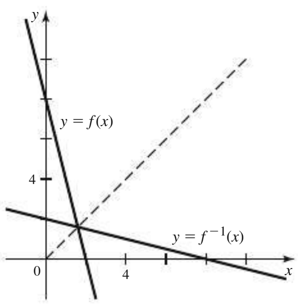
13. $g_1(x) = x^2 + 1$; $D = [0, \infty)$; $R = [1, \infty)$; $g_1^{-1}(x) = \sqrt{x - 1}$; $D = [1, \infty)$; $R = [0, \infty)$

15. The expression $\log_b x$ represents the power to which b must be raised to obtain x. **17.** $D = (0, \infty)$; $R = \mathbb{R}$

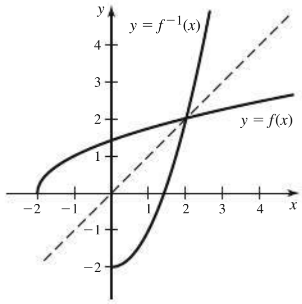
19. a. 3 **b.** 4 **c.** -2 **d.** 3 **e.** $1/2$ **21.** $(-\infty, \infty)$

23. $(-\infty, 5) \cup (5, \infty)$ **25.** $(-\infty, 0), (0, \infty)$

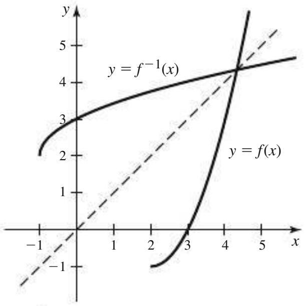
27. $f^{-1}(x) = -\dfrac{1}{4}x + 2$



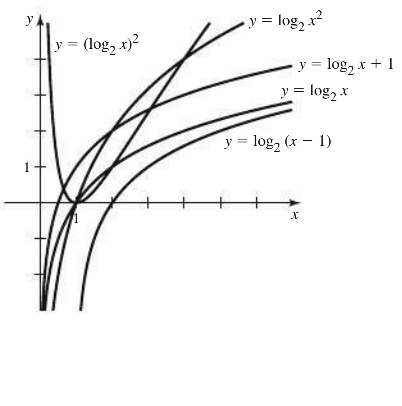
29. $f^{-1}(x) = x^2 - 2$



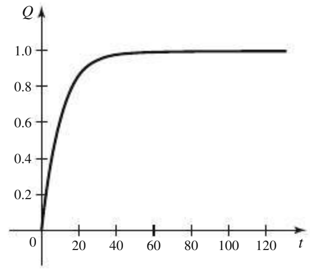
31. $f^{-1}(x) = 2 + \sqrt{x + 1}$



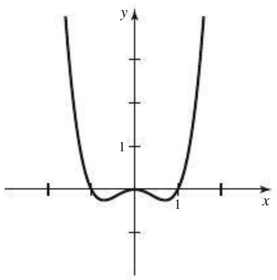
33. $f^{-1}(x) = \sqrt{\frac{2}{x} - 1}$ **35.** $f^{-1}(x) = \frac{1}{2}\ln x - 3$
37. $f^{-1}(x) = \frac{e^x - 1}{3}$ **39.** $f^{-1}(x) = -\frac{1}{2}\log_{10}x$
41. $f^{-1}(x) = \ln\left(\frac{2x}{1 - x}\right)$
43. a. $f_1(x) = \sqrt{1 - x^2};\ 0 \le x \le 1$
$f_2(x) = \sqrt{1 - x^2};\ -1 \le x \le 0$
$f_3(x) = -\sqrt{1 - x^2};\ -1 \le x \le 0$
$f_4(x) = -\sqrt{1 - x^2};\ 0 \le x \le 1$
b. $f_1^{-1}(x) = \sqrt{1 - x^2};\ 0 \le x \le 1$
$f_2^{-1}(x) = -\sqrt{1 - x^2};\ 0 \le x \le 1$
$f_3^{-1}(x) = -\sqrt{1 - x^2};\ -1 \le x \le 0$
$f_4^{-1}(x) = \sqrt{1 - x^2};\ -1 \le x \le 0$
45. -0.2 **47.** 1.19 **49.** $-0.09\overline{6}$ **51.** 1000 **53.** 2 **55.** $1/e$
57. $\ln 21/\ln 7$ **59.** $\ln 5/(3\ln 3) + 5/3$ **61.** 451 years
63. 9.53 years **65. a.** No **b.** $f^{-1}(h) = 2 - \frac{1}{4}\sqrt{64 - h}$
c. $f^{-1}(h) = 2 + \frac{1}{4}\sqrt{64 - h}$ **d.** 0.542 s **e.** 3.837 s
67. $\frac{\ln 15}{\ln 2} \approx 3.9069$ **69.** $\frac{\ln 40}{\ln 4} \approx 2.6610$ **71.** $e^{x\ln 2}$
73. $\log_5 |x|/\log_5 e$ **75.** e **77. a.** False **b.** False **c.** False **d.** True **e.** False **f.** False **g.** True **79.** A is $y = \log_2 x$; B is $y = \log_4 x$; C is $y = \log_{10} x$.
81.



83. a.



b. Vertical scaling; steady state equals a. **c.** Horizontal scaling; steady state remains constant. **d.** a
85. $f^{-1}(x) = \sqrt{x - 5} + 1, x \ge 5$ **87.** $f^{-1}(x) = \sqrt[3]{x} - 1, D = \mathbb{R}$
89. $f_1^{-1}(x) = \sqrt{2/x - 2}, D_1 = (0, 1]$; $f_2^{-1}(x) = -\sqrt{2/x - 2}$, $D_2 = (0, 1]$
95. a.



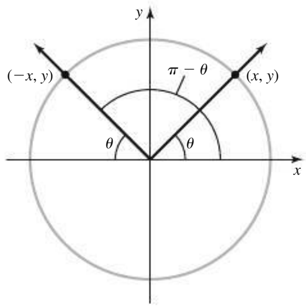
f is one-to-one on the intervals $(-\infty, -1/\sqrt{2}]$, $[-1/\sqrt{2}, 0]$, $[0, 1/\sqrt{2}]$, and $[1/\sqrt{2}, \infty)$.

b. $x = \sqrt{\frac{1 \pm \sqrt{4y + 1}}{2}}, -\sqrt{\frac{1 \pm \sqrt{4y + 1}}{2}}$

Section 1.4 Exercises, pp. 48–51

1. $\sin\theta = \text{opp}/\text{hyp}$; $\cos\theta = \text{adj}/\text{hyp}$; $\tan\theta = \text{opp}/\text{adj}$; $\cot\theta = \text{adj}/\text{opp}$; $\sec\theta = \text{hyp}/\text{adj}$; $\csc\theta = \text{hyp}/\text{opp}$ **3.** 3 s
5. The radian measure of an angle θ is the length s of an arc on the unit circle associated with θ. **7.** $\sin^2\theta + \cos^2\theta = 1$, $1 + \cot^2\theta = \csc^2\theta$, $\tan^2\theta + 1 = \sec^2\theta$ **9.** $\theta = 3\pi/2$
11. $\{x: x \text{ is an odd multiple of } \pi/2\}$
13. Sine is not one-to-one on its domain. **15.** $3\pi/4$
17. Horizontal asymptotes at $y = \pi/2$ and $y = -\pi/2$
19. $-1/2$ **21.** 1 **23.** $-1/\sqrt{3}$ **25.** $1/\sqrt{3}$ **27.** 1 **29.** -1
31. Undefined **33.** $\frac{\sqrt{2 + \sqrt{3}}}{2}$ or $\frac{\sqrt{6} + \sqrt{2}}{4}$
35. $\pi/4 + n\pi, n = 0, \pm 1, \pm 2, \ldots$
37. $\pi/6, 5\pi/6, 7\pi/6, 11\pi/6$
39. $\pi/4 + 2n\pi, 3\pi/4 + 2n\pi, n = 0, \pm 1, \pm 2, \ldots$
41. $0, \pi/2, \pi, 3\pi/2$
43. $\pi/12, 5\pi/12, 3\pi/4, 13\pi/12, 17\pi/12, 7\pi/4$
45. 0.1007; 1.4701 **47.** 17.3°; 72.7° **49.** $\pi/2$ **51.** $-\pi/6$
53. $\pi/3$ **55.** $2\pi/3$ **57.** -1 **59.** $\sin\theta = \frac{12}{13}$; $\tan\theta = \frac{12}{5}$
61. $\sqrt{1 - x^2}$ **63.** $\frac{\sqrt{4 - x^2}}{2}$ **65.** $2x\sqrt{1 - x^2}$
67. $\sec\theta = \frac{r}{x} = \frac{1}{x/r} = \frac{1}{\cos\theta}$
69. Dividing both sides of $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ gives $1 + \tan^2\theta = \sec^2\theta$. **71.** Because $\cos(\pi/2 - \theta) = \sin\theta$, for all θ, $1/\cos(\pi/2 - \theta) = 1/\sin\theta$, excluding integer multiples of π, and $\sec(\pi/2 - \theta) = \csc\theta$.

73. $\cos^{-1} x + \cos^{-1}(-x) = \theta + (\pi - \theta) = \pi$



75. $\pi/3$ **77.** $\pi/3$ **79.** $\pi/4$ **81.** $\pi/2 - 2$

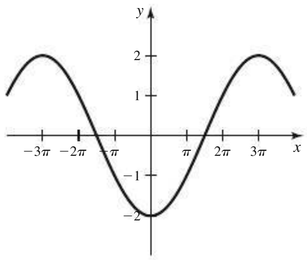
83. $\dfrac{1}{\sqrt{x^2 + 1}}$ **85.** $1/x$ **87.** $x/\sqrt{x^2 + 16}$

89. $\theta = \sin^{-1} \dfrac{x}{6} = \tan^{-1}\left(\dfrac{x}{\sqrt{36 - x^2}}\right) = \sec^{-1}\left(\dfrac{6}{\sqrt{36 - x^2}}\right)$

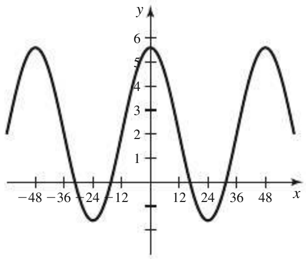
91. a. False **b.** False **c.** False **d.** False **e.** True **f.** False **g.** True **h.** False **93.** $\sin\theta = \frac{12}{13}$; $\tan\theta = \frac{12}{5}$; $\sec\theta = \frac{13}{5}$; $\csc\theta = \frac{13}{12}$; $\cot\theta = \frac{5}{12}$ **95.** $\sin\theta = \frac{12}{13}$; $\cos\theta = \frac{5}{13}$; $\tan\theta = \frac{12}{5}$; $\sec\theta = \frac{13}{5}$; $\cot\theta = \frac{5}{12}$ **97.** Amp $= 3$; period $= 6\pi$

99. Amp $= 3.6$; period $= 48$ **103.** Area of circle is πr^2; $\theta/(2\pi)$ represents the proportion of area swept out by a central angle θ. Therefore, the area of such a sector is $(\theta/2\pi)\pi r^2 = r^2\theta/2$.

105. Stretch the graph of $y = \cos x$ horizontally by a factor of 3, stretch vertically by a factor of 2, and reflect across the x-axis.



107. Stretch the graph of $y = \cos x$ horizontally by a factor of $24/\pi$; then stretch it vertically by a factor of 3.6 and shift it up 2 units.



109. $y = 3\sin(\pi x/12 - 3\pi/4) + 13$ **111.** About 6 ft

113. $d(t) = 10\cos(4\pi t/3)$ **115.** h

Chapter 1 Review Exercises, pp. 51–55

1. a. True **b.** False **c.** False **d.** True **e.** False **f.** False **g.** True **3.** f is one-to-one but not g.

5. $D = \{w: w \neq 2\}$; $R = \{y: y \neq 5\}$

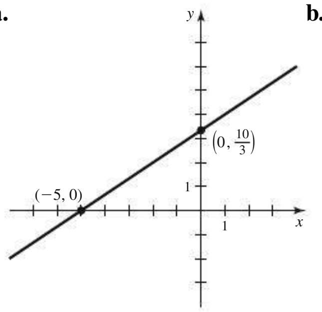
7. $D = (-\infty, -1] \cup [3, \infty)$; $R = [0, \infty)$ **9.** Yes; no **11.** 8

13. 7 **15.** 8 **17.** -2 **19. a.** 1 **b.** $\sqrt{x^3}$ **c.** $\sin^3 \sqrt{x}$ **d.** $\mathbb{R}$ **e.** $[-1, 1]$ **21.** $2x + h - 2$; $x + a - 2$

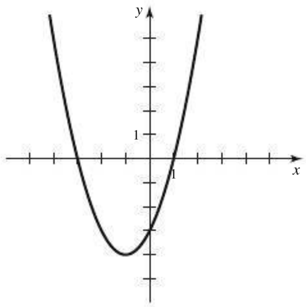
23. $3x^2 + 3xh + h^2$; $x^2 + ax + a^2$ **25. a.** $y = \dfrac{5}{2}x - 8$

b. $y = \dfrac{3}{4}x + 3$ **c.** $y = \dfrac{1}{2}x - 2$ **27.** $B = -\dfrac{1}{500}a + 212$

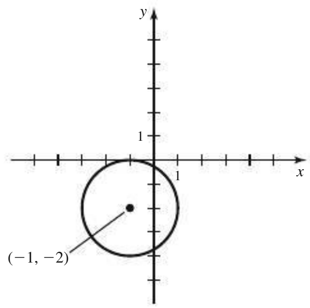
29. a.



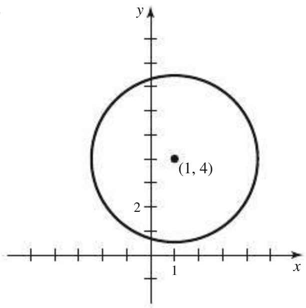
b.



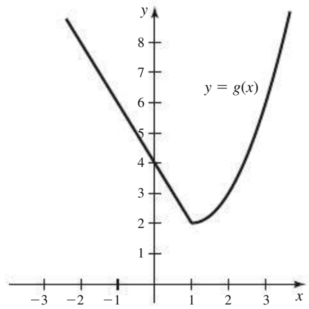
c.



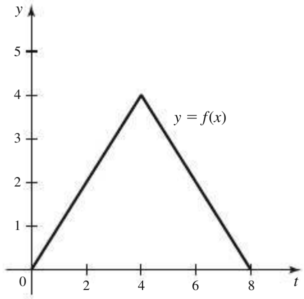
d.



31.

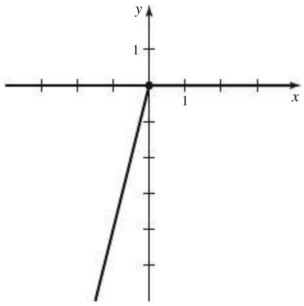


33. a.



b. 2; 14 **c.** $A(x) = \begin{cases} x^2/2 & \text{if } 0 \le x \le 4 \\ -x^2/2 + 8x - 16 & \text{if } 4 < x \le 8 \end{cases}$

35. $f(x) = \begin{cases} 4x & \text{if } x < 0 \\ 0 & \text{if } x \ge 0 \end{cases}$

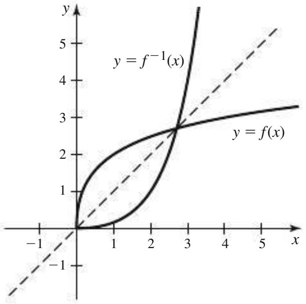


37. $D_f = \mathbb{R}$, $R_f = \mathbb{R}$; $D_g = [0, \infty)$, $R_g = [0, \infty)$

39. Shift $y = x^2$ left 3 units and down 12 units.

41. a. y-axis **b.** y-axis **c.** x-axis, y-axis, origin **43.** $x = 2$

45. $t = \frac{e^4 - 4}{5}$ **47.** $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ **49.** $\theta = \pm\frac{\pi}{12}, \pm\frac{5\pi}{12}$
51. Approx. 35 years **53.** $(-\infty, 0]$, $[0, 2]$, and $[2, \infty)$
55. $f^{-1}(x) = -\frac{1}{4}x + \frac{3}{2}$ **57.** $f^{-1}(x) = 2 + \sqrt{x - 1}$
59. $f^{-1}(x) = -\sqrt{\frac{x - 1}{3}}$ **61.** $f^{-1}(x) = \sqrt{\ln x - 1}$
63. $f^{-1}(x) = \frac{4x^2}{(6 - x)^2}$, for $0 \le x < 6$

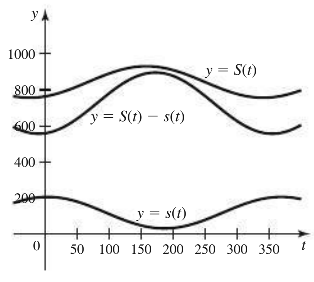


65. a. $f(t) = -2\cos\frac{\pi t}{3}$ **b.** $f(t) = 5\sin\frac{\pi t}{12} + 15$
67. a. F **b.** E **c.** D **d.** B **e.** C **f.** A
69. $(7\pi/6, -1/2)$; $(11\pi/6, -1/2)$ **71.** $-\frac{\sqrt{2 + \sqrt{2}}}{2}$
73. $\pi/6$ **75.** $-\pi/2$ **77.** x, provided $-1 \le x \le 1$
79. $\cos\theta = \frac{5}{13}$; $\tan\theta = \frac{12}{5}$; $\cot\theta = \frac{5}{12}$; $\sec\theta = \frac{13}{5}$; $\csc\theta = \frac{13}{12}$
81. $\frac{\sqrt{16 - x^2}}{4}$ **83.** $\pi/2 - \theta$ **85.** 0
87. $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$
$= \frac{2\sin\theta\cos\theta/\cos^2\theta}{(\cos^2\theta - \sin^2\theta)/\cos^2\theta} = \frac{2\tan\theta}{1 - \tan^2\theta}$

89. a.

n	1	2	3	4	5	6	7	8	9	10
$T(n)$	1	5	14	30	55	91	140	204	285	385

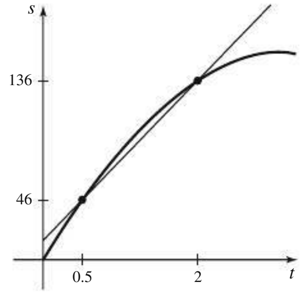
b. $D = \{n: n \text{ is a positive integer}\}$ **c.** 14
91. $s(t) = 117.5 - 87.5\sin\left(\frac{\pi}{182.5}(t - 95)\right)$
$S(t) = 844.5 + 87.5\sin\left(\frac{\pi}{182.5}(t - 67)\right)$



CHAPTER 2

Section 2.1 Exercises, pp. 61–62

1. $\frac{s(b) - s(a)}{b - a}$ **3.** 20 **5. a.** 36 **b.** 44 **c.** 52 **d.** 60
7. 47.84, 47.984, 47.9984; instantaneous velocity appears to be 48
9. $\frac{f(b) - f(a)}{b - a}$ **11.** The instantaneous velocity at $t = a$ is the slope of the line tangent to the position curve at $t = a$. **13. a.** 48 **b.** 64 **c.** 80 **d.** $16(6 - h)$ **15.** $m_{sec} = 60$; the slope is the average velocity of the object over the interval $[0.5, 2]$.



17.

Time interval	Average velocity
[1, 2]	80
[1, 1.5]	88
[1, 1.1]	94.4
[1, 1.01]	95.84
[1, 1.001]	95.984
$v_{inst} = 96$	

19.

Time interval	Average velocity
[2, 3]	20
[2.9, 3]	5.60
[2.99, 3]	4.16
[2.999, 3]	4.016
[2.9999, 3]	4.002
$v_{inst} = 4$	

21.

Time interval	Average velocity
[3, 3.5]	−24
[3, 3.1]	−17.6
[3, 3.01]	−16.16
[3, 3.001]	−16.016
[3, 3.0001]	−16.002
$v_{inst} = -16$	

23.

Time interval	Average velocity
[0, 1]	36.372
[0, 0.5]	67.318
[0, 0.1]	79.468
[0, 0.01]	79.995
[0, 0.001]	80.000
$v_{inst} = 80$	

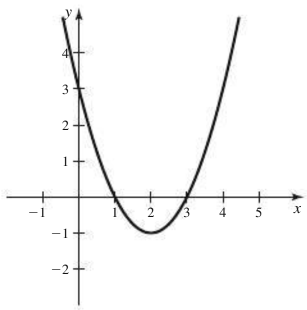
25.

Interval	Slope of secant line
[1, 2]	6
[1.5, 2]	7
[1.9, 2]	7.8
[1.99, 2]	7.98
[1.999, 2]	7.998
$m_{tan} = 8$	

27.

Interval	Slope of secant line
[0, 1]	1.718
[0, 0.5]	1.297
[0, 0.1]	1.052
[0, 0.01]	1.005
[0, 0.001]	1.001
$m_{tan} = 1$	

29. a.

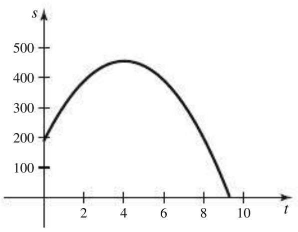


b. $(2, -1)$

c.

Interval	Slope of secant line
[2, 2.5]	0.5
[2, 2.1]	0.1
[2, 2.01]	0.01
[2, 2.001]	0.001
[2, 2.0001]	0.0001
$m_{tan} = 0$	

31. a.



b. $t = 4$

c.

Interval	Average velocity
$[4, 4.5]$	-8
$[4, 4.1]$	-1.6
$[4, 4.01]$	-0.16
$[4, 4.001]$	-0.016
$[4, 4.0001]$	-0.0016
$v_{\text{inst}} = 0$	

d. $0 \le t < 4$ **e.** $4 < t \le 9$ **33.** 0.6366, 0.9589, 0.9996, 1

Section 2.2 Exercises, pp. 67–71

1. As x approaches a from either side, the values of $f(x)$ approach L.
3. a. 5 **b.** 3 **c.** Does not exist **d.** 1 **e.** 2
5. a. -1 **b.** 1 **c.** 2 **d.** 2
7. a.

x	$f(x)$	x	$f(x)$
1.9	3.9	2.1	4.1
1.99	3.99	2.01	4.01
1.999	3.999	2.001	4.001
1.9999	3.9999	2.0001	4.0001

b. 4

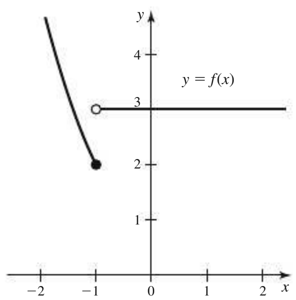
9. a.

t	$g(t)$	t	$g(t)$
8.9	5.983287	9.1	6.016621
8.99	5.998333	9.01	6.001666
8.999	5.999833	9.001	6.000167

b. 6

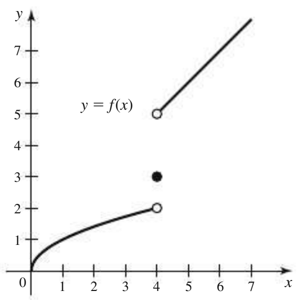
11. As x approaches a from the right, the values of $f(x)$ approach L.
13. $L = M$ **15. a.** 0 **b.** 1 **c.** 0
d. Does not exist; $\lim_{x\to 1^-} f(x) \ne \lim_{x\to 1^+} f(x)$ **17. a.** 3 **b.** 2
c. 2 **d.** 2 **e.** 2 **f.** 4 **g.** 1 **h.** Does not exist
i. 3 **j.** 3 **k.** 3 **l.** 3

19.



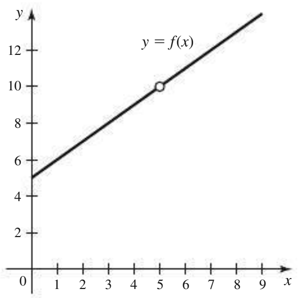
2; 2; 3; does not exist

21.



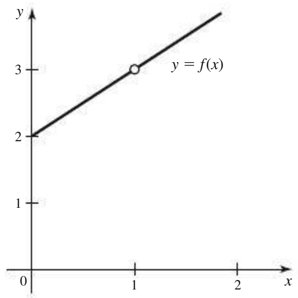
3; 2; 5; does not exist

23.



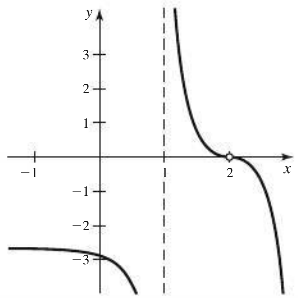
undefined; 10; 10; 10

25.



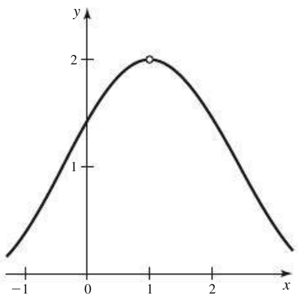
undefined; 3; 3; 3

27. From the graph and table, the limit appears to be 0.



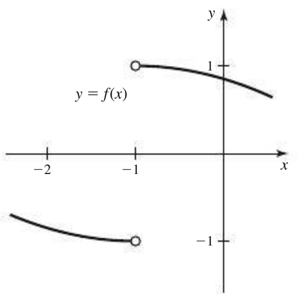
x	1.99	1.999	1.9999
$f(x)$	0.0021715	0.00014476	0.000010857
x	2.0001	2.001	2.01
$f(x)$	-0.000010857	-0.00014476	-0.0021715

29. From the graph and table, the limit appears to be 2.



x	0.9	0.99	0.999
$f(x)$	1.993342	1.999933	1.999999
x	1.001	1.01	1.1
$f(x)$	1.999999	1.999933	1.993342

31.

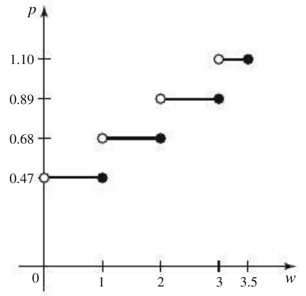


x	-1.1	-1.01	-1.001
$g(x)$	-0.9983342	-0.9999833	-0.9999998
x	-0.999	-0.99	-0.9
$g(x)$	0.9999998	0.9999833	0.9983342

From the table and the graph, it appears that the limit does not exist.

33. a. False **b.** False **c.** False **d.** False **e.** True

35. a.



b. 0.89 **c.** Because $\lim_{w\to 3^-} f(w) = 0.89$ and $\lim_{w\to 3^+} f(w) = 1.1$, we know that $\lim_{w\to 3^-} f(w) \neq \lim_{w\to 3^+} f(w)$. So $\lim_{w\to 3} f(w)$ does not exist.

37. 3 **39.** 16 **41.** 1

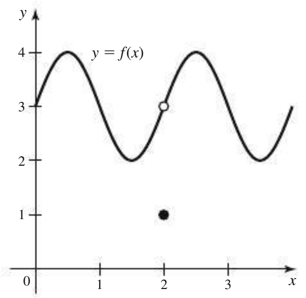
43. a.

x	$\sin(1/x)$
$2/\pi$	1
$2/(3\pi)$	-1
$2/(5\pi)$	1
$2/(7\pi)$	-1
$2/(9\pi)$	1
$2/(11\pi)$	-1

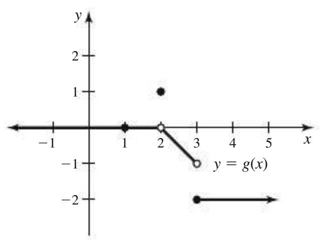
The function values alternate between 1 and -1.

b. The function values alternate between 1 and -1 infinitely many times on the interval $(0, h)$ no matter how small $h > 0$ becomes. **c.** Does not exist

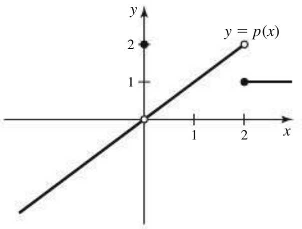
45.



47.



49.



51. a. $-2, -1, 1, 2$ **b.** 2, 2, 2

c. $\lim_{x\to a^-} \lfloor x \rfloor = a - 1$ and $\lim_{x\to a^+} \lfloor x \rfloor = a$, if a is an integer

d. $\lim_{x\to a^-} \lfloor x \rfloor = \lfloor a \rfloor$ and $\lim_{x\to a^+} \lfloor x \rfloor = \lfloor a \rfloor$, if a is not an integer

e. Limit exists provided a is not an integer

53. a. 8 **b.** 5 **55. a.** 2; 3; 4 **b.** p **57.** p/q

Section 2.3 Exercises, pp. 79–82

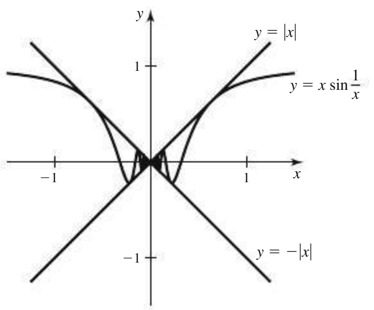
1. $\lim_{x\to a} p(x) = p(a)$ **3.** Those values of a for which the denominator is not zero **5.** $\dfrac{x^2 - 7x + 12}{x - 3} = x - 4$, for $x \neq 3$; -1
7. 32; Constant Multiple Law **9.** 5; Difference Law
11. 8; Quotient and Difference Laws **13.** 4; Root and Power Laws **15.** 5; 1 **17.** 20 **19.** 5 **21.** -45 **23.** 8 **25.** 3
27. 3 **29.** -5 **31.** 8 **33.** 2 **35.** -8 **37.** -1 **39.** -12 **41.** $\frac{1}{6}$
43. $-\frac{1}{36}$ **45.** $2\sqrt{a}$ **47.** $\frac{1}{8}$ **49.** $-\frac{1}{16}$ **51.** 5 **53.** 10 **55.** 2
57. -54 **59.** 0 **61.** 1 **63.** 1 **65.** $1/2$ **67.** Does not exist
69. Does not exist **71. a.** False **b.** False **c.** False **d.** False **e.** False **73. a.** 2 **b.** 0 **c.** Does not exist **75. a.** 0
b. $\sqrt{x-2}$ is undefined for $x < 2$. **77.** 0.0435 N/C
79. $\lim_{S\to 0^+} r(S) = 0$; the radius of the cylinder approaches 0 as the surface area of the cylinder approaches 0.

81. a. Because $\left|\sin \frac{1}{x}\right| \leq 1$, for all $x \neq 0$, we have that

$$|x|\left|\sin \frac{1}{x}\right| \leq |x|.$$

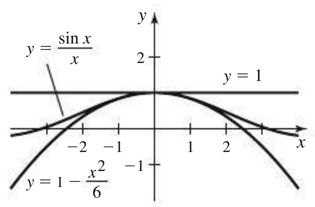
That is, $\left|x \sin \frac{1}{x}\right| \leq |x|$, so $-|x| \leq x \sin \frac{1}{x} \leq |x|$, for all $x \neq 0$.

b.



c. $\lim_{x\to 0} -|x| = 0$ and $\lim_{x\to 0} |x| = 0$; by part (a) and the Squeeze Theorem, $\lim_{x\to 0} x \sin \frac{1}{x} = 0$.

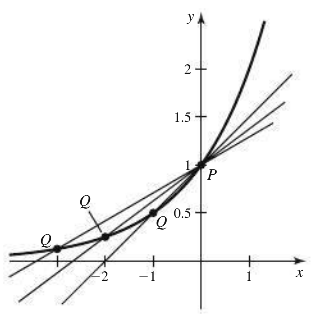
83. a.



b. $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$

85. Because $\lim_{x\to 0^-} |x| = \lim_{x\to 0^-} (-x) = 0$ and $\lim_{x\to 0^+} |x| = \lim_{x\to 0^+} x = 0$, we know that $\lim_{x\to 0} |x| = 0$. **87.** 1 **89.** $a = -13$; $\lim_{x\to -1} g(x) = 6$
91. 6 **93.** $5a^4$

95. a.



b. $\dfrac{2^x - 1}{x}$

c.

x	$\frac{2^x - 1}{x}$
-1	0.5
-0.1	0.6697
-0.01	0.6908
-0.001	0.6929
-0.0001	0.6931
-0.00001	0.6931
Limit $\approx$ 0.693	

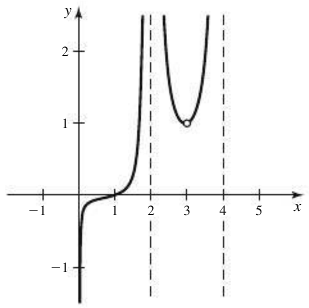
97. 6; 5 **99.** $\frac{1}{3}$ **101.** $f(x) = x - 1, g(x) = \frac{5}{x - 1}$
103. $b = 2$ and $c = -8$; yes **105.** 6; 4

Section 2.4 Exercises, pp. 88–91

1. As x approaches a from the right, the values of $f(x)$ are negative and become arbitrarily large in magnitude.
3. A vertical asymptote for a function f is a vertical line $x = a$, where one (or more) of the following is true:

$$\lim_{x\to a^-} f(x) = \pm\infty;\ \lim_{x\to a^+} f(x) = \pm\infty.$$

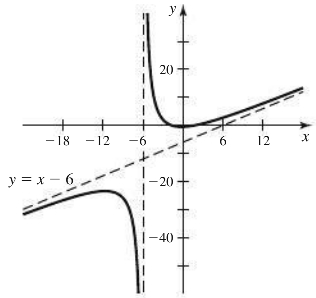
5. ∞ **7. a.** ∞ **b.** ∞ **c.** ∞ **d.** ∞ **e.** $-\infty$ **f.** Does not exist **9. a.** $-\infty$ **b.** $-\infty$ **c.** $-\infty$ **d.** ∞ **e.** $-\infty$ **f.** Does not exist **11. a.** ∞ **b.** $-\infty$ **c.** $-\infty$ **d.** ∞ **13.** $-\infty$ **15.** No; there is a vertical asymptote at $x = 2$ but not at $x = 1$.
17.



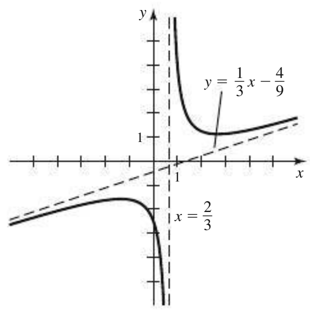
19. a and b are correct. **21. a.** ∞ **b.** $-\infty$ **c.** Does not exist **23. a.** $-\infty$ **b.** $-\infty$ **c.** $-\infty$ **25. a.** ∞ **b.** $-\infty$ **c.** Does not exist **27. a.** $-\infty$ **b.** $-\infty$ **c.** $-\infty$ **29. a.** ∞ **b.** Does not exist **c.** Does not exist **31. a.** ∞ **b.** $1/54$ **c.** Does not exist **33.** -5 **35.** ∞ **37.** $-\infty$ **39.** ∞ **41.** $-\infty$ **43.** ∞
45. a. $1/10$ **b.** $-\infty$ **c.** ∞; vertical asymptote: $x = -5$
47. $x = 3$; $\lim_{x\to 3^+} f(x) = -\infty$; $\lim_{x\to 3^-} f(x) = \infty$; $\lim_{x\to 3} f(x)$ does not exist **49.** $x = 0$ and $x = 2$; $\lim_{x\to 0^+} f(x) = \infty$; $\lim_{x\to 0^-} f(x) = -\infty$; $\lim_{x\to 0} f(x)$ does not exist; $\lim_{x\to 2^+} f(x) = \infty$; $\lim_{x\to 2^-} f(x) = \infty$; $\lim_{x\to 2} f(x) = \infty$
51. a. $-\infty$ **b.** ∞ **c.** $-\infty$ **d.** ∞ **53. a.** False **b.** True **c.** False **55.** $r(x) = \frac{(x-1)^2}{(x-1)(x-2)^2}$ **57.** $f(x) = \frac{1}{x - 6}$
59. $x = 0$ **61.** $x = -1$ **63.** $\theta = 10k + 5$, for any integer k
65. $x = 0$ **67. a.** $a = 4$ or $a = 3$ **b.** Either $a > 4$ or $a < 3$ **c.** $3 < a < 4$ **69. a.** $\frac{1}{\sqrt[3]{h}}$ regardless of the sign of h
b. $\lim_{h\to 0^+} \frac{1}{\sqrt[3]{h}} = \infty$; $\lim_{h\to 0^-} \frac{1}{\sqrt[3]{h}} = -\infty$; the tangent line at $(0, 0)$ is vertical.

Section 2.5 Exercises, pp. 100–102

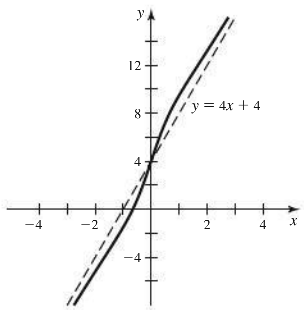
1. As $x < 0$ becomes arbitrarily large in magnitude, the corresponding values of f approach 10. **3.** ∞ **5.** 0 **7.** 0 **9.** 3 **11.** 0
13. ∞; 0; 0 **15.** 3; 3 **17.** 0 **19.** 0 **21.** ∞ **23.** $-\infty$
25. $2/3$ **27.** $-\infty$ **29.** 5 **31.** -4 **33.** $-3/2$ **35.** 0
37. $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = \frac{1}{5}$; $y = \frac{1}{5}$
39. $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 2$; $y = 2$
41. $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$; $y = 0$
43. $\lim_{x\to\infty} f(x) = \infty$; $\lim_{x\to-\infty} f(x) = -\infty$; none
45. $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = \frac{4}{9}$; $y = \frac{4}{9}$
47. $\lim_{x\to\infty} f(x) = \frac{2}{3}$; $\lim_{x\to-\infty} f(x) = -2$; $y = \frac{2}{3}$; $y = -2$
49. $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = \frac{1}{4 + \sqrt{3}}$; $y = \frac{1}{4 + \sqrt{3}}$
51. a. $y = x - 6$ **b.** $x = -6$
c.



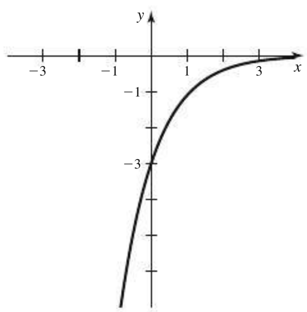
53. a. $y = \frac{1}{3}x - \frac{4}{9}$ **b.** $x = \frac{2}{3}$
c.



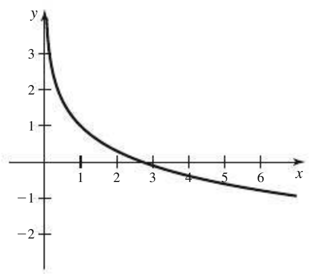
55. a. $y = 4x + 4$ **b.** No vertical asymptote
c.



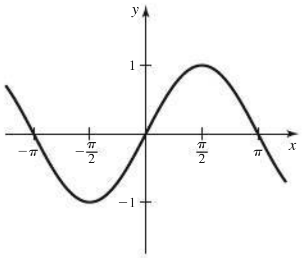
57. $\lim_{x\to\infty} (-3e^{-x}) = 0$; $\lim_{x\to-\infty} (-3e^{-x}) = -\infty$



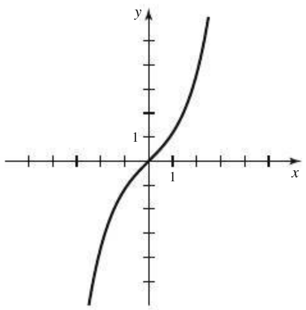
59. $\lim_{x\to\infty} (1 - \ln x) = -\infty$; $\lim_{x\to 0^+} (1 - \ln x) = \infty$



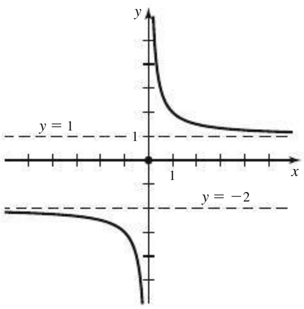
61. $\lim_{x\to\infty} \sin x$ does not exist; $\lim_{x\to-\infty} \sin x$ does not exist



63. a. False **b.** False **c.** True **d.** False **65.** 3500
67. No steady state **69.** 2
71. a. $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 2$; $y = 2$
b. $x = 0$; $\lim_{x\to0^+} f(x) = \infty$; $\lim_{x\to0^-} f(x) = -\infty$
73. a. $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 3$; $y = 3$
b. $x = -3$ and $x = 4$; $\lim_{x\to-3^-} f(x) = \infty$; $\lim_{x\to-3^+} f(x) = -\infty$; $\lim_{x\to4^-} f(x) = -\infty$; $\lim_{x\to4^+} f(x) = \infty$
75. a. $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 1$; $y = 1$
b. $x = 0$; $\lim_{x\to0^+} f(x) = \infty$; $\lim_{x\to0^-} f(x) = -\infty$
77. a. $\lim_{x\to\infty} f(x) = 1$; $\lim_{x\to-\infty} f(x) = -1$; $y = 1$ and $y = -1$
b. No vertical asymptote
79. a. $\lim_{x\to\infty} f(x) = 0$; $\lim_{x\to-\infty} f(x) = 0$; $y = 0$
b. No vertical asymptote **81. a.** $\lim_{x\to\infty} f(x) = 2$; $\lim_{x\to-\infty} f(x)$ does not exist; $y = 2$ **b.** $x = 0$; $\lim_{x\to0^+} f(x) = \infty$; $\lim_{x\to0^-} f(x)$ does not exist
83. a. $\frac{\pi}{2}$ **b.** $\frac{\pi}{2}$ **85. a.** $\lim_{x\to\infty} \sinh x = \infty$; $\lim_{x\to-\infty} \sinh x = -\infty$
b. $\sinh 0 = 0$

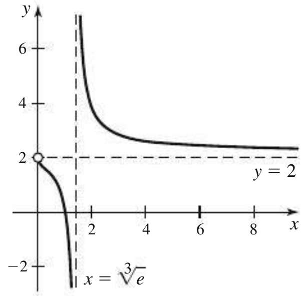


87.



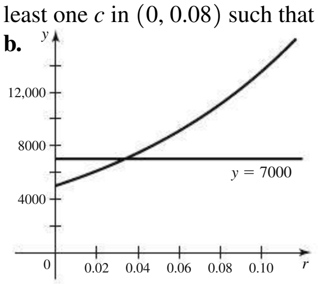
89. 1 **91.** 0 **93. a.** No; f has a horizontal asymptote if $m = n$, and it has a slant asymptote if $m = n + 1$. **b.** Yes; $f(x) = x^4/\sqrt{x^6 + 1}$ **95.** $y = 3$ and $y = 2$

97. $y = 2$; $x = \sqrt[3]{e}$

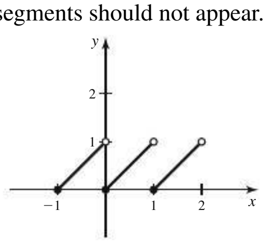


Section 2.6 Exercises, pp. 112–115

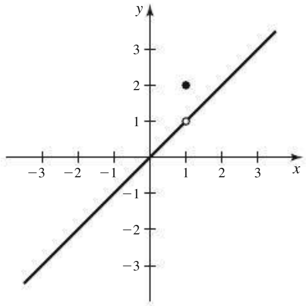
1. a, c **3.** A function is continuous on an interval if it is continuous at each point of the interval. If the interval contains endpoints, then the function must be right- or left-continuous at those points. **5.** $a = 1$, item 1; $a = 2$, item 3; $a = 3$, item 2 **7.** $a = 1$, item 1; $a = 2$, item 2; $a = 3$, item 1 **9. a.** $\lim_{x\to a^-} f(x) = f(a)$ **b.** $\lim_{x\to a^+} f(x) = f(a)$
11. $(0, 1), (1, 2), (2, 3], (3, 5)$; left-continuous at 3
13. $(0, 1), (1, 2), [2, 3), (3, 5)$; right-continuous at 2
15. $\{x: x \neq 0\}, \{x: x \neq 0\}$ **17.** No; $f(-5)$ is undefined.
19. No; $f(1)$ is undefined. **21.** No; $\lim_{x\to1} f(x) = 2$ but $f(1) = 3$.
23. No; $f(4)$ is undefined. **25.** $(-\infty, \infty)$
27. $(-\infty, -3), (-3, 3), (3, \infty)$ **29.** $(-\infty, -2), (-2, 2), (2, \infty)$
31. 1 **33.** $2\sqrt{6}$ **35.** 16 **37.** ln 2 **39. a.** $\lim_{x\to1} f(x)$ does not exist.
b. Continuous from the right **c.** $(-\infty, 1), [1, \infty)$ **41.** $(-\infty, 5]$; left-continuous at 5 **43.** $(-\infty, -2\sqrt{2}], [2\sqrt{2}, \infty)$; left-continuous at $-2\sqrt{2}$; right-continuous at $2\sqrt{2}$ **45.** $(-\infty, \infty)$ **47.** $(-\infty, \infty)$
49. 3 **51.** 1 **53.** 4 **55.** 2 **57.** $-\frac{1}{2}$ **59.** 4
61. $(n\pi, (n + 1)\pi)$, where n is an integer; $\sqrt{2}$; $-\infty$
63. $\left(\frac{n\pi}{2}, \left(\frac{n}{2} + 1\right)\frac{\pi}{2}\right)$, where n is an odd integer; ∞; $\sqrt{3} - 2$
65. $(-\infty, 0), (0, \infty)$; ∞; $-\infty$ **67. b.** $x \approx 0.835$
69. b. $x \approx -0.285$; $x \approx 0.778$; $x \approx 4.507$ **71. b.** $x \approx -0.567$
73. a. True **b.** True **c.** False **d.** False **75. a.** $A(r)$ is continuous on $[0, 0.08]$, and 7000 is between $A(0) = 5000$ and $A(0.08) = 11{,}098.20$. By the Intermediate Value Theorem, there is at least one c in $(0, 0.08)$ such that $A(c) = 7000$.
b. $c \approx 0.034$ or 3.4%



77. $[0, \pi/2]$; 0.45 **79.** $(-\infty, \infty)$ **81.** $[0, 16), (16, \infty)$
83. The vertical line segments should not appear.

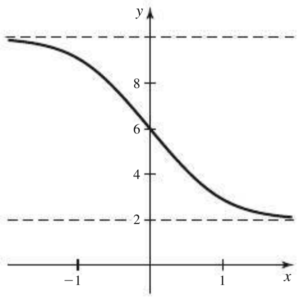


85. **a, b.**



87. **a.** 2 **b.** 8 **c.** No; $\lim_{x\to 1^-} g(x) = 2$ and $\lim_{x\to 1^+} g(x) = 8$.

89. $\lim_{x\to 0} f(x) = 6$, $\lim_{x\to -\infty} f(x) = 10$, and $\lim_{x\to\infty} f(x) = 2$; no vertical asymptote; $y = 2$ and $y = 10$ are the horizontal asymptotes.



91. $x_1 = \frac{1}{7}; x_2 = \frac{1}{2}; x_3 = \frac{3}{5}$ **93.** Yes. Imagine there is a clone of the monk who walks down the path at the same time the monk walks up the path. The monk and his clone must cross paths at some time between dawn and dusk. **95.** No; f cannot be made continuous at $x = a$ by redefining $f(a)$. **97.** $\lim_{x\to 2} f(x) = -3$; define $f(2)$ to be -3.
99. $a = 0$ removable discontinuity; $a = 1$ infinite discontinuity
101. **a.** Yes **b.** No **103.** **a.** For example, $f(x) = 1/(x - 1)$, $g(x) = x + 1$ **b.** For continuity, g must be continuous at 0, and f must be continuous at $g(0)$.

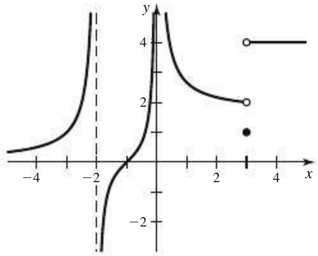
Section 2.7 Exercises, pp. 124–128

1. 1 **3.** c **5.** Given any $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$. **7.** $0 < \delta \le 2$
9. **a.** $\delta = 1$ **b.** $\delta = \frac{1}{2}$ **11.** **a.** $\delta = 2$ **b.** $\delta = \frac{1}{2}$
13. **a.** $0 < \delta \le 1$ **b.** $0 < \delta \le 0.79$ **15.** **a.** $0 < \delta \le 1$
b. $0 < \delta \le \frac{1}{2}$ **c.** $0 < \delta \le \varepsilon$ **17.** **a.** $0 < \delta \le 1$ **b.** $0 < \delta \le \frac{1}{2}$
c. $0 < \delta \le \frac{\varepsilon}{2}$ **19.** $\delta = \varepsilon/8$ **21.** $\delta = \varepsilon$ **23.** $\delta = \varepsilon$
25. $\delta = \varepsilon/3$ **27.** $\delta = \sqrt{\varepsilon}$ **29.** $\delta = \min\{1, \varepsilon/8\}$ **31.** $\delta = \varepsilon/2$
33. $\delta = \min\{1, 6\varepsilon\}$ **35.** $\delta = \min\{1/20, \varepsilon/200\}$
37. $\delta = \min\{1, \sqrt{\varepsilon/2}\}$ **39.** $\delta = \varepsilon/|m|$ if $m \ne 0$; use any $\delta > 0$ if $m = 0$ **41.** $\delta = \min\{1, 8\varepsilon/15\}$ **45.** $\delta = 1/\sqrt{N}$
47. $\delta = 1/\sqrt{N - 1}$ **49.** **a.** False **b.** False **c.** True **d.** True
51. For $x > a$, $|x - a| = x - a$. **53.** **a.** $\delta = \varepsilon/2$ **b.** $\delta = \varepsilon/3$
c. Because $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = -4$, $\lim_{x\to 0} f(x) = -4$.
55. $\delta = \varepsilon^2$ **57.** **a.** For each $N > 0$ there exists $\delta > 0$ such that $f(x) > N$ whenever $0 < x - a < \delta$. **b.** For each $N < 0$ there exists $\delta > 0$ such that $f(x) < N$ whenever $0 < a - x < \delta$.
c. For each $N > 0$ there exists $\delta > 0$ such that $f(x) > N$ whenever $0 < a - x < \delta$. **59.** $\delta = 1/N$ **61.** $\delta = (-10/M)^{1/4}$
65. $N = 1/\varepsilon$ **67.** $N = M - 1$

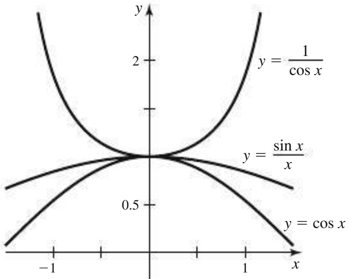
Chapter 2 Review Exercises, pp. 128–130

1. **a.** False **b.** False **c.** False **d.** True **e.** False **f.** False
g. False **h.** True **3.** 12 ft/s **5.** $x = -1$; $\lim_{x\to -1} f(x)$ does not exist; $x = 1$; $\lim_{x\to 1} f(x) \ne f(1)$; $x = 3$; $f(3)$ is undefined.

7. **a.** 1.414 **b.** $\sqrt{2}$
9.

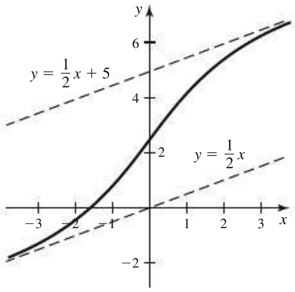


11. $\sqrt{11}$ **13.** 13 **15.** 2 **17.** $\frac{1}{3}$ **19.** $-\frac{1}{16}$ **21.** 108 **23.** $\frac{1}{108}$
25. 0 **27.** $-\infty$ **29.** ∞ **31.** 4 **33.** $-\infty$ **35.** $\frac{1}{2}$ **37.** $-3/\sqrt{a}$
39. $2/(1 - a)$ **41.** $3\pi/2 + 2$ **43.** $1; \infty$ **45.** $2/3$
47. $-1/3; 2/7$ **49.** 5 **51.** $-\infty$
53. **a.**

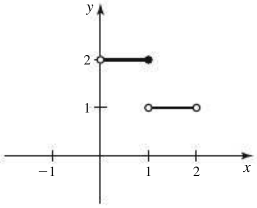


b.
$$\lim_{x\to 0} \cos x \le \lim_{x\to 0} \frac{\sin x}{x} \le \lim_{x\to 0} \frac{1}{\cos x};$$
$$1 \le \lim_{x\to 0} \frac{\sin x}{x} \le 1;$$
$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

55. $\lim_{x\to\infty} f(x) = -4$; $\lim_{x\to -\infty} f(x) = -4$
57. $\lim_{x\to\infty} f(x) = 1$; $\lim_{x\to -\infty} f(x) = -\infty$
59. $\lim_{x\to\infty} f(x) = 2$; $\lim_{x\to -\infty} f(x) = 5$ **61.** **a.** $\infty; -\infty$
b. $y = 3x + 2$ is the slant asymptote.
63. **a.** $-\infty; \infty$ **b.** $y = -x - 2$ is the slant asymptote.
65. **a.** $\infty, -\infty$ **b.** $y = 4x + 5$ is the slant asymptote.
67. Horizontal asymptotes at $y = 2/\pi$ and $y = -2/\pi$; vertical asymptote at $x = 0$
69.



71. No; $f(5)$ does not exist. **73.** Yes; $h(5) = \lim_{x\to 5} h(x) = 4$
75. $(-\infty, -\sqrt{5}]$ and $[\sqrt{5}, \infty)$; left-continuous at $-\sqrt{5}$ and right-continuous at $\sqrt{5}$ **77.** $(-\infty, -5)$, $(-5, 0)$, $(0, 5)$, and $(5, \infty)$
79. $a = 3, b = 0$
81.

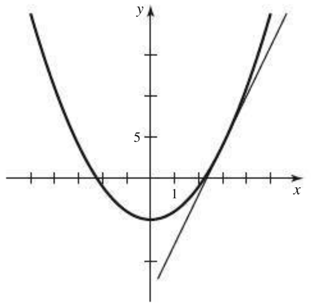


83. a. Let $f(x) = x - \cos x$; $f(0) < 0 < f\left(\frac{\pi}{2}\right)$ **b.** $x \approx 0.739$
85. a. $m(0) < 30 < m(5)$ and $m(5) > 30 > m(15)$
b. $m = 30$ when $t \approx 2.4$ hr and $t \approx 10.8$ hr **c.** No; the maximum amount is approximately $m(5.5) \approx 38.5$. **87.** $\delta = \varepsilon$
89. $\delta = \min\left\{1, \frac{\varepsilon}{15}\right\}$ **91.** $\delta = 1/\sqrt[4]{N}$

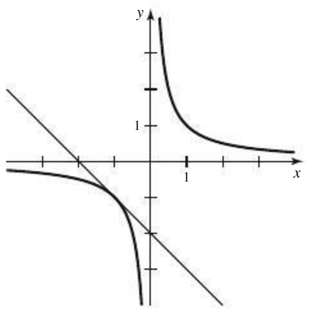
CHAPTER 3

Section 3.1 Exercises, pp. 137–140

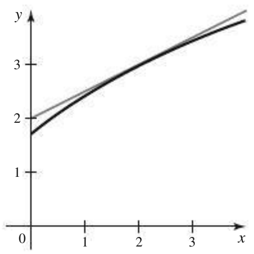
1. Given the point $(a, f(a))$ and any point $(x, f(x))$ near $(a, f(a))$, the slope of the secant line joining these points is $\frac{f(x) - f(a)}{x - a}$. The limit of this quotient as x approaches a is the slope of the tangent line at the point. **3.** The average rate of change over the interval $[a, x]$ is $\frac{f(x) - f(a)}{x - a}$. The value of $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ is the slope of the tangent line; it is also the limit of average rates of change, which is the instantaneous rate of change at $x = a$. **5.** $f'(a)$ is the slope of the tangent line at $(a, f(a))$ or the instantaneous rate of change in f at a.
7. $f(2) = 7$; $f'(2) = 4$ **9.** $y = 3x - 1$ **11.** -5 **13.** 68 ft/s
15. a. 6 **b.** $y = 6x - 14$
c.



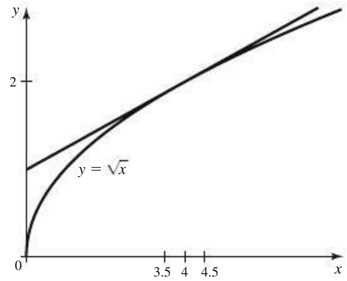
17. a. -1 **b.** $y = -x - 2$
c.



19. a. $\frac{1}{2}$ **b.** $y = \frac{1}{2}x + 2$
c.



21. a. 2 **b.** $y = 2x + 1$ **23. a.** 2 **b.** $y = 2x - 3$
25. a. 4 **b.** $y = 4x - 8$ **27. a.** 3 **b.** $y = 3x - 2$
29. a. $\frac{2}{25}$ **b.** $y = \frac{2}{25}x + \frac{7}{25}$ **31. a.** $\frac{1}{4}$ **b.** $y = \frac{1}{4}x + \frac{7}{4}$
33. a. 8 **b.** $y = 8x$ **35. a.** -14 **b.** $y = -14x - 16$
37. a. -4 **b.** $y = -4x + 3$ **39. a.** $\frac{1}{3}$ **b.** $y = \frac{1}{3}x + \frac{5}{3}$
41. a. $-\frac{1}{100}$ **b.** $y = -\frac{x}{100} + \frac{3}{20}$ **43.** $-\frac{1}{4}$ **45.** $\frac{1}{5}$ **47. a.** True **b.** False **c.** True **49.** $d'(4) = 128$ ft/s; the object falls with an instantaneous speed of 128 ft/s four seconds after being dropped.
51. $v'(3) = -4$ m/s per second; the instantaneous rate of change in the car's speed is -4 m/s^2 at $t = 3$ s.
53. a. $L'(1.5) \approx 4.3$ mm/week; the talon is growing at a rate of approximately 4.3 mm/week at $t = 1.5$ weeks (answers will vary). **b.** $L'(a) \approx 0$, for $a \geq 4$; the talon stops growing at $t = 4$ weeks. **55.** $D'(60) \approx 0.05$ hr/day; the number of daylight hours is increasing at about 0.05 hr/day, 60 days after Jan 1. $D'(170) \approx 0$ hr/day; the number of daylight hours is neither increasing nor decreasing 170 days after Jan 1. **57.** $f(x) = 5x^2$; $a = 2$; 20
59. $f(x) = x^4$; $a = 2$; 32 **61.** $f(x) = |x|$; $a = -1$; -1
63. a.



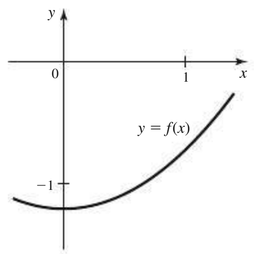
b.

h	Approximation	Error
0.1	0.25002	2.0×10^{-5}
0.01	0.25000	2.0×10^{-7}
0.001	0.25000	2.0×10^{-9}

c. Values of x on both sides of 4 are used in the formula.
d. The centered difference approximations are more accurate than the forward and backward difference approximations. **65. a.** 0.39470, 0.41545 **b.** 0.02, 0.0003

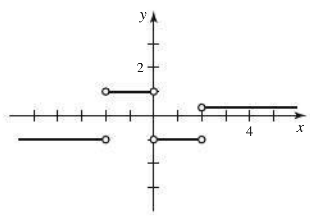
Section 3.2 Exercises, pp. 148–152

1. f' is the slope function of f. **3.** $\frac{dy}{dx}$ is the limit of $\frac{\Delta y}{\Delta x}$ as $\Delta x \to 0$.
5.



7. Yes

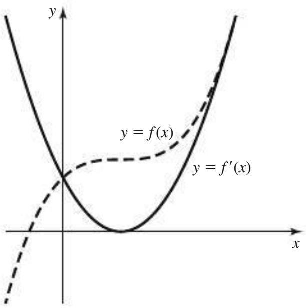
9. A line with a y-intercept of 1 and a slope of 3
11. $f'(x) = 7$ **13.** $\frac{dy}{dx} = 2x$; $\left.\frac{dy}{dx}\right|_{x=3} = 6$; $\left.\frac{dy}{dx}\right|_{x=-2} = -4$
15. a–C; b–C; c–A; d–B
17.



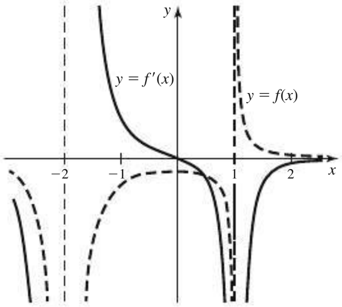
19. a. Not continuous at $x = 1$ **b.** Not differentiable at $x = 0, 1$
21. a. $f'(x) = 5$ **b.** $f'(1) = 5$; $f'(2) = 5$
23. a. $f'(x) = 8x$ **b.** $f'(2) = 16$; $f'(4) = 32$
25. a. $f'(x) = -\frac{1}{(x + 1)^2}$ **b.** $f'\left(-\frac{1}{2}\right) = -4$; $f'(5) = -\frac{1}{36}$
27. a. $f'(t) = -\frac{1}{2t^{3/2}}$ **b.** $f'(9) = -\frac{1}{54}$; $f'\left(\frac{1}{4}\right) = -4$
29. a. $f'(s) = 12s^2 + 3$ **b.** $f'(-3) = 111$; $f'(-1) = 15$
31. a. $v(t) = -32t + 100$ **b.** $v(1) = 68$ ft/s; $v(2) = 36$ ft/s
33. $\frac{dy}{dx} = \frac{1}{(x + 2)^2}$; $\left.\frac{dy}{dx}\right|_{x=2} = \frac{1}{16}$ **35. a.** $6x + 2$
b. $y = 8x - 13$ **37. a.** $\frac{3}{2\sqrt{3x + 1}}$ **b.** $y = 3x/10 + 13/5$

39. a. $\dfrac{6}{(3x+1)^2}$ **b.** $y = -3x/2 - 5/2$
41. a. Approximately 10 kW; approximately −5 kW
b. $t = 6, 18$ **c.** $t = 12$ **43. a.** $2ax + b$ **b.** $8x - 3$ **c.** 5
45. a. C, D **b.** A, B, E **c.** A, B, E, D, C **47.** a–D; b–C; c–B; d–A

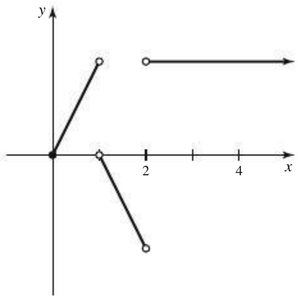
49.



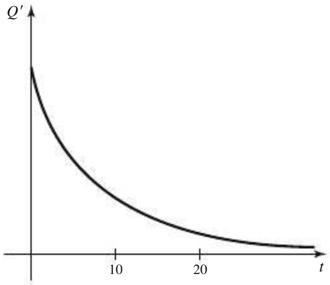
51.



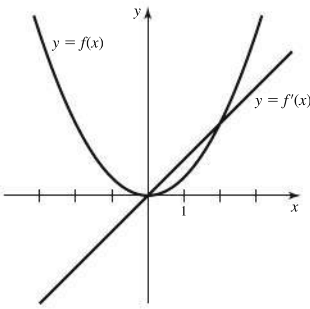
53. a. $x = 1$ **b.** $x = 1, x = 2$ **c.**



55. a. $t = 0$ **b.** Positive **c.** Decreasing
d.



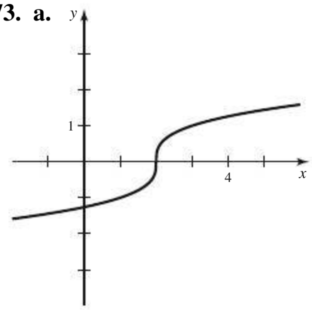
57. a. True **b.** True **c.** False **59.** $a = 4$
61. Yes



63. $y = -\dfrac{x}{3} - \dfrac{2}{3}$ **65.** $y = \dfrac{x}{2} + \dfrac{3}{2}$ **67.** $(1, 2), (5, 26)$

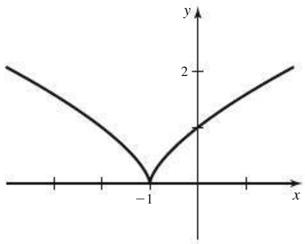
69. $(1, 1), \left(-\dfrac{1}{2}, -2\right)$ **71. b.** $f'_+(2) = 1; f'_-(2) = -1$
c. f is continuous but not differentiable at $x = 2$.

73. a.



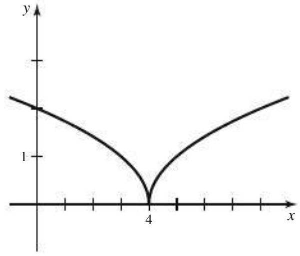
Vertical tangent line $x = 2$

b.



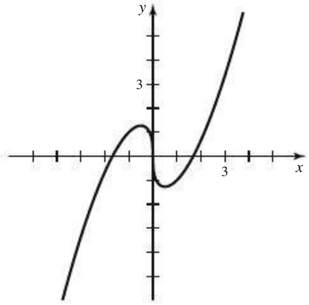
Vertical tangent line $x = -1$

c.



Vertical tangent line $x = 4$

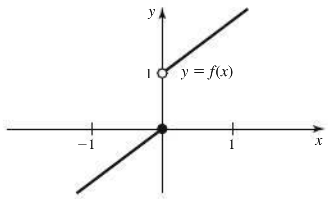
d.



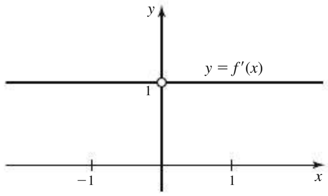
Vertical tangent line $x = 0$

75. $f'(x) = \dfrac{1}{3}x^{-2/3}$ and $\lim_{x\to 0^-} |f'(x)| = \lim_{x\to 0^+} |f'(x)| = \infty$

77. a.



b. 1 **c.** 1 **d.**



e. f is not differentiable at 0 because it is not continuous at 0.

Section 3.3 Exercises, pp. 159–162

1. Using the definition can be tedious. **3.** $f(x) = e^x$ **5.** Take the product of the constant and the derivative of the function. **7.** 4

9. $-\dfrac{1}{2}$ **11.** −2 **13.** 7.5 **15.** $10t^9; 90t^8; 720t^7$ **17.** $\dfrac{2}{5}$ **19.** $5x^4$
21. 0 **23.** $15x^2$ **25.** t **27.** 8 **29.** $200t$ **31.** $12x^3 + 7$
33. $40x^3 - 32$ **35.** $6w^2 + 6w + 10$ **37.** $3e^x + 5$

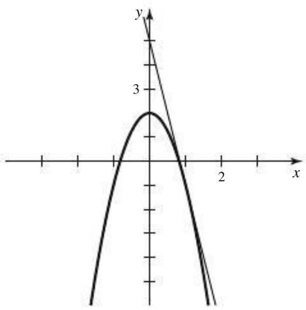
39. $\begin{cases} 2x & \text{if } x < 0 \\ 4x + 1 & \text{if } x > 0 \end{cases}$ **41. a.** $d'(t) = 32t$; ft/s; the velocity of the stone **b.** 576 ft; approx. 131 mi/hr **43. a.** $A'(t) = -\frac{1}{25}t + 2$ measures the rate at which the city grows in mi^2/yr. **b.** 1.6 mi^2/yr
c. 1200 people/yr

45. $w'(x) = \begin{cases} 0.4 & \text{if } 19 < x < 21 \\ 0.8 & \text{if } 21 < x < 32 \\ 1.5 & \text{if } x > 32 \end{cases}$ **47.** $18x^2 + 6x + 4$

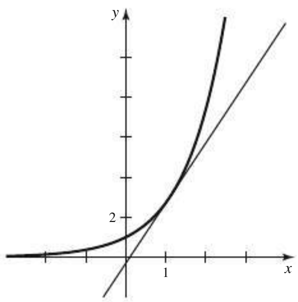
49. $2w$, for $w \neq 0$ **51.** $4x^3 + 4x$ **53.** 1, for $x \neq 1$

55. $\dfrac{1}{2\sqrt{x}}$, for $x \neq a$ **57.** e^w

59. a. $y = -6x + 5$ **b.**



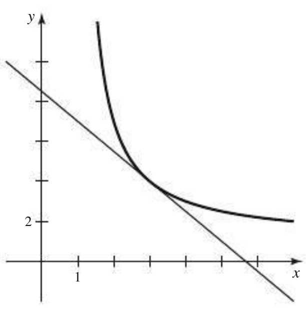
61. a. $y = 3x + 3 - 3 \ln 3$ **b.**



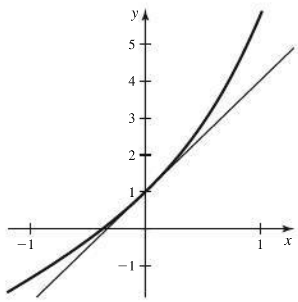
63. a. $x = 3$ **b.** $x = 4$
65. a. $(-1, 11), (2, -16)$ **b.** $(-3, -41), (4, 36)$
67. a. $(4, 4)$ **b.** $(16, 0)$ **69.** $f'(x) = 20x^3 + 30x^2 + 3$; $f''(x) = 60x^2 + 60x$; $f'''(x) = 120x + 60$
71. $f'(x) = 1$; $f''(x) = f'''(x) = 0$, for $x \neq -1$
73. a. False **b.** True **c.** False **d.** False **e.** False
75. a. $y = 7x - 1$ **b.** $y = -2x + 5$ **c.** $y = 16x + 4$
77. $b = 2, c = 3$ **79.** -10 **81.** 4 **83. a.** $f(x) = x + e^x; a = 0$ **b.** 2 **85. a.** $f(x) = \sqrt{x}; a = 9$ **b.** $\frac{1}{6}$ **87. a.** $f(x) = e^x; a = 3$ **b.** e^3 **89.** 3 **91.** 1 **95. d.** $\frac{n}{2} x^{n/2 - 1}$ **97. c.** $2e^{2x}$

Section 3.4 Exercises, pp. 168–170

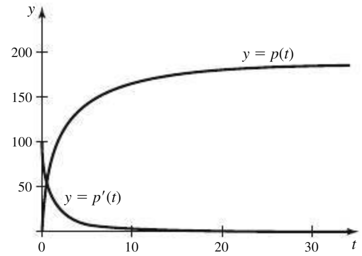
1. $\frac{d}{dx}(f(x)\, g(x)) = f'(x)\, g(x) + f(x)\, g'(x)$ **3.** $6x + 5$
5. $\frac{5}{(3x + 2)^2}$ **7. a.** $2x - 1$ **9. a.** $6x + 1$ **11. a.** $2w$, for $w \neq 0$
13. 1, for $x \neq a$ **15.** $23; \frac{7}{4}$ **17.** $\frac{2}{27}; \frac{3}{8}$ **19.** $36x^5 - 12x^3$
21. $\frac{1}{(x + 1)^2}$ **23.** $e^t t^{2/3}\left(t + \frac{5}{3}\right)$ **25.** $\frac{e^x}{(e^x + 1)^2}$ **27.** $e^{-x}(1 - x)$
29. $-\frac{1}{(t - 1)^2}$ **31.** $4x^3$ **33.** $e^w(w^3 + 3w^2 - 1)$ **35.** $t^2 e^t$
37. $\frac{e^x(x^2 - 2x - 1)}{(x^2 - 1)^2}$ **39.** $-27x^{-10}$ **41.** $6t - 42/t^8$
43. $-3/t^2 - 2/t^3$ **45.** $\frac{e^x(x^2 - x - 5)}{(x - 2)^2}$
47. $\frac{e^x(x^2 + x + 1)}{(x + 1)^2}$ **49.** $\frac{\sqrt{w}}{(\sqrt{w} - w)^2}$ **51.** $\frac{5w^{2/3}}{3(w^{5/3} + 1)^2}$
53. $8x - \frac{2}{(5x + 1)^2}$ **55.** $\frac{r - 6\sqrt{r} - 1}{2\sqrt{r}(r + 1)^2}$
57. $300x^9 + 135x^8 + 105x^6 + 120x^3 + 45x^2 + 15$ **59.** $e^x + 8x$
61. a. $y = -3x/2 + 17/2$ **b.**



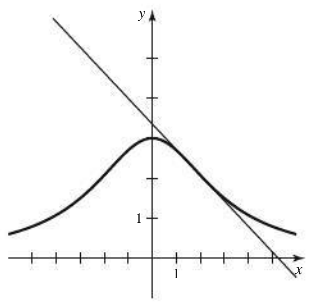
63. a. $y = 3x + 1$ **b.**



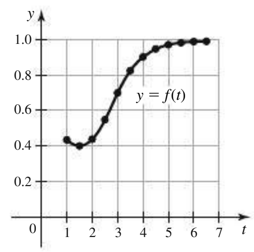
65. a. $p'(t) = \left(\frac{20}{t + 2}\right)^2$ **b.** $p'(5) \approx 8.16$ **c.** $t = 0$
d. $\lim_{t \to \infty} p(t) = 200$; the population approaches a steady state.
e.



67. a. $F'(x) = -\frac{1.8 \times 10^{10}\, Qq}{x^3}$ N/m **b.** -1.8×10^{19} N/m
c. $|F'(x)|$ decreases as x increases. **69. a.** False **b.** False **c.** False **d.** False **71.** $4x - \frac{1}{x^2}; 2\left(\frac{1}{x^3} + 2\right); -\frac{6}{x^4}$
73. $\frac{x^2 + 2x - 7}{(x + 1)^2}; \frac{16}{(x + 1)^3}$
75. a. $y = -\frac{108}{169}x + \frac{567}{169}$ **b.**



77. $-\frac{3}{2}$ **79.** $\frac{1}{9}$ **81.** $\frac{7}{8}$
83. a.

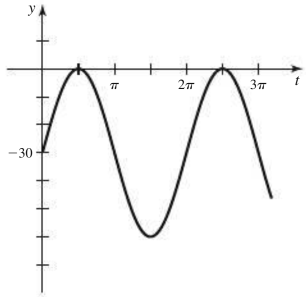


b. $t \approx 3$
c. $f'(3) \approx 0.28 \frac{\text{mm/g}}{\text{week}}$; at a young age, the bird's wings are growing quickly relative to its weight.
d. $f'(6.5) \approx 0.003 \frac{\text{mm/g}}{\text{week}}$; the rate of change of the ratio of wing chord length to mass is nearly 0. **85.** $\frac{15}{2}$ **87.** $-\frac{5}{2}$ **89.** 1
91. a. $y = -2x + 16$ **b.** $y = -\frac{5}{9}x + \frac{23}{9}$
93. -90 **97.** $f''g + 2f'g' + fg''$ **99. a.** $f'gh + fg'h + fgh'$ **b.** $e^x(x^2 + 4x - 1)$

Section 3.5 Exercises, pp. 175–178

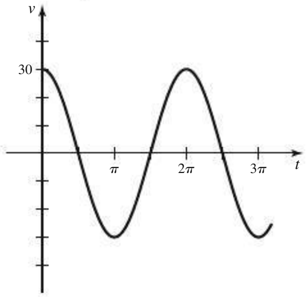
1. $\frac{\sin x}{x}$ is undefined at $x = 0$. **3.** The tangent and cotangent functions are defined as ratios of the sine and cosine functions. **5.** -1 **7.** $y = x$ **9.** $-\sin x - \cos x$ **11.** 3 **13.** $\frac{7}{3}$ **15.** 5 **17.** 7 **19.** $\frac{1}{4}$ **21.** a/b **23.** $\cos x - \sin x$ **25.** $e^{-x}(\cos x - \sin x)$ **27.** $\sin x + x\cos x$ **29.** $-\frac{1}{1 + \sin x}$ **31.** $\cos^2 x - \sin^2 x = \cos 2x$ **33.** $-2\sin x\cos x = -\sin 2x$ **35.** $w^2\cos w$ **37.** $x\cos 2x + \frac{1}{2}\sin 2x$ **39.** $\frac{1}{1 + \cos x}$ **41.** $\frac{2\sin x}{(1 + \cos x)^2}$ **43.** $\sec x\tan x - \csc x\cot x$ **45.** $e^x\csc x(1 - \cot x)$ **47.** $-\frac{\csc x}{1 + \csc x}$ **49.** $\cos^2 z - \sin^2 z = \cos 2z$ **51.** $2\sin^2 x$

55. a.

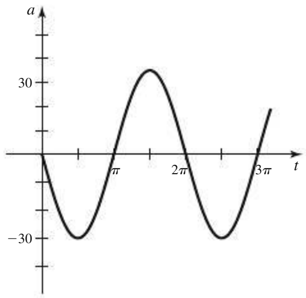


b. $v(t) = 30\cos t$

c.

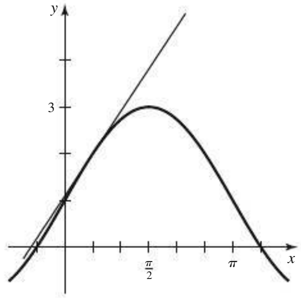


d. $v(t) = 0$, for $t = (2k + 1)\frac{\pi}{2}$, where k is any nonnegative integer; the position is $y\left((2k + 1)\frac{\pi}{2}\right) = 0$ if k is even or $y\left((2k + 1)\frac{\pi}{2}\right) = -60$ if k is odd. **e.** $v(t)$ has a maximum at $t = 2k\pi$, where k is a nonnegative integer; the position is $y(2k\pi) = -30$. **f.** $a(t) = -30\sin t$

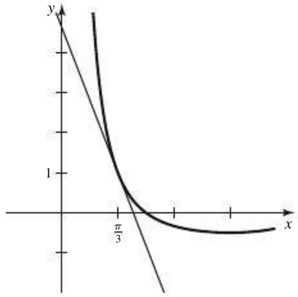


57. $2\cos x - x\sin x$ **59.** $2e^x\cos x$ **61.** $2\csc^2 x\cot x$ **63.** $2(\sec^2 x\tan x + \csc^2 x\cot x)$ **65. a.** False **b.** False **c.** True **d.** True **67.** 2 **69.** $-\frac{1}{2}$ **71.** $\frac{4}{3}$

73. a. $y = \sqrt{3}x + 2 - \frac{\pi\sqrt{3}}{6}$ **b.**

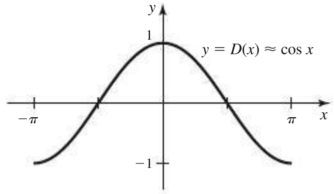


75. a. $y = -2\sqrt{3}x + \frac{2\sqrt{3}\pi}{3} + 1$ **b.**



77. $x = 7\pi/6 + 2k\pi$ and $x = 11\pi/6 + 2k\pi$, where k is an integer **85.** $a = 0$ **87. a.** $2\sin x\cos x$ **b.** $3\sin^2 x\cos x$ **c.** $4\sin^3 x\cos x$ **d.** $n\sin^{n-1} x\cos x$; the conjecture is true for $n = 1$. If it holds for $n = k$, then when $n = k + 1$, we have $\frac{d}{dx}(\sin^{k+1} x) = \frac{d}{dx}(\sin^k x \cdot \sin x) = \sin^k x\cos x + \sin x \cdot k\sin^{k-1} x\cos x = (k + 1)\sin^k x\cos x$.

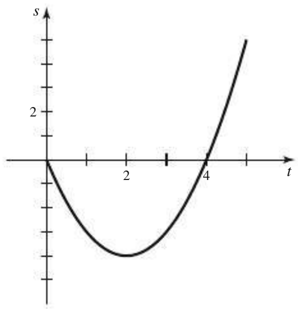
89. Because D is a difference quotient for f (and $h = 0.01$ is small), D is a good approximation to f'. Therefore, the graph of D is nearly indistinguishable from the graph of $f'(x) = \cos x$.



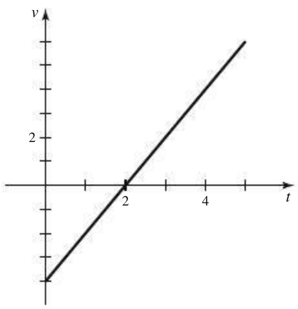
Section 3.6 Exercises, pp. 186–191

1. The average rate of change is $\frac{f(x + \Delta x) - f(x)}{\Delta x}$, whereas the instantaneous rate of change is the limit as Δx goes to zero in this quotient. **3.** Small **5.** At 15 weeks, the puppy grows at a rate of 1.75 lb/week. **7.** If the position of the object at time t is $s(t)$, then the acceleration at time t is $a(t) = d^2s/dt^2$. **9.** $v'(T) = 0.6$; the speed of sound increases by approximately 0.6 m/s for each increase of 1°C. **11. a.** 40 mi/hr **b.** 40 mi/hr; yes **c.** -60 mi/hr; -60 mi/hr; south **d.** The police car drives away from the police station going north until about 10:08, when it turns around and heads south, toward the police station. It continues south until it passes the police station at about 11:02 and keeps going south until about 11:40, when it turns around and heads north. **13.** The first 200 stoves cost, on average, $70 to produce. When 200 stoves have already been produced, the 201st stove costs $65 to produce.

15. a.

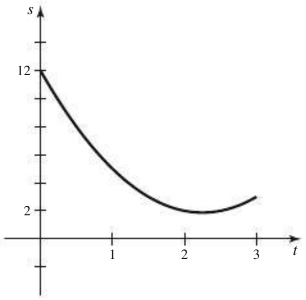


b. $v(t) = 2t - 4$; stationary at $t = 2$, to the right on $(2, 5]$, to the left on $[0, 2)$

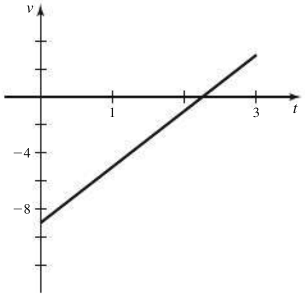


c. $v(1) = -2$ ft/s; $a(1) = 2$ ft/s^2 **d.** $a(2) = 2$ ft/s^2 **e.** $(2, 5]$

17. a.

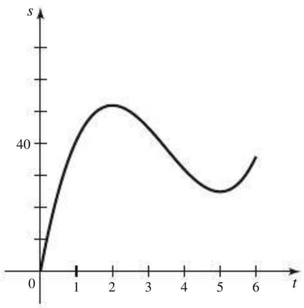


b. $v(t) = 4t - 9$; stationary at $t = \frac{9}{4}$, to the right on $\left(\frac{9}{4}, 3\right]$, to the left on $\left[0, \frac{9}{4}\right)$

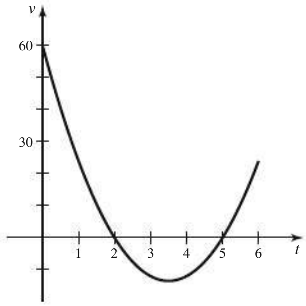


c. $v(1) = -5$ ft/s; $a(1) = 4$ ft/s^2
d. $a\left(\frac{9}{4}\right) = 4$ ft/s^2 **e.** $\left(\frac{9}{4}, 3\right]$

19. a.

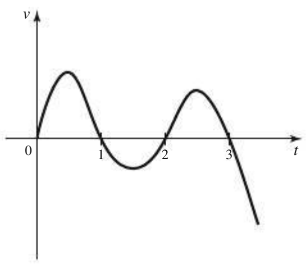


b. $v(t) = 6t^2 - 42t + 60$; stationary at $t = 2$ and $t = 5$, to the right on $[0, 2)$ and $(5, 6]$, to the left on $(2, 5)$



c. $v(1) = 24$ ft/s; $a(1) = -30$ ft/s^2 **d.** $a(2) = -18$ ft/s^2; $a(5) = 18$ ft/s^2 **e.** $\left(2, \frac{7}{2}\right), (5, 6]$ **21.** -64 ft/s; 64 ft/s
23. a. $v(t) = -32t + 32$ **b.** At $t = 1$ s **c.** 64 ft **d.** At $t = 3$ s **e.** -64 ft/s **f.** $(1, 3)$ **25. a.** $v(t) = -32t + 64$ **b.** At $t = 2$ **c.** 96 ft **d.** At $2 + \sqrt{6}$ **e.** $-32\sqrt{6}$ ft/s **f.** $(2, 2 + \sqrt{6})$
27. Approx. 90.5 ft/s **29. a.** $\overline{C}(x) = \dfrac{1000}{x} + 0.1$; $C'(x) = 0.1$
b. $\overline{C}(2000) = \$0.60/\text{item}$; $C'(2000) = \$0.10/\text{item}$
c. The average cost per item when 2000 items are produced is \$0.60/item. The cost of producing the 2001st item is \$0.10.
31. a. $\overline{C}(x) = -0.01x + 40 + 100/x$; $C'(x) = -0.02x + 40$
b. $\overline{C}(1000) = \$30.10/\text{item}$; $C'(1000) = \$20/\text{item}$
c. The average cost per item is about \$30.10 when 1000 items are produced. The cost of producing the 1001st item is \$20. **33. a.** 20 **b.** \$20 **c.** $E(p) = \frac{p}{p - 20}$ **d.** Elastic for $p > 10$; inelastic for $0 < p < 10$ **e.** 2.5% **f.** 2.5% **35. a.** False **b.** True **c.** False **d.** True **37.** 240 ft **39.** 64 ft/s **41. a.** $t = 1, 2, 3$ **b.** It is moving in the positive direction for t in $(0, 1)$ and $(2, 3)$; it is moving in the negative direction for t in $(1, 2)$ and $t > 3$.

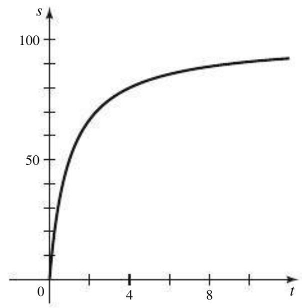
c.



d. $\left(0, \frac{1}{2}\right), \left(1, \frac{3}{2}\right), \left(2, \frac{5}{2}\right), (3, \infty)$

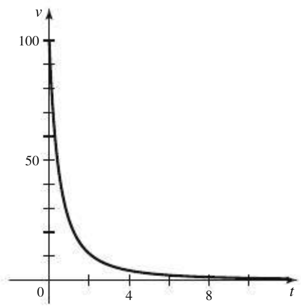
43. a. $P(x) = 0.02x^2 + 50x - 100$
b. $\dfrac{P(x)}{x} = 0.02x + 50 - \dfrac{100}{x}$; $\dfrac{dP}{dx} = 0.04x + 50$
c. $\dfrac{P(500)}{500} = 59.8$; $\dfrac{dp}{dx}(500) = 70$
d. The profit, on average, for each of the first 500 items produced is 59.8; the profit for the 501st item produced is 70.
45. a. $P(x) = 0.04x^2 + 100x - 800$
b. $\dfrac{P(x)}{x} = 0.04x + 100 - \dfrac{800}{x}$; $\dfrac{dp}{dx} = 0.08x + 100$
c. $\dfrac{P(1000)}{1000} = 139.2$; $p'(1000) = 180$
d. The average profit per item for each of the first 1000 items produced is \$139.20. The profit for the 1001st item produced is \$180.
47. About 1935; approximately 890,000 people/yr (answers will vary)

49. a.



b. $v = \dfrac{100}{(t + 1)^2}$

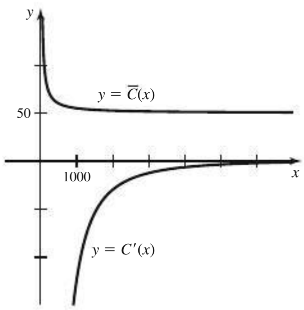
c.



The marble moves fastest at the beginning and slows considerably over the first 5 s. It continues to slow but never actually stops.
d. $t = 4$ s **e.** $t = -1 + \sqrt{2} \approx 0.414$ s

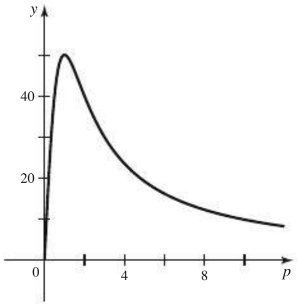
51. a. $C'(x) = -\dfrac{125{,}000{,}000}{x^2} + 1.5$;

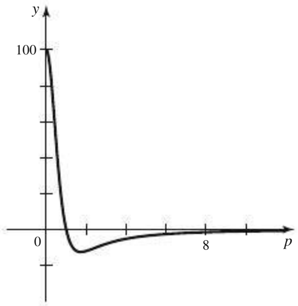
$$\overline{C}(x) = \frac{C(x)}{25{,}000} = 50 + \frac{5000}{x} + 0.00006x$$



b. $C'(5000) = -3.5$; $\overline{C}(5000) = 51.3$ **c.** Marginal cost: If the batch size is increased from 5000 to 5001, then the cost of producing 25,000 gadgets will *decrease* by about \$3.50. Average cost: When batch size is 5000, it costs \$51.30 *per item* to produce all 25,000 gadgets.

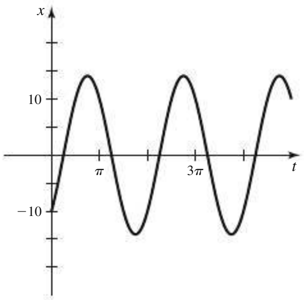
53. a. $R(p) = \dfrac{100p}{p^2 + 1}$ **b.** $R'(p) = \dfrac{100(1 - p^2)}{(p^2 + 1)^2}$





c. $p = 1$

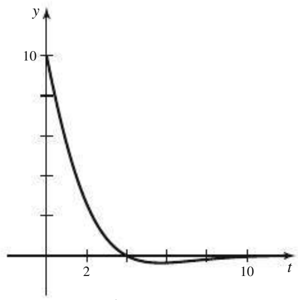
55. a.



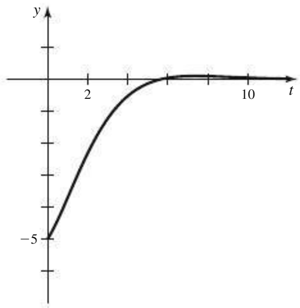
b. $dx/dt = 10 \cos t + 10 \sin t$
c. $t = 3\pi/4 + k\pi$, where k is any positive integer
d. The graph implies that the spring never stops oscillating. In reality, the weight would eventually come to rest.
57. a. Juan starts faster than Jean and opens up a big lead. Then Juan slows down while Jean speeds up. Jean catches up, and the race finishes in a tie. **b.** Same average velocity **c.** Tie **d.** At $t = 2$, $\theta'(2) = \pi/2$ rad/min; $\theta'(4) = \pi$ = Jean's greatest velocity
e. At $t = 2$, $\varphi'(2) = \pi/2$ rad/min; $\varphi'(0) = \pi$ = Juan's greatest velocity **59. a.** $v(t) = -15e^{-t}(\sin t + \cos t)$; $v(1) \approx -7.6$ m/s, $v(3) \approx 0.63$ m/s **b.** Down (0, 2.4) and (5.5, 8.6); up (2.4, 5.5) and (8.6, 10) **c.** ≈ 0.65 m/s **61. a.** $-T'(1) = -80$, $-T'(3) = 80$
b. $-T'(x) < 0$ for $0 \le x < 2$; $-T'(x) > 0$ for $2 < x \le 4$ **c.** Near $x = 0$, with $x > 0$, $-T'(x) < 0$, so heat flows toward the end of the rod. Similarly, near $x = 4$, with $x < 4$, $-T'(x) > 0$.

Section 3.7 Exercises, pp. 196–200

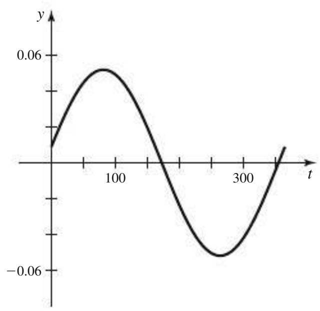
1. $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$; $\dfrac{d}{dx}(f(g(x)) = f'(g(x)) \cdot g'(x)$
3. $u = x^3 + x + 1$; $y = u^4$; $4(x^3 + x + 1)^3(3x^2 + 1)$
5. $u = \cos x$, $y = u^3$, $dy/dx = -3 \cos^2 x \sin x$; $u = x^3$, $y = \cos u$, $dy/dx = -3x^2 \sin x^3$ **7.** $g(x)$, x **9.** $\dfrac{2}{\sqrt{4x + 1}}$
11. 50 **13.** ke^{kx} **15.** $u = 3x + 7$; $f(u) = u^{10}$; $30(3x + 7)^9$
17. $u = \sin x$; $f(u) = u^5$; $5 \sin^4 x \cos x$
19. $u = x^2 + 1$; $f(u) = \sqrt{u}$; $\dfrac{x}{\sqrt{x^2 + 1}}$
21. $u = 4x^2 + 1$; $f(u) = e^u$; $8xe^{4x^2+1}$
23. $u = 5x^2$; $f(u) = \tan u$; $10x \sec^2 5x^2$ **25. a.** 100 **b.** −100 **c.** −16 **d.** 40 **e.** 40 **27.** $10(6x + 7)(3x^2 + 7x)^9$
29. $\dfrac{5}{\sqrt{10x + 1}}$ **31.** $-\dfrac{315x^2}{(7x^3 + 1)^4}$ **33.** $3 \sec(3x + 1) \tan(3x + 1)$
35. $e^x \sec^2 e^x$ **37.** $(12x^2 + 3) \cos(4x^3 + 3x + 1)$
39. $\dfrac{10}{3(5x + 1)^{1/3}}$ **41.** $-\dfrac{3}{2^{7/4}x^{3/4}(4x - 3)^{5/4}}$
43. $5 \sec x (\sec x + \tan x)^5$ **45.** $25(12x^5 - 9x^2)(2x^6 - 3x^3 + 3)^{24}$
47. $9(1 + 2 \tan u)^{3.5} \sec^2 u$ **49.** $-\dfrac{\cot x \csc^2 x}{\sqrt{1 + \cot^2 x}}$
51. $\dfrac{2}{3}e^x - e^{-x}$ **53.** $e^x \cos(\sin e^x) \cos e^x$
55. $-15 \sin^4(\cos 3x)(\sin 3x)(\cos(\cos 3x))$
57. $\dfrac{2e^{2t}}{(1 + e^{2t})^2}$ **59.** $\dfrac{1}{2\sqrt{x + \sqrt{x}}}\left(1 + \dfrac{1}{2\sqrt{x}}\right)$
61. $f'(g(x^2))g'(x^2)\,2x$ **63.** $\dfrac{5x^4}{(x + 1)^6}$
65. $xe^{x^2+1}(2 \sin x^3 + 3x \cos x^3)$ **67.** $\theta(2 + 5\theta \tan 5\theta) \sec 5\theta$
69. $4((x + 2)(x^2 + 1))^3(3x + 1)(x + 1)$ **71.** $\dfrac{4x^3 - 2 \sin 2x}{5(x^4 + \cos 2x)^{4/5}}$
73. $2(p + 3)(\sin p^2 + p(p + 3) \cos p^2)$
75. $f'(x)/\left(2\sqrt{f(x)}\right)$ **77. a.** True **b.** True **c.** True **d.** False **79.** −0.297 hPa/min **81.** Approx. 0.33 g/day; mass is increasing by 0.33 g/day 65 days after the diet switch.
83. a. \$297.77 **b.** \$11.85/yr **c.** $y = 11.85t + 179.27$
85. a. $x = -\frac{1}{2}$ **b.** The line tangent to the graph of $f(x)$ at $x = -\frac{1}{2}$ is horizontal. **87.** $2 \cos x^2 - 4x^2 \sin x^2$ **89.** $4e^{-2x^2}(4x^2 - 1)$
91. $y = 6x - 1$ **93. a.** $h(4) = 9$, $h'(4) = -6$ **b.** $y = -6x + 33$
95. $y = 6x + 3 - 3 \ln 3$ **97. a.** -3π **b.** -5π
99. a. $\dfrac{d^2y}{dt^2} = -\dfrac{y_0 k}{m} \cos\left(t\sqrt{\dfrac{k}{m}}\right)$
101. a.



b. $v(t) = -5e^{-t/2}\left(\dfrac{\pi}{4} \sin \dfrac{\pi t}{8} + \cos \dfrac{\pi t}{8}\right)$

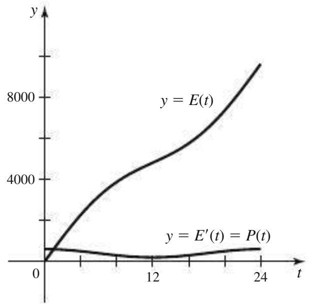


103. a. 10.88 hr **b.** $D'(t) = \dfrac{6\pi}{365} \sin\left(\dfrac{2\pi(t + 10)}{365}\right)$
c. 2.87 min/day; on March 1, the length of day is increasing at a rate of about 2.87 min/day.
d.



e. Most rapidly: approximately March 22 and September 22; least rapidly: approximately December 21 and June 21

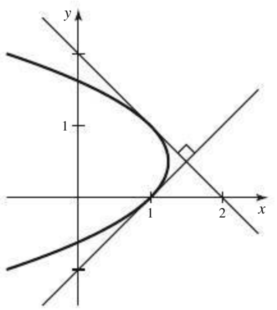
105. a. $E'(t) = 400 + 200\cos\dfrac{\pi t}{12}$ MW
b. At noon; $E'(0) = 600$ MW **c.** At midnight; $E'(12) = 200$ MW
d.



109. a. $g(x) = (x^2 - 3)^5$; $a = 2$ **b.** 20
111. a. $g(x) = \sin x^2$; $a = \pi/2$ **b.** $\pi\cos(\pi^2/4)$ **113.** $10 f'(25)$

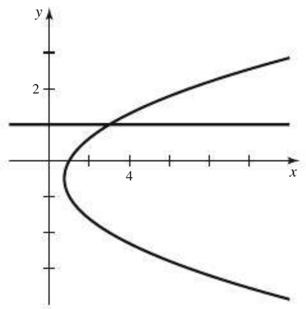
Section 3.8 Exercises, pp. 205–208

1. There may be more than one expression for y or y'.
3. When derived implicitly, dy/dx is usually given in terms of both x and y. **5.** $\dfrac{1}{2y}$ **7.** $\dfrac{1}{\cos y}$ **9. a.** $(0, 0), (0, -1), (0, 1)$
c. Slope at $(0, 0)$ is 2; slope at $(0, -1)$ and $(1, 0)$ is -1.
11. $\dfrac{d^2y}{dx^2} = -\dfrac{2}{9y^5}$ **13. a.** $-\dfrac{x^3}{y^3}$ **b.** 1 **15. a.** $\dfrac{2}{y}$ **b.** 1
17. a. $\dfrac{20x^3}{\cos y}$ **b.** -20 **19. a.** $-\dfrac{1}{\sin y}$ **b.** -1 **21. a.** $-\dfrac{y}{x}$ **b.** -7
23. a. $-\dfrac{1}{4x^{2/3}y^{1/3}}$ **b.** $-\dfrac{1}{4}$ **25. a.** $-\dfrac{3y}{x + 3y^{2/3}}$ **b.** $-\dfrac{24}{13}$
27. $\dfrac{\cos x}{1 - \cos y}$ **29.** $-\dfrac{1}{1 + \sin y}$ **31.** $\dfrac{1 - y\cos xy}{x\cos xy - 1}$ **33.** $\dfrac{1}{2y\sin y^2 + e^y}$
35. $\dfrac{3x^2(x - y)^2 + 2y}{2x}$ **37.** $\dfrac{13y - 18x^2}{21y^2 - 13x}$ **39.** $\dfrac{5\sqrt{x^4 + y^2} - 2x^3}{y - 6y^2\sqrt{x^4 + y^2}}$
41. a. $\dfrac{dK}{dL} = -\dfrac{K}{2L}$ **b.** -4 **43.** $\dfrac{dr}{dh} = \dfrac{h - 2r}{h}$; -3
45. b. $y = -5x$ **47. b.** $y = -5x/4 + 7/2$ **49. b.** $y = \dfrac{x}{2}$
51. $-\dfrac{1}{4y^3}$ **53.** $\dfrac{\sin y}{(\cos y - 1)^3}$ **55.** $\dfrac{4e^{2y}}{(1 - 2e^{2y})^3}$ **57. a.** False
b. True **c.** False **d.** False **59. a.** $\dfrac{y(3\sqrt{x} + 2y^{3/2})}{x(\sqrt{x} - 2y^{3/2})}$ **b.** -5
61. a. $y = x - 1$ and $y = -x + 2$
b.

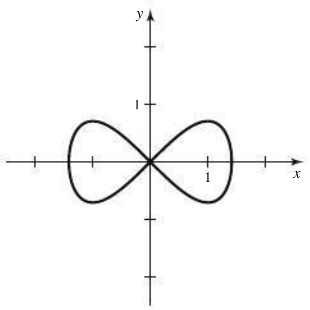


63. a. $y' = -\dfrac{2xy}{x^2 + 4}$ **b.** $y = \dfrac{1}{2}x + 2$, $y = -\dfrac{1}{2}x + 2$
c. $-\dfrac{16x}{(x^2 + 4)^2}$ **65. a.** $\left(\dfrac{5}{4}, \dfrac{1}{2}\right)$ **b.** No
67. Horizontal: $y = -6$, $y = 0$; vertical: $x = 1$, $x = 3$

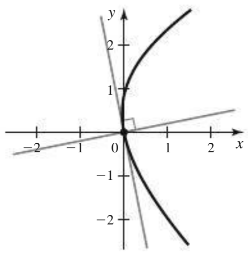
69. a. $\dfrac{dy}{dx} = 0$ on the $y = 1$ branch; $\dfrac{dy}{dx} = \dfrac{1}{2y + 1}$ on the other two branches. **b.** $f_1(x) = 1$, $f_2(x) = \dfrac{-1 + \sqrt{4x - 3}}{2}$, $f_3(x) = \dfrac{-1 - \sqrt{4x - 3}}{2}$ **c.**



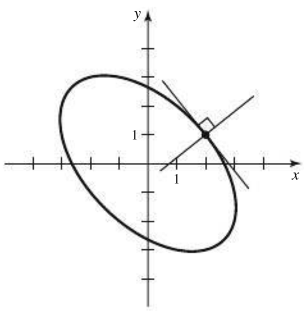
71. a. $\dfrac{dy}{dx} = \dfrac{x - x^3}{y}$ **b.** $f_1(x) = \sqrt{x^2 - \dfrac{x^4}{2}}$; $f_2(x) = -\sqrt{x^2 - \dfrac{x^4}{2}}$
c.



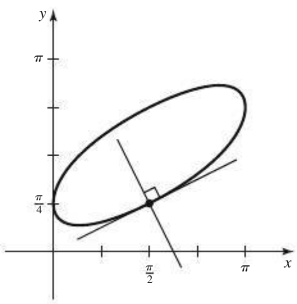
73. $y = \dfrac{x}{5}$



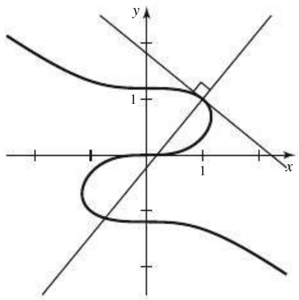
75. $y = \dfrac{4x}{5} - \dfrac{3}{5}$



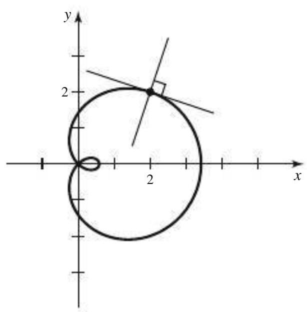
77. $y = -2x + \dfrac{5\pi}{4}$



79. a. Tangent line $y = -\dfrac{9x}{11} + \dfrac{20}{11}$; normal line $y = \dfrac{11x}{9} - \dfrac{2}{9}$
b.



81. a. Tangent line $y = -\dfrac{x}{3} + \dfrac{8}{3}$; normal line $y = 3x - 4$
b.



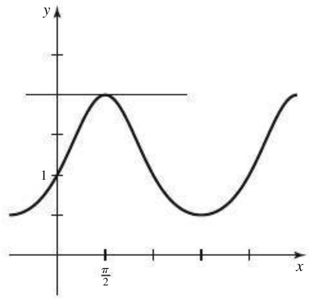
83. For $y = mx$, $dy/dx = m$; for $x^2 + y^2 = a^2$, $dy/dx = -x/y$.
85. For $xy = a$, $dy/dx = -y/x$; for $x^2 - y^2 = b$, $dy/dx = x/y$. Because $(-y/x)\cdot(x/y) = -1$, the families of curves form orthogonal trajectories. **87.** $\dfrac{7y^2 - 3x^2 - 4xy^2 - 4x^3}{2y(2x^2 + 2y^2 - 7x)}$

89. $\dfrac{2y^2(5 + 8x\sqrt{y})}{(1 + 2x\sqrt{y})^3}$ **91.** No horizontal tangent line; vertical tangent lines at $(2, 1), (-2, 1)$ **93.** No horizontal tangent line; vertical tangent lines at $(0, 0), \left(\frac{3\sqrt{3}}{2}, \sqrt{3}\right), \left(-\frac{3\sqrt{3}}{2}, -\sqrt{3}\right)$

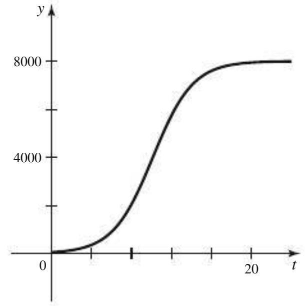
Section 3.9 Exercises, pp. 215–218

1. $x = e^y \Rightarrow 1 = e^y y'(x) \Rightarrow y'(x) = 1/e^y = 1/x$
3. $\dfrac{d}{dx}(\ln kx) = \dfrac{d}{dx}(\ln k + \ln x) = \dfrac{d}{dx}(\ln x)$ **5.** $f'(x) = \dfrac{1}{x \ln b}$; if $b = e$, then $f'(x) = \dfrac{1}{x}$. **7.** $(x^2 + 1)^x$ **9.** $\dfrac{x}{x^2 + 1}$
11. $f(x) = e^{h(x) \ln g(x)}$ **13.** $\dfrac{1 + x}{x}$ **15.** $\dfrac{1}{x}$ **17.** $2/x$ **19.** $\cot x$
21. $\dfrac{4x^3}{x^4 + 1}$ **23.** $2/(1 - x^2)$ **25.** $(x^2 + 1)/x + 2x \ln x$
27. $-2x \ln x^2$ or $-4x \ln x$ **29.** $1/(x \ln x)$ **31.** $\dfrac{1}{x(\ln x + 1)^2}$
33. ex^{e-1} **35.** $\pi(2^x + 1)^{\pi - 1} 2^x \ln 2$ **37.** $8^x \ln 8$ **39.** $5 \cdot 4^x \ln 4$
41. $2^{3 + \sin x}(\ln 2)\cos x$ **43.** $3^x \cdot x^2 (x \ln 3 + 3)$
45. $1000(1.045)^{4t} \ln 1.045$ **47.** $\dfrac{2^x \ln 2}{(2^x + 1)^2}$
49. $x^{\cos x - 1}(\cos x - x \ln x \sin x)$; $-\ln(\pi/2)$
51. $x^{\sqrt{x}}\left(\dfrac{2 + \ln x}{2\sqrt{x}}\right)$; $4(2 + \ln 4)$
53. $\dfrac{(\sin x)^{\ln x}(\ln(\sin x) + x(\ln x) \cot x)}{x}$; 0
55. $(4 \sin x + 2)^{\cos x}\left(\dfrac{2 \cos^2 x}{2 \sin x + 1} - \sin x \ln(4 \sin x + 2)\right)$; 1
57. a. Approx. 28.7 s **b.** -46.512 s/1000 ft
c. $dT/da = -2.74 \cdot 2^{-0.274a} \ln 2$
At $a = 8$, $\dfrac{dT}{da} = -0.4156$ min/1000 ft
$= -24.938$ s/1000 ft.
If a plane travels at 30,000 feet and increases its altitude by 1000 feet, the time of useful consciousness decreases by about 25 seconds.
59. $y = x \sin 1 + 1 - \sin 1$ **61.** $y = e^{2/e}$ and $y = e^{-2/e}$
63. $\dfrac{8x}{(x^2 - 1) \ln 3}$ **65.** $-\sin x (\ln(\cos^2 x) + 2)$
67. $-\dfrac{\ln 4}{x \ln^2 x}$ **69.** $\dfrac{12}{3x + 1}$ **71.** $\dfrac{1}{2x}$
73. $\dfrac{2}{2x - 1} + \dfrac{3}{x + 2} + \dfrac{8}{1 - 4x}$ **75.** $10x^{10x}(1 + \ln x)$
77. $\dfrac{(x + 1)^{10}}{(2x - 4)^8}\left(\dfrac{10}{x + 1} - \dfrac{8}{x - 2}\right)$ **79.** $2x^{\ln x - 1} \ln x$
81. $\dfrac{(x + 1)^{3/2}(x - 4)^{5/2}}{(5x + 3)^{2/3}}\left(\dfrac{3}{2(x + 1)} + \dfrac{5}{2(x - 4)} - \dfrac{10}{3(5x + 3)}\right)$
83. $(\sin x)^{\tan x}(1 + (\sec^2 x) \ln \sin x)$
85. $\left(1 + \dfrac{1}{x}\right)^x\left(\ln\left(1 + \dfrac{1}{x}\right) - \dfrac{1}{x + 1}\right)$
87. a. False **b.** False **c.** False **d.** False **e.** True **f.** True
89. $-\dfrac{1}{x^2 \ln 10}$ **91.** $\dfrac{2}{x}$ **93.** $3^x \ln 3$

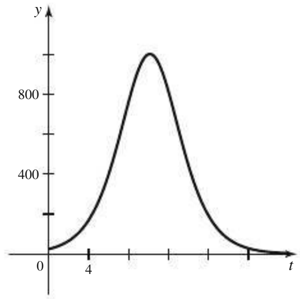
95. $y = 2$



97. a.



b. $t = 2 \ln 265 \approx 11.2$ years; approx. 14.5 years
c. $P'(0) \approx 25$ fish/year; $P'(5) \approx 264$ fish/year
d.



The population is growing fastest after about 10 years.
99. b. $r(11) \approx 0.0133$; $r(21) \approx 0.0118$; the relative growth rate is decreasing. **c.** $\lim_{t \to \infty} r(t) = 0$; as the population gets close to carrying capacity, the relative growth rate approaches zero.
101. a. $A(5) = \$17{,}443$
$A(15) = \$72{,}705$
$A(25) = \$173{,}248$
$A(35) = \$356{,}178$
\$5526.20/year, \$10,054.30/year, \$18,293/year
b. $A(40) = \$497{,}873$
c. $\dfrac{dA}{dt} = 600{,}000 \ln(1.005)((1.005)^{12t})$
$\approx (2992.5)(1.005)^{12t}$
A increases at an increasing rate.
103. $p = e^{1/e}$; (e, e) **105.** $1/e$ **107.** $27(1 + \ln 3)$

Section 3.10 Exercises, pp. 225–227

1. $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1 - x^2}}$; $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$; $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2 - 1}}$ **3.** $\dfrac{1}{5}$ **5.** $\dfrac{1}{4}$ **7. a.** $\dfrac{1}{2}$ **b.** $\dfrac{2}{3}$
c. Cannot be determined **d.** $\dfrac{3}{2}$ **9.** $y = \dfrac{1}{7}x + \dfrac{13}{7}$ **11.** $\dfrac{2}{\sqrt{3}}$
13. $\dfrac{2}{\sqrt{1 - 4x^2}}$ **15.** $-\dfrac{4w}{\sqrt{1 - 4w^2}}$ **17.** $-\dfrac{2e^{-2x}}{\sqrt{1 - e^{-4x}}}$
19. $\dfrac{10}{100x^2 + 1}$ **21.** $\dfrac{4y}{1 + (2y^2 - 4)^2}$ **23.** $-\dfrac{1}{2\sqrt{z}(1 + z)}$

25. $6x^2 \cot^{-1} x$ **27.** $\frac{2w^5}{1 + w^4}$ **29.** $\frac{1}{|x|\sqrt{x^2 - 1}}$

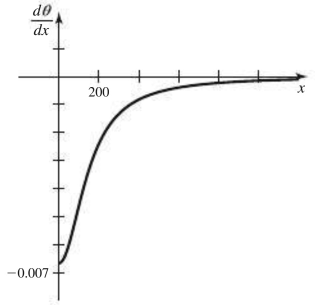
31. $-\frac{1}{|2u + 1|\sqrt{u^2 + u}}$ **33.** $\frac{2y}{(y^2 + 1)^2 + 1}$

35. $\frac{1}{x|\ln x|\sqrt{(\ln x)^2 - 1}}$ **37.** $-\frac{e^x \sec^2 e^x}{|\tan e^x|\sqrt{\tan^2 e^x - 1}}$

39. $-\frac{e^s}{1 + e^{2s}}$ **41.** $y = x + \frac{\pi}{4} - \frac{1}{2}$ **43.** $y = -\frac{4}{\sqrt{6}}x + \frac{\pi}{3} + \frac{2}{\sqrt{3}}$

45. a. Approx. -0.00055 rad/m

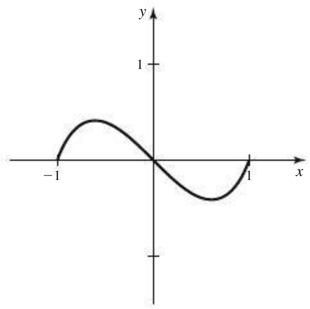
b.



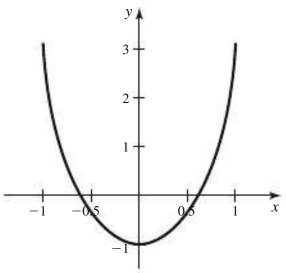
The magnitude of the change in angular size, $|d\theta/dx|$, is greatest when the boat is at the skyscraper (that is, at $x = 0$).

47. $\frac{1}{3}$ **49.** $\frac{e}{5}$ **51.** $\frac{1}{2}$ **53.** 4 **55.** $\frac{1}{12}$ **57.** $\frac{1}{4}$ **59.** $\frac{5}{4}$ **61. a.** True **b.** False **c.** True **d.** True **e.** True

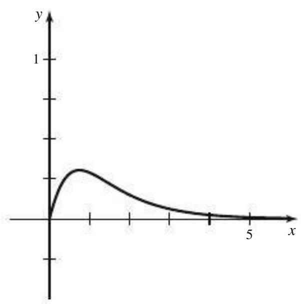
63. a.



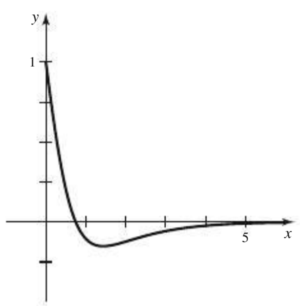
b. $f'(x) = 2x \sin^{-1} x + \frac{x^2 - 1}{\sqrt{1 - x^2}}$



65. a.



b. $f'(x) = \frac{e^{-x}}{1 + x^2} - e^{-x} \tan^{-1} x$



67. $\frac{1}{3}$ **69.** $1/(2\sqrt{x + 4})$ **71.** $\frac{1}{3x}$ **73.** $\frac{1}{12x \ln 10}$ **75.** $2x$
77. $-2/x^3$ **79. b.** -0.0041, -0.0289, and -0.1984
c. $\lim_{\ell \to 10^+} d\theta/d\ell = -\infty$ **d.** The length ℓ is decreasing.
81. a. $1/\sqrt{D^2 - c^2}$ **b.** $1/D$ **85.** Use the identity $\cot^{-1} x + \tan^{-1} x = \pi/2$.

Section 3.11 Exercises, pp. 231–236

1. As the side length s of a cube changes, the surface area $6s^2$ changes as well. **3.** The other two opposite sides decrease in length.

5. a. $V = 200h; \frac{dV}{dt} = 200\frac{dh}{dt}$ **b.** $50 \text{ ft}^3/\text{min}$
c. $\frac{1}{20}$ ft/min **7. a.** $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ **b.** $128\pi \text{ in}^3/\text{min}$
c. $\frac{1}{10\pi}$ in/min **9.** 59 **11. a.** $40 \text{ m}^2/\text{s}$ **b.** $80 \text{ m}^2/\text{s}$
13. a. $4 \text{ m}^2/\text{s}$ **b.** $\sqrt{2} \text{ m}^2/\text{s}$ **c.** $2\sqrt{2}$ m/s **15. a.** $\frac{1}{4\pi}$ cm/s
b. $\frac{1}{2}$ cm/s **17.** $-40\pi \text{ ft}^2/\text{min}$ **19.** $\frac{3}{80\pi}$ in/min
23. 720.3 mi/hr **25.** $\frac{3\sqrt{5}}{2}$ ft/s **27.** 57.89 ft/s **29.** 4.66 in/s
31. $\frac{\pi}{2} \text{ ft}^3/\text{min}$ **33.** $-75\pi \text{ cm}^3/\text{s}$ **35.** $2592\pi \text{ cm}^3/\text{s}$
37. $9\pi \text{ ft}^3/\text{min}$ **39.** $\frac{1}{25\pi}$ m/min **41.** $\frac{5}{24}$ ft/s
43. $-\frac{8}{3}$ ft/s, $-\frac{32}{3}$ ft/s **45.** $\frac{d\theta}{dt} = \frac{1}{5}$ rad/s, $\frac{d\theta}{dt} = \frac{1}{8}$ rad/s
47. -0.0201 rad/s **49.** $10 \tan 20°$ km/hr ≈ 3.6 km/hr
51. a. 187.5 ft/s **b.** 0.938 rad/s **53. a.** $P = \frac{1}{2}v^2\frac{dm}{dt}$
c. 17,388.7 W **d.** 4347.2 W **55.** 11.06 m/hr
57. $\frac{1}{500}$ m/min; 2000 min **59.** 0.543 rad/hr
61. $\frac{d\theta}{dt} = 0$ rad/s, for all $t \geq 0$ **63. a.** $-\frac{\sqrt{3}}{10}$ m/hr **b.** $-1 \text{ m}^2/\text{hr}$

Chapter 3 Review Exercises, pp. 236–240

1. a. False **b.** False **c.** False **d.** False **e.** True
3. $-\frac{2x}{(x^2 + 5)^2}$ **9.** $2x^2 + 2\pi x + 7$ **11.** $2^x \ln 2$
13. $2e^{2\theta}$ **15.** $6x^3\sqrt{1 + x^4}$ **17.** $5t^2 \cos t + 10t \sin t$
19. $-x^2e^{-x}$ **21.** $\frac{2 \sec 2w \tan 2w}{(\sec 2w + 1)^2}$ **23.** $3 \tan 3x$
25. $1000t(5t^2 + 10)^{99}$ **27.** $3x^2 \cot x^3$ **29.** $\frac{1}{t\sqrt{t^2 - 1}}$
31. $(8\theta + 12) \sec^2(\theta^2 + 3\theta + 2)$ **33.** $\frac{1 - 5 \ln w}{w^6}$
35. $\frac{32u^2 + 8u + 1}{(8u + 1)^2}$ **37.** $(\sec^2 \sin \theta) \cos \theta$
39. $-\frac{\cos\sqrt{\cos^2 x + 1} \cos x \sin x}{\sqrt{\cos^2 x + 1}}$ **41.** $\frac{e^t}{2(e^t + 1)}$
43. $2 \tan^{-1}(\cot x)$ **45.** $(2 + \ln x) \ln x$ **47.** $(2x - 1)\, 2^{x^2 - x} \ln 2$
49. $(x^2 + 1)^{\ln x}\left(\frac{\ln(x^2 + 1)}{x} + \frac{2x \ln x}{x^2 + 1}\right)$ **51.** $-\frac{1}{|x|\sqrt{x^2 - 1}}$
53. $6 \cot^{-1} 3x$ **55.** $1 + \csc(x - y)$
57. $\frac{y \cos x}{e^y - 1 - \sin x}$ **59.** $-\frac{xy}{x^2 + 2y^2}$

61. $\dfrac{(3x+5)^{10}\sqrt{x^2+5}}{(x^3+1)^{50}}\left(\dfrac{30}{3x+5}+\dfrac{x}{x^2+5}-\dfrac{150x^2}{x^3+1}\right)$

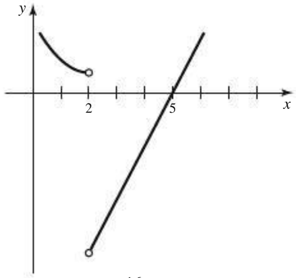
63. $\sqrt{3}+\pi/6$ **65.** 1 **67.** $2^x \ln 2(x\ln 2+2)$ **69.** $\dfrac{6\ln x-5}{x^4}$

71. $\dfrac{2(xy+y^2)}{(x+2y)^3}=\dfrac{2}{(x+2y)^3}$ **73.** $y=x$ **75.** $y=-\dfrac{4x}{5}+\dfrac{24}{5}$

77. $x^2f'(x)+2xf(x)$ **79.** $\dfrac{g(x)(xf'(x)+f(x))-xf(x)g'(x)}{g^2(x)}$

81. a–D; b–C; c–B; d–A

83.



85. a. 27 **b.** $\frac{16}{27}$ **c.** 72 **d.** 1215 **e.** $\frac{1}{9}$ **87.** $\frac{6}{13}$

89. $(f^{-1})'(x)=-3/x^4$ **91. a.** $\frac{1}{4}$ **b.** 1 **c.** $\frac{1}{3}$

93. $y=24x-118$ **95. a.** 84 ft/s **b.** 7 s **c.** 384 ft

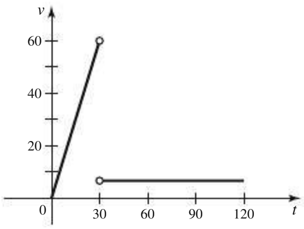
d. 96 ft/s **97. a.** \$200,366; \$21,552/yr

b. 14 yr; \$12,551/yr **99. a.** 2.70 million people/yr

b. The slope of the secant line through the two points is approximately equal to the slope of that tangent line at $t=55$.

c. 2.217 million people/yr **101. a.** 40 m/s **b.** 20/3 m/s

c. 15 m/s **d.**



e. The skydiver deployed the parachute. **103.** $x=4$; $x=6$

105. $f(x)=\tan(\pi\sqrt{3x-11})$, $a=5$; $f'(5)=3\pi/4$

107. a. $\overline{C}(3000)=\$341.67$; $C'(3000)=\$280$ **b.** The average cost of producing the first 3000 lawn mowers is \$341.67 per mower. The cost of producing the 3001st lawn mower is \$280.

109. a. 6550 people/yr **b.** $p'(40)=4800$ people/yr

111. 50 mi/hr **113.** $-5\sin 65°$ ft/s ≈ -4.5 ft/s

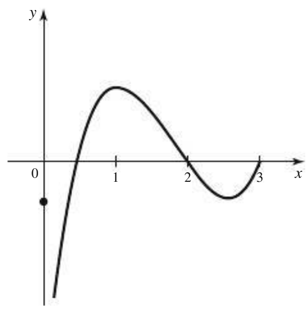
115. -0.166 rad/s **117.** 1.5 ft/s **119. a.** $(f^{-1})'(1/\sqrt{2})=\sqrt{2}$

CHAPTER 4

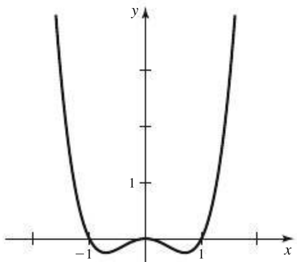
Section 4.1 Exercises, pp. 247–250

1. f has an absolute maximum at c in $[a,b]$ if $f(x)\le f(c)$ for all x in $[a,b]$. f has an absolute minimum at c in $[a,b]$ if $f(x)\ge f(c)$ for all x in $[a,b]$. **3.** The function must be continuous on a closed interval.

5.

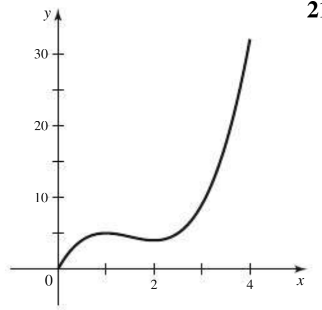


7.

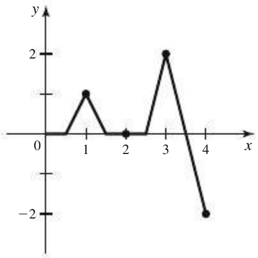


9. Evaluate the function at the critical points and at the endpoints of the interval. **11.** Abs. min at $x=c_2$; abs. max at $x=b$ **13.** Abs. min at $x=a$; no abs. max **15.** Local min at $x=q,s$; local max at $x=p,r$; abs. min at $x=a$; abs. max at $x=b$ **17.** Local max at $x=p,r$; local min at $x=q$; abs. max at $x=p$; abs. min at $x=b$

19.



21.



23. $x=\frac{2}{3}$ **25.** $x=\pm 3$ **27.** $x=-\frac{2}{3},\frac{1}{3}$ **29.** $x=\pm\dfrac{2a}{\sqrt{3}}$

31. $t=\pm 1$ **33.** $x=0$ **35.** $x=1$ **37.** $x=-4,0$

39. If $a\ge 0$, there is no critical point. If $a<0$, $x=2a/3$ is the only critical point. **41.** $t=\pm a$ **43.** Abs. max: -1 at $x=3$; abs. min: -10 at $x=0$ **45.** Abs. max: 0 at $x=0,3$; abs. min: -4 at $x=-1,2$ **47.** Abs. max: 234 at $x=3$; abs. min: -38 at $x=-1$ **49.** Abs. max: 1 at $x=0,\pi$; abs. min: 0 at $x=\pi/2$ **51.** Abs. max: 1 at $x=\pi/6$; abs. min: -1 at $x=-\pi/6$

53. Abs. min: $(\sqrt{1/e})^{1/e}$ at $x=1/(2e)$; abs. max: 2 at $x=1$

55. Abs. max: $1+\pi$ at $x=-1$; abs. min: 1 at $x=1$

57. Abs. max: 11 at $x=1$; abs. min: -16 at $x=4$

59. Abs. max: 27 at $x=-3$; abs. min: $-\frac{19}{12}$ at $x=\frac{1}{2}$

61. Abs. max: $\dfrac{1}{100{,}000}$ at $x=1$; abs. min: $-\dfrac{1}{100{,}000}$ at $x=-1$

63. Abs. max: $\sqrt{2}$ at $x=\pm\pi/4$; abs. min: 1 at $x=0$

65. Abs. max: $27/e^3$ at $x=3$; abs. min: $-e$ at $x=-1$

67. Abs. max: 3 at $x=\pm 1$; abs. min: 0 at $x=-2,0,2$

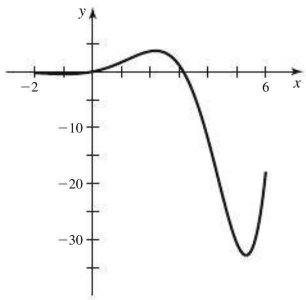
69. a. The velocity of the downstream wind v_2 is less than or equal to the velocity of the upstream wind, so $0\le v_2\le v_1$, or $0\le\dfrac{v_2}{v_1}\le 1$.

b. $R(1)=0$ **c.** $R(0)=\frac{1}{2}$ **d.** 0.593 is the maximum fraction of power that can be extracted from a wind stream by a wind turbine.

71. $t=2$ s **73.** $t=2$ s **75. a.** 50 **b.** 45 **77. a.** False **b.** False **c.** False **d.** True **79. a.** $x=-0.96, 2.18, 5.32$

b. Abs. max: 3.72 at $x=2.18$; abs. min: -32.80 at $x=5.32$

c.



81. a. $x=\tan^{-1}2+k\pi$, for $k=-2,-1,0,1$

b. $x=\tan^{-1}2+k\pi$, for $k=-2,0$, correspond to local max; $x=\tan^{-1}2+k\pi$, for $k=-1,1$, correspond to local min.

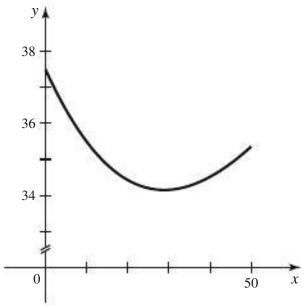
c. Abs. max: 2.24; abs. min: -2.24 **83. a.** $x=5-4\sqrt{2}$

b. $x=5-4\sqrt{2}$ corresponds to a local max. **c.** No abs. max or min

85. Abs. max: 4 at $x=-1$; abs. min: -8 at $x=3$

87. a. $T(x)=\dfrac{\sqrt{2500+x^2}}{2}+\dfrac{50-x}{4}$ **b.** $x=50/\sqrt{3}$

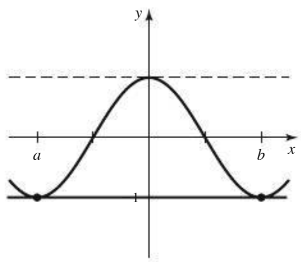
c. $T(50/\sqrt{3}) \approx 34.15, T(0) = 37.50, T(50) \approx 35.36$
d.



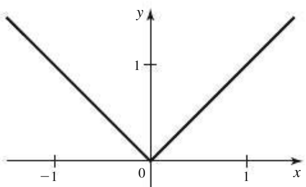
89. a. 1, 3, 0, 1 **b.** Because $h'(2) \neq 0$, h does not have a local extreme value at $x = 2$. However, g may have a local extremum at $x = 2$ (because $g'(2) = 0$). **91. a.** $f(x) - f(c) \leq 0$ for all x near c
b. $\lim_{x\to c^+} \dfrac{f(x) - f(c)}{x - c} \leq 0$ **c.** $\lim_{x\to c^-} \dfrac{f(x) - f(c)}{x - c} \geq 0$
d. Because $f'(c)$ exists, $\lim_{x\to c^+} \dfrac{f(x) - f(c)}{x - c} = \lim_{x\to c^-} \dfrac{f(x) - f(c)}{x - c}$. By parts (b) and (c), we must have that $f'(c) = 0$.

Section 4.2 Exercises, pp. 254–257

1. If f is a continuous function on the closed interval $[a, b]$ and is differentiable on (a, b), and the slope of the secant line that joins $(a, f(a))$ to $(b, f(b))$ is zero, then there is at least one value c in (a, b) at which the slope of the line tangent to f at $(c, f(c))$ is also zero.

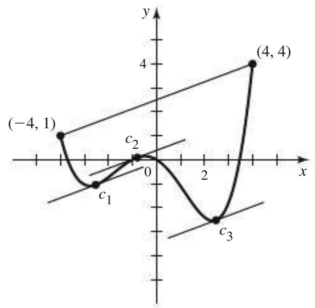


3. $f(x) = |x|$ is not differentiable at 0. **5. b.** $c = 1$
7. b. $c = \pm 2/\sqrt[4]{5} \approx \pm 1.34$
9.



11. $x = \frac{1}{3}$

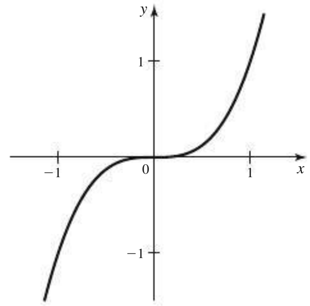
13. $x = \pi/4$ **15.** Does not apply **17.** $x = \frac{5}{3}$ **19.** Average lapse rate $= -6.3°/\text{km}$. You cannot conclude that the lapse rate at a point exceeds the threshold value. **21. a.** Yes **b.** $c = \frac{1}{2}$ **23. a.** Does not apply **25. a.** Yes **b.** $c = \ln(e - 1)$ **27. a.** Yes **b.** $c \approx \pm 0.881$ **29. a.** Yes **b.** $c = \sqrt{1 - 9/\pi^2}$ **31. a.** Does not apply **33. a.** False **b.** True **c.** False **37.** h and p
39.



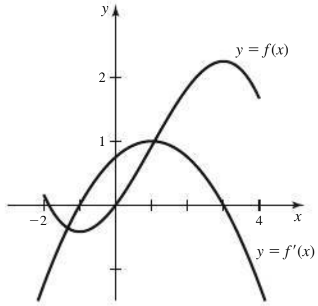
41. No such point exists; function is not continuous at 2.
43. The car's average velocity is $(30 - 0)/(28/60) = 64.3$ mi/hr. By the MVT, the car's instantaneous velocity was 64.3 mi/hr at some time.
45. Average speed $= 11.6$ mi/hr. By the MVT, the speed was exactly 11.6 mi/hr at least once. By the Intermediate Value Theorem, all speeds between 0 and 11.6 mi/hr were reached. Because the initial and final speeds were 0 mi/hr, the speed of 11 mi/hr was reached at least twice.
47. $\dfrac{f(b) - f(a)}{b - a} = A(a + b) + B$ and $f'(x) = 2Ax + B$; $2Ax + B = A(a + b) + B$ implies that $x = \dfrac{a + b}{2}$, the midpoint of $[a, b]$. **49.** $\tan^2 x$ and $\sec^2 x$ differ by a constant; in fact, $\tan^2 x - \sec^2 x = -1$. **53.** *Hint:* By the MVT, there is a value of c in $(2, 4)$ for which $\dfrac{f(4) - f(2)}{4 - 2} = f'(c)$. **57. b.** $c = \frac{1}{2}$

Section 4.3 Exercises, pp. 267–270

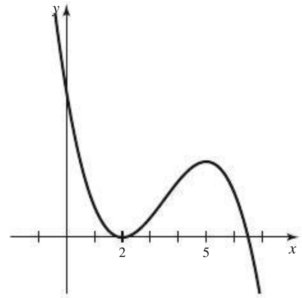
1. f is increasing on I if $f'(x) > 0$ for all x in I; f is decreasing on I if $f'(x) < 0$ for all x in I. **3. a.** $x = 3$ **b.** Increasing on $(3, \infty)$; decreasing on $(-\infty, 3)$
5.



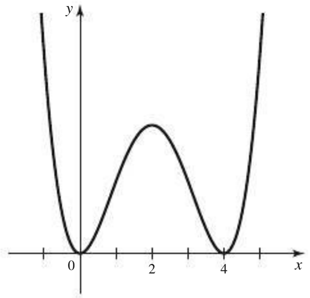
7.



9.



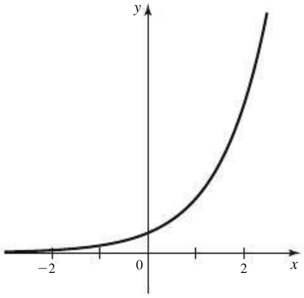
11.



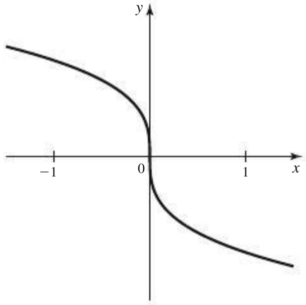
13. a. Concave up on $(-\infty, 2)$; concave down on $(2, \infty)$
b. Inflection point at $x = 2$ **15.** Yes; consider the graph of $y = \sqrt{x}$ on $(0, \infty)$. **17.** $f(x) = x^4$ **19.** Increasing on $(-\infty, 0)$; decreasing on $(0, \infty)$ **21.** Decreasing on $(-\infty, 1)$; increasing on $(1, \infty)$
23. Increasing on $(-\infty, 1)$ and $(4, \infty)$; decreasing on $(1,4)$
25. Increasing on $(-\infty, 1/2)$; decreasing on $(1/2, \infty)$
27. Increasing on $(-\infty, 0)$, $(1, 2)$; decreasing on $(0, 1)$, $(2, \infty)$
29. Increasing on $\left(-\dfrac{1}{\sqrt{e}}, 0\right), \left(\dfrac{1}{\sqrt{e}}, \infty\right)$; decreasing on $\left(-\infty, -\dfrac{1}{\sqrt{e}}\right), \left(0, \dfrac{1}{\sqrt{e}}\right)$ **31.** Increasing on $\left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right)$; decreasing on $\left(0, \dfrac{\pi}{6}\right), \left(\dfrac{5\pi}{6}, 2\pi\right)$ **33.** Increasing on $(-\pi, -2\pi/3)$, $(-\pi/3, 0)$, $(\pi/3, 2\pi/3)$; decreasing on $(-2\pi/3, -\pi/3)$, $(0, \pi/3)$, $(2\pi/3, \pi)$ **35.** Increasing on $(-1, 0)$, $(1, \infty)$; decreasing on $(-\infty, -1)$, $(0, 1)$ **37.** Increasing on $(-3, 1)$; decreasing on $(1,3)$ **39.** Increasing on $(1, 4)$; decreasing on $(-\infty, 1)$, $(4, \infty)$ **41.** Increasing on $\left(-\infty, -\frac{1}{2}\right), \left(0, \frac{1}{2}\right)$; decreasing on $\left(-\frac{1}{2}, 0\right), \left(\frac{1}{2}, \infty\right)$ **43.** Increasing on $(-1, 1)$; decreasing on $(-\infty, -1)$, $(1, \infty)$ **45. a.** $x = 0$ **b.** Local min at $x = 0$ **c.** Abs. min: 3 at $x = 0$; abs. max: 12 at $x = -3$

47. a. $x = \pm\sqrt{2}$ **b.** Local min at $x = -\sqrt{2}$; local max at $x = \sqrt{2}$ **c.** Abs. max: 2 at $x = \sqrt{2}$; abs. min: -2 at $x = -\sqrt{2}$ **49. a.** $x = \pm\sqrt{3}$ **b.** Local min at $x = -\sqrt{3}$; local max at $x = \sqrt{3}$ **c.** Abs. max: 28 at $x = -4$; abs. min: $-6\sqrt{3}$ at $x = -\sqrt{3}$ **51. a.** $x = 2$ and $x = 0$ **b.** Local max at $x = 0$; local min at $x = 2$ **c.** Abs. min: $-10\sqrt[3]{25}$ at $x = -5$; abs. max: 0 at $x = 0, 5$ **53. a.** $x = e^{-2}$ **b.** Local min at $x = e^{-2}$ **c.** Abs. min: $-2/e$ at $x = e^{-2}$; no abs. max **55.** Abs. max: $1/e$ at $x = 1$ **57.** Abs. min: $36\sqrt[3]{\pi/6}$ at $r = \sqrt[3]{6/\pi}$

59.

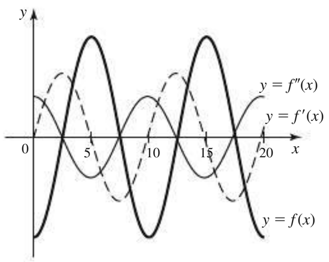


61.



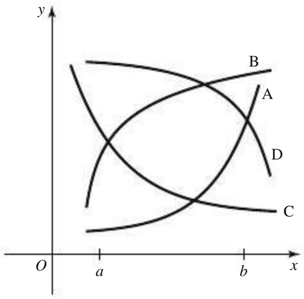
63. Concave up on $(-\infty, 0)$, $(1, \infty)$; concave down on $(0, 1)$; inflection points at $x = 0$ and $x = 1$ **65.** Concave up on $(-\infty, 0)$, $(2, \infty)$; concave down on $(0, 2)$; inflection points at $x = 0$ and $x = 2$ **67.** Concave down on $(-\infty, 1)$; concave up on $(1, \infty)$; inflection point at $x = 1$ **69.** Concave up on $(-1/\sqrt{3}, 1/\sqrt{3})$; concave down on $(-\infty, -1/\sqrt{3})$, $(1/\sqrt{3}, \infty)$; inflection points at $t = \pm 1/\sqrt{3}$ **71.** Concave up on $(-\infty, -1)$, $(1, \infty)$; concave down on $(-1, 1)$; inflection points at $x = \pm 1$ **73.** Concave up on $(0, 1)$; concave down on $(1, \infty)$; inflection point at $x = 1$ **75.** Concave up on $(0, 2)$, $(4, \infty)$; concave down on $(-\infty, 0)$, $(2, 4)$; inflection points at $x = 0, 2, 4$ **77.** Critical pts. $x = 0$ and $x = 2$; local max at $x = 0$, local min at $x = 2$ **79.** Critical pt. $x = 0$; local max at $x = 0$ **81.** Critical pt. $x = 6$; local min at $x = 6$ **83.** Critical pts. $x = 0$ and $x = 1$; local max at $x = 0$; local min at $x = 1$
85. Critical pts. $x = 0$ and $x = 2$; local min at $x = 0$; local max at $x = 2$ **87.** Critical pt. $x = e^5$; local min at $x = e^5$ **89.** Critical pts. $t = -3$ and $t = 2$; local max at $t = -3$; local min at $t = 2$
91. Critical pt. $x = -a$; local min at $x = -a$
93. Critical pts. $x = \pm\sqrt[3]{e}$; local min at $x = \pm\sqrt[3]{e}$
95. a. True **b.** False **c.** True **d.** False **e.** False

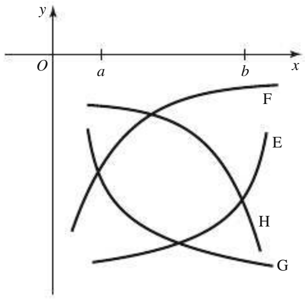
97.



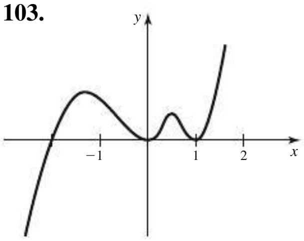
99. a–C–i, b–B–iii, c–A–ii

101.

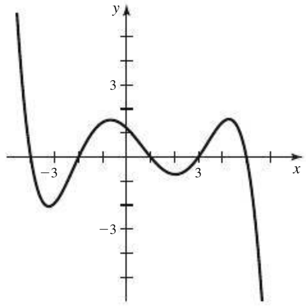




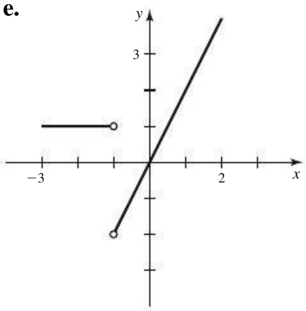
103.



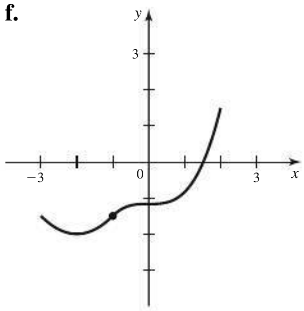
105.



107. a. Increasing on $(-2, 2)$; decreasing on $(-3, -2)$
b. Critical pts. $x = -2$ and $x = 0$; local min at $x = -2$; neither a local max nor min at $x = 0$ **c.** Inflection pts. at $x = -1$ and $x = 0$
d. Concave up on $(-3, -1)$, $(0, 2)$; concave down on $(-1, 0)$
e.



f.

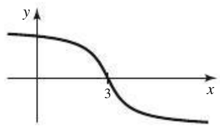


109. a. $E = \dfrac{p}{p - 50}$ **b.** -1.4% **c.** $E'(p) = -\dfrac{ab}{(a - bp)^2} < 0$, for $p \ge 0$, $p \ne a/b$ **d.** $E(p) = -b$, for $p \ge 0$
111. a. 300 **b.** $t = \sqrt{10}$ **c.** $t = \sqrt{b/3}$

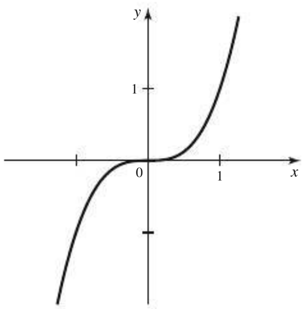
Section 4.4 Exercises, pp. 277–280

1. We need to know on which interval(s) to graph f. **3.** No; the domain of any polynomial is $(-\infty, \infty)$; there is no vertical asymptote. Also, $\lim_{x \to \pm\infty} p(x) = \pm\infty$, where p is any polynomial; there is no horizontal asymptote. **5.** Evaluate the function at the critical points and at the endpoints. Then find the largest and smallest values among those candidates.

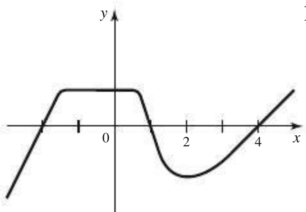
7.



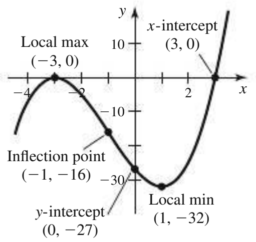
9.



11.



13.



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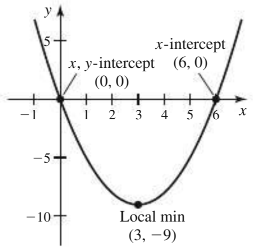

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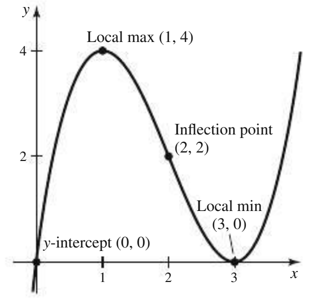
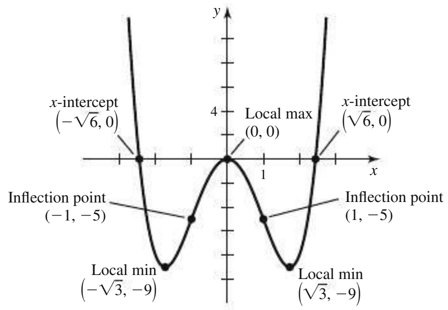


19.



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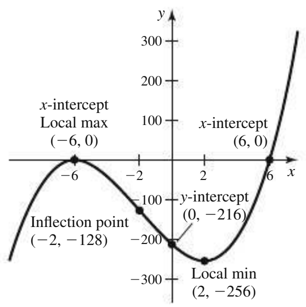
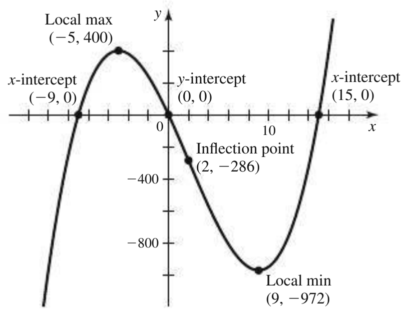
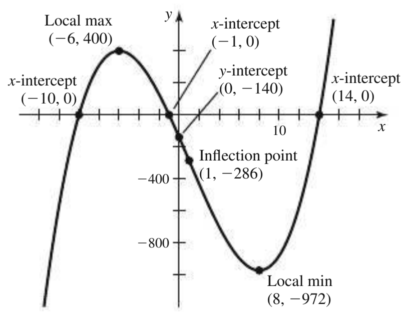


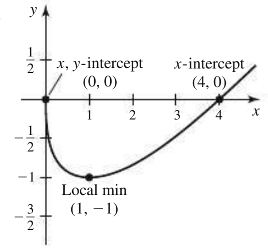
23.



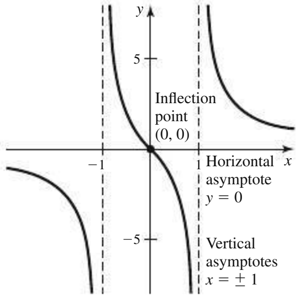
25.



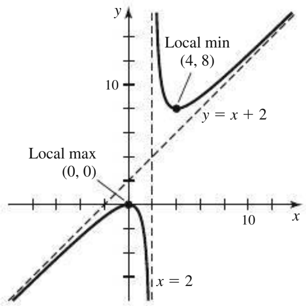
27.



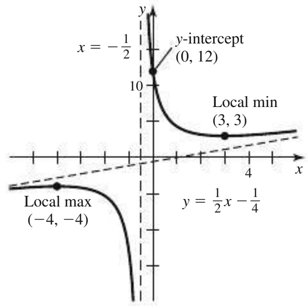
29.



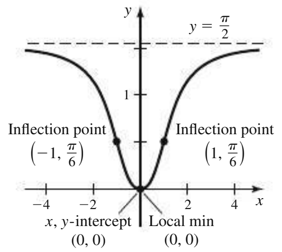
31.



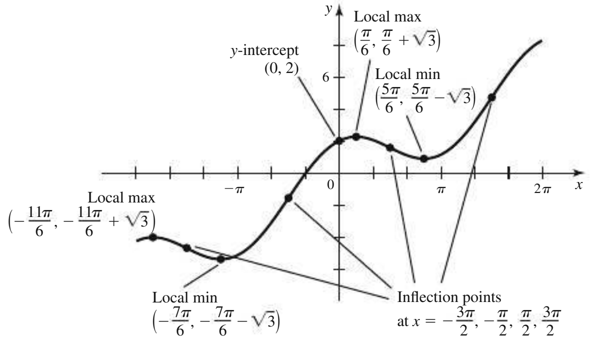
33.



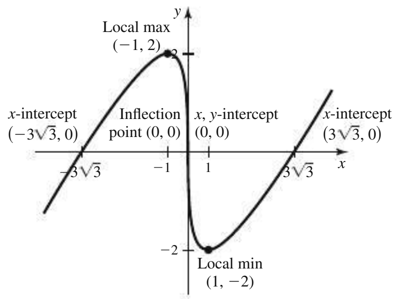
35.



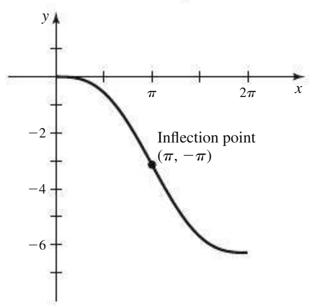
37.



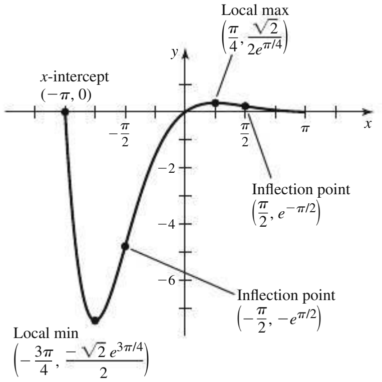
39.



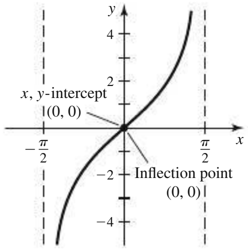
41.



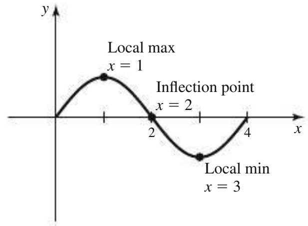
43.



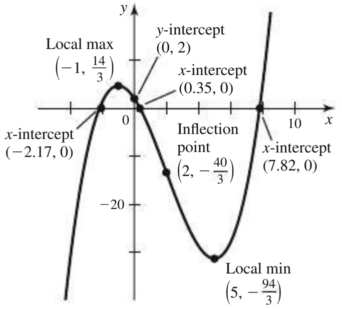
45.



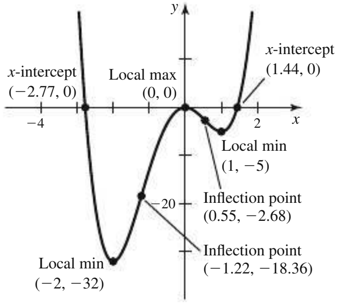
47. Critical pts. at $x = 1, 3$; local max at $x = 1$; local min at $x = 3$; inflection pt. at $x = 2$; increasing on $(0, 1)$, $(3, 4)$; decreasing on $(1, 3)$; concave up on $(2, 4)$; concave down on $(0, 2)$



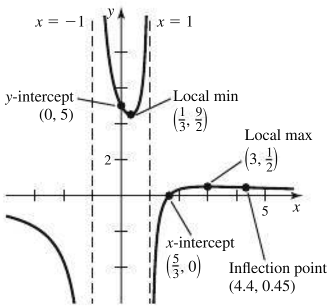
49.



51.

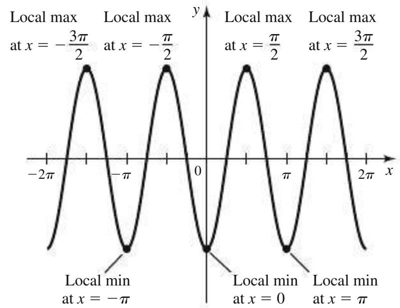


53.

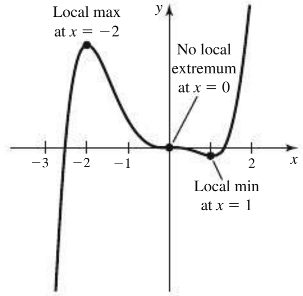


55. **a.** False **b.** False **c.** False **d.** True

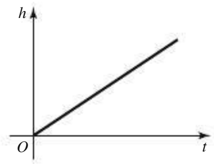
57.



59.

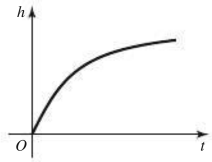


61. (A) a.



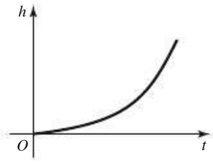
b. Water is being added at all times. **c.** No concavity **d.** h' has an abs. max at all points of $[0, 10]$.

(B) a.



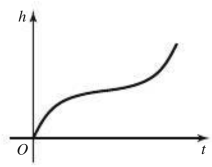
c. Concave down **d.** h' has abs. max at $t = 0$.

(C) a.



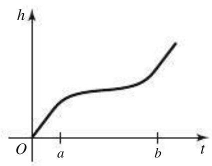
c. Concave up **d.** h' has abs. max at $t = 10$.

(D) a.



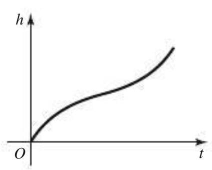
c. Concave down on $(0, 5)$, then concave up on $(5, 10)$; inflection pt. at $t = 5$ **d.** h' has abs. max at $t = 0$ and $t = 10$.

(E) a.



c. First, no concavity; then concave down, no concavity, concave up, and, finally, no concavity
d. h' has abs. max at all points of an interval $[0, a]$ and $[b, 10]$.

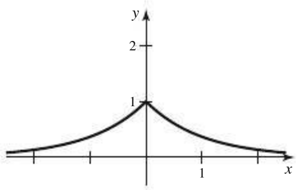
(F) a.



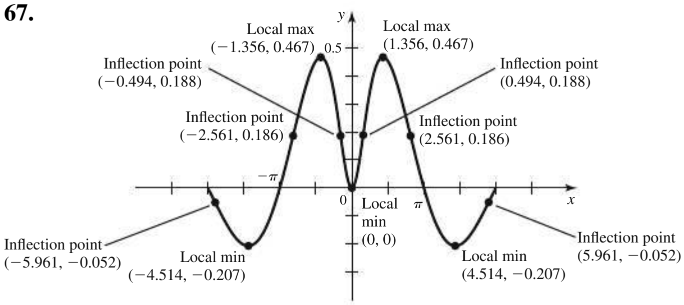
c. Concave down on $(0, 5)$; concave up on $(5, 10)$; inflection pt. at $t = 5$ **d.** h' has abs. max at $t = 0$ and $t = 10$.

63. Local max of $e^{1/e}$ at $x = e$

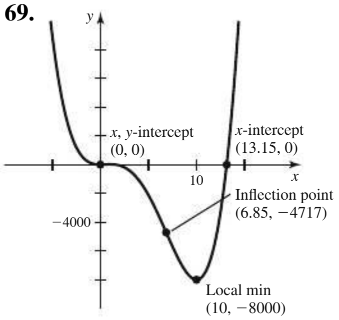
65. $f'(0)$ does not exist.

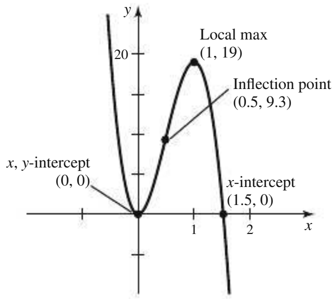


67.

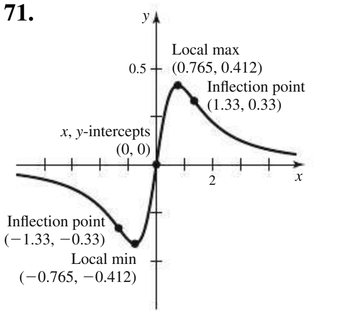


69.





71.



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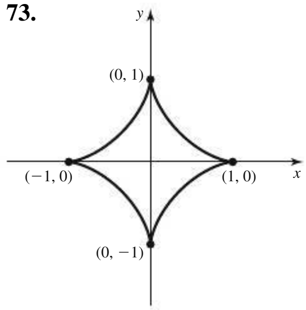
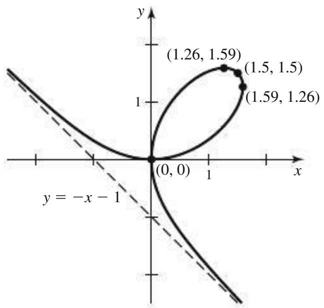
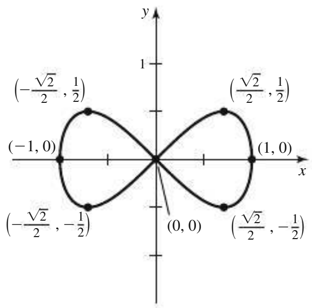


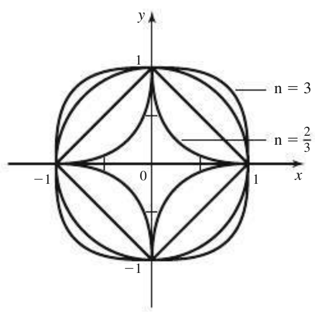
75.



77.



79.



Section 4.5 Exercises, pp. 284–291

1. Objective function, constraint(s) **3.** $Q = x^2(10 - x)$; $Q = (10 - y)^2 y$ **5. a.** $P(x) = 100x - 10x^2$ **b.** Abs. max: 250 at $x = 5$ **7.** $\frac{23}{2}$ and $\frac{23}{2}$ **9.** $5\sqrt{2}$ and $5\sqrt{2}$ **11.** Width = length = $\frac{5}{2}$ **13.** Width = length = 10 **15.** $x = \sqrt{6}, y = 2\sqrt{6}$

17. $\dfrac{10}{\sqrt{2}}$ cm by $\dfrac{5}{\sqrt{2}}$ cm **19.** Length = width = height = 2

21. $\dfrac{4}{\sqrt[3]{5}}$ ft by $\dfrac{4}{\sqrt[3]{5}}$ ft by $5^{2/3}$ ft **23.** Approx. $(0.59, 0.65)$

25. $(5, 15)$, distance ≈ 47.4 **27. a.** A point $8/\sqrt{5}$ mi from the point on the shore nearest the woman in the direction of the restaurant **b.** $9/\sqrt{13}$ mi/hr **29.** A point $7\sqrt{3}/6$ mi from the point on shore nearest the island, in the direction of the power station

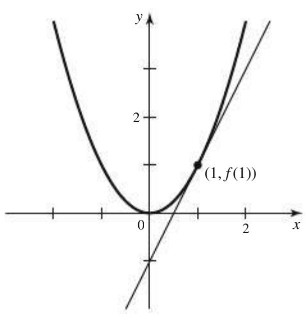
31. 18.2 ft **33.** $h = \dfrac{20}{\sqrt{3}}$; $r = 20\sqrt{\frac{2}{3}}$

35. a. $r = \sqrt[3]{177/\pi} \approx 3.83$ cm; $h = 2\sqrt[3]{177/\pi} \approx 7.67$ cm **b.** $r = \sqrt[3]{177/2\pi} \approx 3.04$ cm; $h = 2\sqrt[3]{708/\pi} \approx 12.17$ cm; part (b) is closer to the real can. **37.** $\sqrt{15}$ m by $2\sqrt{15}$ m **39.** 12″ by 6″ by 3″; 216 in^3 **41.** Lower rectangular pane is approximately 5.6 ft wide by 2.8 ft high. **43.** $r/h = \sqrt{2}$ **45.** $r = \sqrt{2}R/\sqrt{3}$; $h = 2R/\sqrt{3}$ **47.** 3:1 **49. a.** 0, 30, 25 **b.** 42.5 mi/hr **c.** The units of $p/g(v)$ are \$/mi and so are the units of w/v. Therefore, $L\left(\dfrac{p}{g(v)} + \dfrac{w}{v}\right)$ gives the total cost of a trip of L miles. **d.** Approx. 62.9 mi/hr **e.** Neither; the zeros of $C'(v)$ are independent of L. **f.** Decreased slightly, to 62.5 mi/hr **g.** Decreased to 60.8 mi/hr **51.** $\sqrt{30} \approx 5.5$ ft **53.** The point $12/\left(\sqrt[3]{2} + 1\right) \approx 5.3$ m from the weaker source **55. b.** Because the speed of light is constant, travel time is minimized when distance is minimized. **57.** $r = h = \sqrt[3]{450/\pi}$ m **59. a.** $\dfrac{a_1 + a_2}{2}$ **b.** $\dfrac{a_1 + a_2 + a_3}{3}$ **c.** $\dfrac{a_1 + a_2 + \cdots + a_n}{n}$ **61.** $\dfrac{\pi}{3}$

63. When the seat is at its lowest point **65.** For $L \le 4r$, max at $\theta = 0$ and $\theta = 2\pi$; min at $\theta = \cos^{-1}(-L/(4r))$ and $\theta = 2\pi - \cos^{-1}(-L/(4r))$. For $L > 4r$, max at $\theta = 0$ and $\theta = 2\pi$; min at $\theta = \pi$. **67. a.** $P = 2/\sqrt{3}$ units from the midpoint of the base **69.** You can run 12 mi/hr if you run toward the point 3/16 mi ahead of the locomotive (when it passes the point nearest you). **71. a.** $r = 2R/3$; $h = \frac{1}{3}H$ **b.** $r = R/2$; $h = H/2$ **73.** $(1 + \sqrt{3})$ mi ≈ 2.7 mi **75. (i)** $\left(p - \frac{1}{2}, \sqrt{p - \frac{1}{2}}\right)$ **(ii)** $(0, 0)$ **77.** Let the angle of the cuts be φ_1 and φ_2, where $\varphi_1 + \varphi_2 = \theta$. The volume of the notch is proportional to $\tan\varphi_1 + \tan\varphi_2 = \tan\varphi_1 + \tan(\theta - \varphi_1)$, which is minimized when $\varphi_1 = \varphi_2 = \dfrac{\theta}{2}$. **79.** $x \approx 38.81, y \approx 55.03$

Section 4.6 Exercises, pp. 298–300

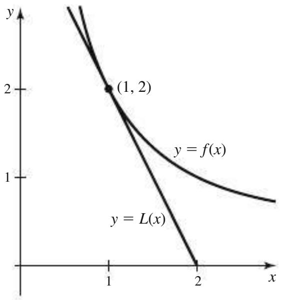
1.



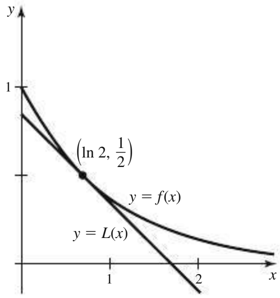
3. $f(x) \approx f(a) + f'(a)(x - a)$

5. $L(x) = 3x - 1$; 2.3 **7.** 2.7 **9.** $dy = f'(x)\,dx$

11. Approx. 25 **13.** 61 mi/hr; 61.02 mi/hr
15. $L(x) = T(0) + T'(0)(x - 0) = D - (D/60)x = D(1 - x/60)$
17. 84 min; 84.21 min **19.** $L(x) = 9x - 4$ **21.** $L(t) = t + 5$
23. $L(x) = 3x - 5$ **25. a.** $L(x) = -4x + 16$ **b.** 7.6
c. 0.13% error **27. a.** $L(x) = x$ **b.** 0.9 **c.** 40% error
29. a. $L(x) = 1$ **b.** 1 **c.** 0.005% error **31. a.** $L(x) = \frac{1}{2} - \frac{x}{48}$
b. 0.5 **c.** 0.003% error **33. a.** $L(x) = 1 - x$; **b.** $1/1.1 \approx 0.9$
c. 1% error **35. a.** $L(x) = 1 - x$ **b.** $e^{-0.03} \approx 0.97$
c. 0.05% error **37.** $a = 200$; $\frac{1}{203} \approx 0.004925$
39. $a = 144$; $\sqrt{146} \approx \frac{145}{12}$ **41.** $a = 1$; $\ln 1.05 \approx 0.05$
43. $a = 0$; $e^{0.06} \approx 1.06$ **45.** $a = 512$; $\frac{1}{\sqrt[3]{510}} \approx 0.125$
47. a. $L(x) = -2x + 4$ **b.**



c. Underestimates **d.** $f''(1) = 4 > 0$
49. a. $L(x) = -\frac{1}{2}x + \frac{1}{2}(1 + \ln 2)$
b.



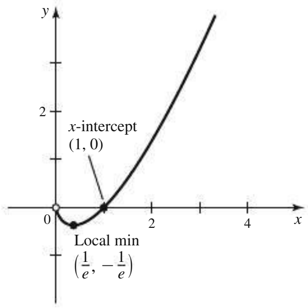
c. Underestimates **d.** $f''(\ln 2) = \frac{1}{2} > 0$
51. $E(x) \le 1$ when $-7.26 \le x \le 8.26$, which corresponds to driving times for 1 mi from about 53 s to 68 s. Therefore, $L(x)$ gives approximations to $s(x)$ that are within 1 mi/hr of the true value when you drive 1 mi in t seconds, where $53 < t < 68$.
53. a. True **b.** False **c.** True **d.** True
55. $\Delta V \approx 10\pi \text{ ft}^3$ **57.** $\Delta V \approx -40\pi \text{ cm}^3$
59. $\Delta S \approx -\frac{59\pi}{5\sqrt{34}} \text{ m}^2$ **61.** $dy = 2\,dx$ **63.** $dy = -\frac{3}{x^4}\,dx$
65. $dy = a \sin x\,dx$ **67.** $dy = (9x^2 - 4)\,dx$
69. $dy = \sec^2 x\,dx$
71. $L(x) = 2 + (x - 8)/12$

x	Linear approx.	Exact value	Percent error
8.1	$2.008\overline{3}$	2.00829885	1.7×10^{-3}
8.01	$2.0008\overline{3}$	2.000832986	1.7×10^{-5}
8.001	$2.00008\overline{3}$	2.00008333	1.7×10^{-7}
8.0001	$2.000008\overline{3}$	2.000008333	1.7×10^{-9}
7.9999	$1.999991\overline{6}$	1.999991667	1.7×10^{-9}
7.999	$1.99991\overline{6}$	1.999916663	1.7×10^{-7}
7.99	$1.9991\overline{6}$	1.999166319	1.7×10^{-5}
7.9	$1.991\overline{6}$	1.991631701	1.8×10^{-3}

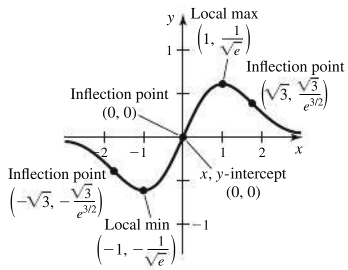
73. a. f; the rate at which f' is changing at 1 is smaller than the rate at which g' is changing at 1. The graph of f bends away from the linear function more slowly than the graph of g. **b.** The larger the value of $|f''(a)|$, the greater the deviation of the curve $y = f(x)$ from the tangent line at points near $x = a$.

Section 4.7 Exercises, pp. 310–312

1. If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, then we say $\lim_{x\to a} f(x)/g(x)$ is an indeterminate form 0/0. **3.** Take the limit of the quotient of the derivatives of the functions.
5. a. $\lim_{x\to 0}\left(x^2 \cdot \frac{1}{x^2}\right) = 1$ **b.** $\lim_{x\to 0}\left(2x^2 \cdot \frac{1}{x^2}\right) = 2$
7. If $\lim_{x\to a} f(x)g(x)$ has the indeterminate form $0 \cdot \infty$, then $\lim_{x\to a}\left(\frac{f(x)}{1/g(x)}\right)$ has the indeterminate form 0/0 or ∞/∞.
9. $\frac{1}{5}$ **11.** If $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = \infty$, then $f(x)^{g(x)}$ has the form 1^∞ as $x \to a$, which is meaningless; so direct substitution does not work. **13.** $\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$ **15.** $\ln x, x^3, 2^x, x^x$
17. −1 **19.** $\frac{3}{4}$ **21.** $\frac{1}{2}$ **23.** $\frac{1}{2}$ **25.** $\frac{1}{e}$ **27.** −1 **29.** $\frac{12}{5}$
31. 4 **33.** $\frac{9}{16}$ **35.** $\frac{1}{2}$ **37.** $-\frac{2}{3}$ **39.** $\frac{1}{24}$ **41.** 1 **43.** 4
45. 1 **47.** $-\frac{1}{2}$ **49.** $\frac{1}{\pi^2}$ **51.** $\frac{1}{3}$ **53.** 1 **55.** $\frac{7}{6}$ **57.** 1
59. $-\frac{1}{2}$ **61.** 0 **63.** −8 **65.** 0 **67.** $\frac{1}{2}$ **69.** $\frac{\ln 3}{\ln 2}$
71. $-\frac{9}{4}$ **73.** $\frac{1}{6}$ **75.** 1 **77.** 1 **79.** e **81.** e^{a+1} **83.** e
85. b. $\lim_{m\to\infty} (1 + r/m)^m = \lim_{m\to\infty}\left(1 + \frac{1}{(m/r)}\right)^{(m/r)r} = e^r$
87. 3 **89.** $\frac{1}{2}$ **91.** e **93.** $\ln a - \ln b$ **95.** $e^{0.01x}$ **97.** Comparable growth rates **99.** x^x **101.** 1.00001^x **103.** e^{x^2} **105. a.** False **b.** False **c.** False **d.** False **e.** True **f.** True
107.



109.



111. $\sqrt{a/c}$ **113.** $\lim_{x\to\infty} \frac{x^p}{b^x} = \lim_{t\to\infty} \frac{\ln^p t}{t \ln^p b} = 0$, where $t = b^x$
115. Show $\lim_{x\to\infty} \frac{\log_a x}{\log_b x} = \frac{\ln b}{\ln a} \ne 0$. **117.** 1/3 **121. a.** $b > e$
b. e^{ax} grows faster than e^x as $x \to \infty$, for $a > 1$; e^{ax} grows slower than e^x as $x \to \infty$, for $0 < a < 1$.

Section 4.8 Exercises, pp. 318–321

1. Newton's method generates a sequence of x-intercepts of lines tangent to the graph of f to approximate the roots of f.
3. $x_1 = 2, x_2 = 1, x_3 = 0$ **5.** $x_1 = 0.75$ **7.** Generally, if two successive Newton approximations agree in their first p digits, then those approximations have p digits of accuracy. The method is terminated when the desired accuracy is reached.
9. $x_{n+1} = x_n - \dfrac{x_n^2 - 6}{2x_n} = \dfrac{x_n^2 + 6}{2x_n}$; $x_1 = 2.5, x_2 = 2.45$
11. $x_{n+1} = x_n - \dfrac{e^{-x_n} - x_n}{e^{-x_n} - 1}$; $x_1 = 0.564382, x_2 = 0.567142$

13.

n	x_n
0	3
1	3.1667
2	3.16228
3	3.16228

$r \approx 3.16228$

15.

n	x_n
0	0.5
1	0.51096
2	0.51097
3	0.51097

$r \approx 0.51097$

17.

n	x_n
0	1.2
1	1.16935
2	1.16561
3	1.16556
4	1.16556

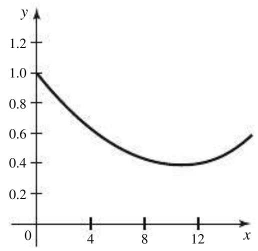
$r \approx 1.16556$

19.

n	x_n
0	0.75
1	0.73915
2	0.73909
3	0.73909

$r \approx 0.73909$

21. $x \approx -0.335408, 1.333057$ **23.** $x \approx 0.179295$
25. $x \approx 0.620723, 3.03645$ **27.** $x \approx 0, 1.895494, -1.895494$
29. $x \approx -2.114908, 0.254102, 1.860806$
31. $x \approx 0.062997, 2.230120$
33.



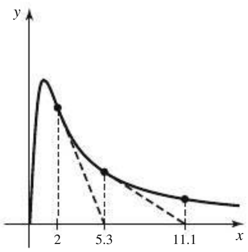
The tumor decreases in size and then starts growing again. It decreases to half its size after about 6.4 days.

35. b. $r \approx 7.3\%$ **37. a.** $t = \pi/4$ **b.** $t \approx 1.33897$
c. $t \approx 2.35619$ **d.** $t \approx 2.90977$
39. $p(x) = x^4 - 7$; $r \approx 1.62658$
41. $p(x) = x^3 + 9$; $r \approx -2.08008$ **43.** $x \approx 2.798386$
45. $x \approx -0.666667, 1.5, 1.666667$ **47. a.** True **b.** False
c. False **49.** $x \approx 0.739085$ **51.** $x = 0$ and $x \approx 1.047198$

53. a.

n	x_n
0	2
1	5.33333
2	11.0553
3	22.2931

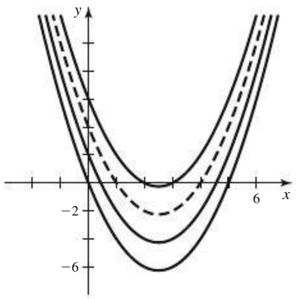
b.

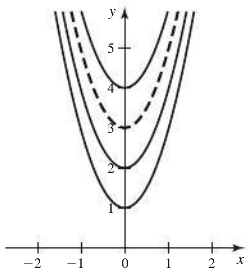


c. The tangent lines intersect the x-axis farther and farther away from the root r. **55. b.** $x \approx 0.142857$ is approximately $\frac{1}{7}$.
57. $\lambda = 1.29011, 2.37305, 3.40918$

Section 4.9 Exercises, pp. 331–334

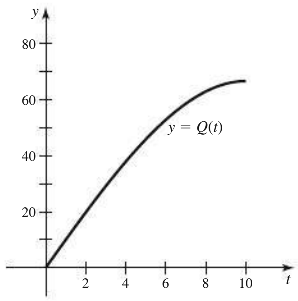
1. The derivative, an antiderivative **3.** $x + C$, where C is an arbitrary constant **5.** $\dfrac{x^{p+1}}{p+1} + C$, where $p \neq -1$ **7.** $\ln|x| + C$
9. 0 **11.** $x^5 + C$ **13.** $-2\cos x + x + C$ **15.** $3\tan x + C$
17. $y^{-2} + C$ **19.** $e^x + C$ **21.** $\tan^{-1}s + C$ **23.** $\frac{1}{2}x^6 - \frac{1}{2}x^{10} + C$
25. $\frac{8}{3}x^{3/2} - 8x^{1/2} + C$ **27.** $\frac{25}{3}s^3 + 15s^2 + 9s + C$
29. $\frac{9}{4}x^{4/3} + 6x^{2/3} + 6x + C$ **31.** $-x^3 + \frac{11}{2}x^2 + 4x + C$
33. $-x^{-3} + 2x + 3x^{-1} + C$ **35.** $x^4 - 3x^2 + C$
37. $\dfrac{1}{2}x^2 + 6x + C$ **39.** $-\cot\theta + 2\theta^3/3 - 3\theta^2/2 + C$
41. $-2\cot y - 3\csc y + C$ **43.** $\tan x - x + C$
45. $\tan\theta + \sec\theta + C$ **47.** $t^3 - 2\cot t + C$
49. $\sec\theta + \tan\theta + \theta + C$ **51.** $\frac{1}{2}\ln|y| + C$ **53.** $3\sin^{-1}x + C$
55. $4\sec^{-1}|x| + C$ **57.** $\frac{1}{6}\sec^{-1}|x| + C$ **59.** $t + \ln|t| + C$
61. $e^{x+2} + C$ **63.** $e^w - 4w + C$ **65.** $\ln|x| + 2\sqrt{x} + C$
67. $\frac{4}{15}x^{15/2} - \frac{24}{11}x^{11/6} + C$ **69.** $\dfrac{1}{6}x^6 - \dfrac{2}{3}x^3 + x + 1$
71. $2x^4 - \cos x + 3$ **73.** $\sec v + 1, -\pi/2 < v < \pi/2$
75. $y^3 + 5\ln y + 2, y > 0$ **77.** $f(x) = x^2 - 3x + 4$
79. $g(x) = \dfrac{7}{8}x^8 - \dfrac{x^2}{2} + \dfrac{13}{8}$ **81.** $f(u) = 4(\sin u + \cos u) - 4$
83. $y(t) = 3\ln t + 6t + 2, t > 0$
85. $y(\theta) = \sqrt{2}\sin\theta + \tan\theta + 1, -\pi/2 < \theta < \pi/2$
87. $f(x) = x^2 - 5x + 4$ **89.** $f(x) = \dfrac{3}{2}x^2 - \cos x + 4$





91. $s(t) = t^2 + 4t$ **93.** $s(t) = \frac{4}{3}t^{3/2} + 1$
95. $s(t) = 2t^3 + 2t^2 - 10t$ **97.** $s(t) = -16t^2 + 20t$
99. $s(t) = \frac{1}{30}t^3 + 1$ **101.** $s(t) = t^2 + 4t - 3\sin t + 10$
103. 200 ft **105.** Runner A overtakes runner B at $t = \pi/2$
107. a. $v(t) = -9.8t + 30$ **b.** $s(t) = -4.9t^2 + 30t$
c. Approx. 45.92 m at time $t \approx 3.06$ **d.** $t \approx 6.12$
109. a. $v(t) = -9.8t + 10$ **b.** $s(t) = -4.9t^2 + 10t + 400$
c. Approx. 405.10 m at time $t \approx 1.02$ **d.** $t \approx 10.11$
111. a. True **b.** False **c.** True **d.** False
e. False **113.** $F(x) = -\cos x + 3x + 3 - 3\pi$
115. $F(x) = 2x^8 + x^4 + 2x + 1$
117. a. $Q(t) = 10t - t^3/30$ gal

b.

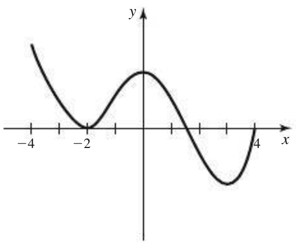


c. $\dfrac{200}{3}$ gal

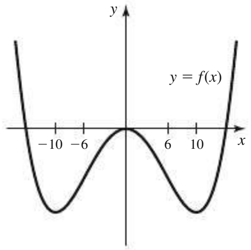
Chapter 4 Review Exercises, pp. 334–337

1. a. False **b.** False **c.** True **d.** True **e.** True **f.** False

3.

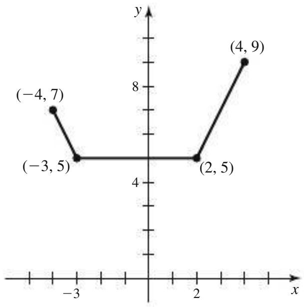


5. a. $x = 0, \pm 10$; increasing on $(-10, 0)$ and $(10, \infty)$, decreasing on $(-\infty, -10)$ and $(0, 10)$ **b.** $x = \pm 6$; concave up on $(-\infty, -6)$ and $(6, \infty)$, concave down on $(-6, 6)$ **c.** Local min at $x = -10, 10$; local max at $x = 0$ **d.**



7. Critical pts. $x = 0, \pm 1$; abs. max: 33 at $x = \pm 2$; abs. min: 6 at $x = \pm 1$ **9.** $x = 3$ and $x = -2$; no abs. max or min
11. Critical pt. $x = 1$; abs. max: $\ln 2$ at $x = 0, 2$; abs. min: 0 at $x = 1$
13. Critical pts. $x = \frac{2\pi}{3}, \frac{4\pi}{3}$; abs. max: $\frac{3\sqrt{3}}{8}$ at $x = \frac{4\pi}{3}$; abs. min: $-\frac{3\sqrt{3}}{8}$ at $x = \frac{2\pi}{3}$ **15.** Critical pt. $x = 1/e$; abs. min: $10 - 2/e$ at $x = 1/e$

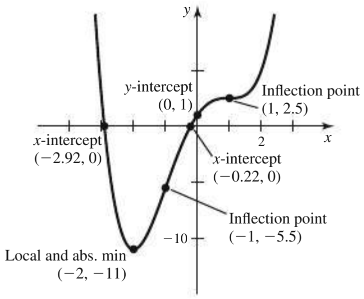
17.



Critical pts.: all x in the interval $[-3, 2]$; abs. max: 9 at $x = 4$; abs. and local min: 5 for x in $[-3, 2]$; local max: 5 for x in $(-3, 2)$

19. a. Increasing on $(-\infty, -1)$ and $(1, \infty)$; decreasing on $(-1, 1)$ **b.** Concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$
21. Inflection pt. $x = 0$ **23.** Critical pts. $x = -a, a/2$; inflection pts. $x = 0, -a$

25.



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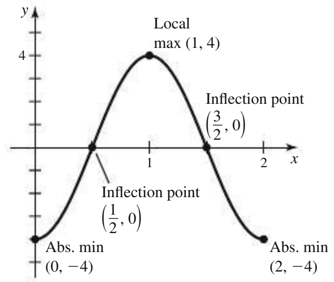

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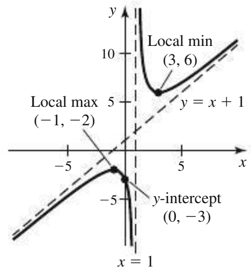

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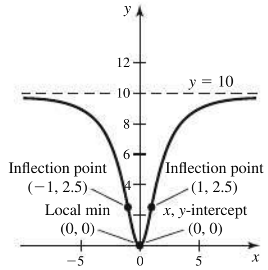
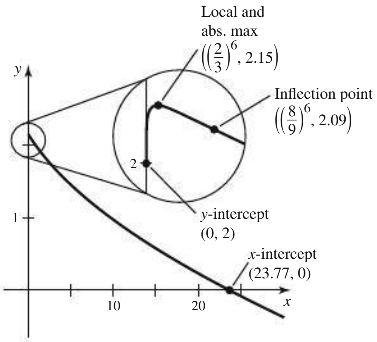


33.

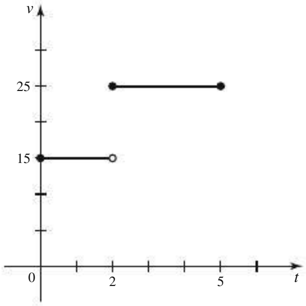


35. Approx. 2.5″ by 3.5″ by 9.5″ **37.** Approx. 59 m from the loudest speaker **39.** $r = 4\sqrt{6}/3$; $h = 4\sqrt{3}/3$ **41.** $x = 7, y = 14$
43. $p = q = 5\sqrt{2}$ **45. a.** $L(x) = \frac{2}{9}x + 3$ **b.** $\frac{85}{9} \approx 9.44$; overestimate **47.** $f(x) = 1/x^2$; $a = 4$; $1/4.2^2 \approx 0.05625$
49. $\Delta h \approx -112$ ft **51.** $c = 2.5$ **53. a.** $\frac{100}{9}$ cells/week
b. $t = 2$ weeks **55.** $-0.434259, 0.767592, 1$ **57.** $0, \pm 0.948683$
59. 0 **61.** 0 **63.** 12 **65.** $\frac{2}{3}$ **67.** ∞ **69.** 0 **71.** 1 **73.** 0
75. 1 **77.** 1 **79.** $1/e^3$ **81.** 1 **83.** $x^{1/2}$ **85.** $\sqrt{x}$ **87.** 3^x
89. Comparable growth rates **91.** $\frac{4}{3}x^3 + 2x^2 + x + C$
93. $-\frac{1}{x} + \frac{4}{3}x^{-3/2} + C$ **95.** $\theta + 3\sin\theta + C$ **97.** $\tan x + C$
99. $12\ln|x| + C$ **101.** $\tan^{-1} x + C$ **103.** $\frac{4}{7}x^{7/4} + \frac{2}{7}x^{7/2} + C$
105. $f(t) = -\cos t + t^2 + 6$ **107.** $h(x) = \frac{1}{3}x^3 - x - \tan^{-1} x + \frac{\pi}{4}$
109. $v(t) = -9.8t + 120$; $s(t) = -4.9t^2 + 120t + 125$. The rocket reaches a height of 859.69 m at time $t \approx 12.24$ s and then falls to the ground, hitting at time $t \approx 25.49$ s. **111. a.** $v(t) = 64 - 32t$
b. $s(t) = -16t^2 + 64t + 128$ **c.** $t = 2$; 192 ft
d. $-64\sqrt{3}$ ft/s ≈ -110.9 ft/s **113.** 1; 1
115. $\lim_{x\to 0^+} f(x) = 1$; $\lim_{x\to 0^+} g(x) = 0$

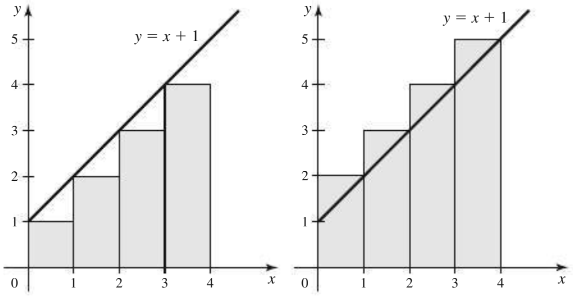
CHAPTER 5

Section 5.1 Exercises, pp. 347–352

1. Displacement = 105 m

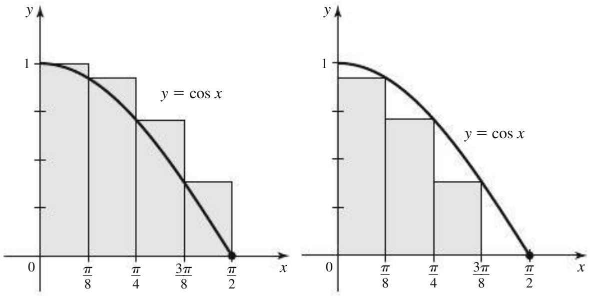


3. a. 440 ft **b.** 400 ft **5. a.** 340 ft **b.** 330 ft
7. Subdivide the interval $[0, \pi/2]$ into several subintervals, which will be the bases of rectangles that fit under the curve. The heights of the rectangles are computed by taking the value of cos x at the right-hand endpoint of each base. We calculate the area of each rectangle and add them to get a lower bound on the area. **9.** Left sum: 34; right sum: 24 **11.** 0.5; 1, 1.5, 2, 2.5, 3; 1, 1.5, 2, 2.5; 1.5, 2, 2.5, 3; 1.25, 1.75, 2.25, 2.75 **13.** Underestimate; the rectangles all fit under the curve. **15. a.** 67 ft **b.** 67.75 ft **17.** 40 m
19. 2.78 m **21.** 148.96 mi **23.** 20; 25
25. a. c.



Left Riemann sum underestimates area. Right Riemann sum overestimates area.

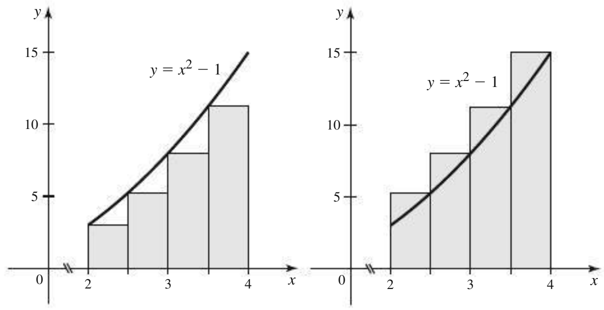
b. $\Delta x = 1; x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4$ **d.** 10, 14
27. a. c.



Left Riemann sum overestimates area. Right Riemann sum underestimates area.

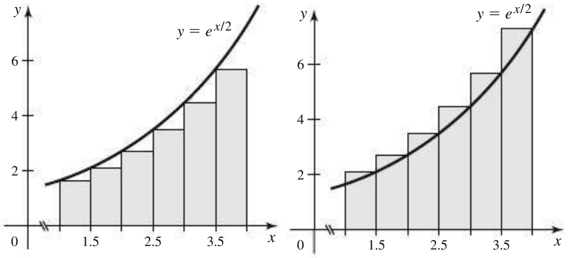
b. $\Delta x = \pi/8; 0, \pi/8, \pi/4, 3\pi/8, \pi/2$ **d.** 1.18; 0.79

29. a. c.



Left Riemann sum underestimates area. Right Riemann sum overestimates area.

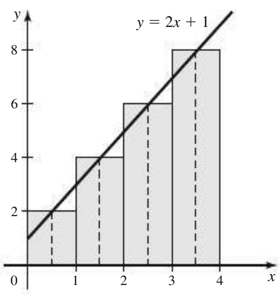
b. $\Delta x = 0.5$; 2, 2.5, 3, 3.5, 4 **d.** 13.75; 19.75
31. a. c.



Left Riemann sum underestimates area. Right Riemann sum overestimates area.

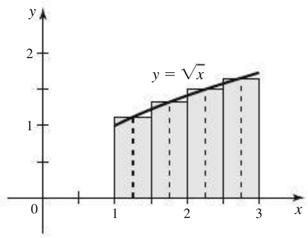
b. $\Delta x = 0.5; x_0 = 1, x_1 = 1.5, x_2 = 2, x_3 = 2.5, x_4 = 3, x_5 = 3.5, x_6 = 4$ **d.** 10.11, 12.98 **33.** 670 **35. a.** 10,500 m; 10,350 m **b.** Left Riemann sum **c.** Increase the number of subintervals in the partition.

37. a. c.



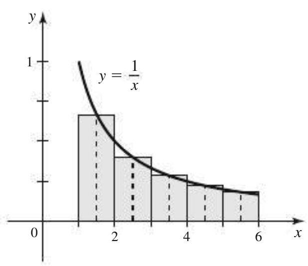
b. $\Delta x = 1$; 0, 1, 2, 3, 4
d. 20

39. a. c.



b. $\Delta x = \frac{1}{2}; 1, \frac{3}{2}, 2, \frac{5}{2}, 3$
d. 2.80

41. a. c.



b. $\Delta x = 1$; 1, 2, 3, 4, 5, 6
d. 1.76

43. 5.5, 3.5 **45. b.** 110, 117.5 **47. a.** $\sum_{k=1}^{5} k$ **b.** $\sum_{k=1}^{6} (k+3)$ **c.** $\sum_{k=1}^{4} k^2$ **d.** $\sum_{k=1}^{4} \frac{1}{k}$ **49. a.** 55 **b.** 48 **c.** 30 **d.** 60 **e.** 6 **f.** 6 **g.** 85 **h.** 0 **51. a.** Left: $\frac{3}{10}\sum_{k=1}^{40}\sqrt{\frac{k-1}{10}} \approx 15.6809$; right: $\frac{3}{10}\sum_{k=1}^{40}\sqrt{\frac{k}{10}} \approx 16.2809$; midpoint: $\frac{3}{10}\sum_{k=1}^{40}\sqrt{\frac{k-0.5}{10}} \approx 16.0055$ **b.** 16 **53. a.** Left: $\frac{1}{25}\sum_{k=1}^{75}\left(\left(2+\frac{k-1}{25}\right)^2 - 1\right) \approx 35.5808$; right: $\frac{1}{25}\sum_{k=1}^{75}\left(\left(2+\frac{k}{25}\right)^2 - 1\right) \approx 36.4208$; midpoint: $\frac{1}{25}\sum_{k=1}^{75}\left(\left(2+\frac{k-0.5}{25}\right)^2 - 1\right) \approx 35.9996$ **b.** 36

55.

n	Right Riemann sum
10	21.96
30	21.9956
60	21.9989
80	21.9994

The sums approach 22.

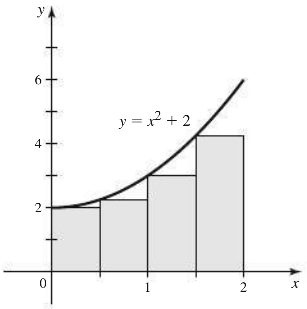
57.

n	Right Riemann sum
10	3.14159
30	3.14159
60	3.14159
80	3.14159

The sums approach π.

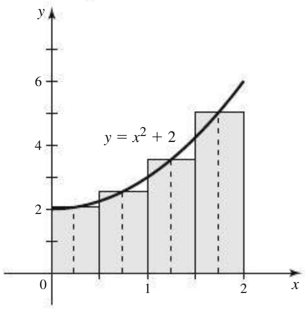
59. a. True **b.** False **c.** True **61.** $\sum_{k=1}^{50}\left(\frac{4k}{50}+1\right)\cdot\frac{4}{50} = 12.16$ **63.** $\sum_{k=1}^{32}\left(3+\frac{2k-1}{8}\right)^3\cdot\frac{1}{4} \approx 3639.1$ **65.** $[1, 5]$; 4 **67.** $[2, 6]$; 4

69. a.



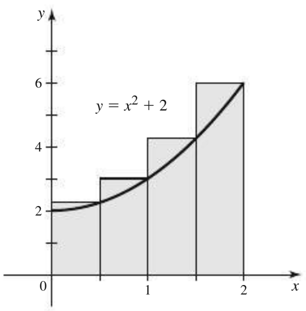
Left Riemann sum is $\frac{23}{4} = 5.75$.

b.



Midpoint Riemann sum is $\frac{53}{8} = 6.625$.

c.



Right Riemann sum is $\frac{31}{4} = 7.75$.

71. a. The object is speeding up on the interval $(0, 1)$, moving at a constant rate on $(1, 3)$, slowing down on $(3, 5)$, and moving at a constant rate on $(5, 6)$. **b.** 30 m **c.** 50 m **d.** $s(t) = 80 + 10t$ **73. a.** 14.5 g **b.** 29.5 g **c.** 44 g **d.** $x = \frac{19}{3}$ cm

75. $s(t) = \begin{cases} 30t & \text{if } 0 \le t \le 2 \\ 50t - 40 & \text{if } 2 < t \le 2.5 \\ 44t - 25 & \text{if } 2.5 < t \le 3 \end{cases}$

77.

n	Midpoint Riemann sum
16	4.7257
32	4.7437
64	4.7485

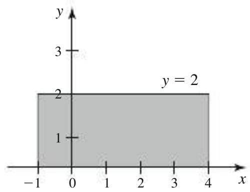
The sums approach 4.75.

81. Underestimates for decreasing functions, independent of concavity; overestimates for increasing functions, independent of concavity

Section 5.2 Exercises, pp. 364–367

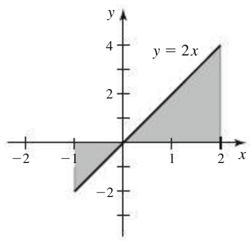
1. The difference between the area bounded by the curve above the x-axis and the area bounded by the curve below the x-axis **3.** 60; 0 **5.** -12; -18; -16

7.



$\int_{-1}^{4} 2\,dx = 10$

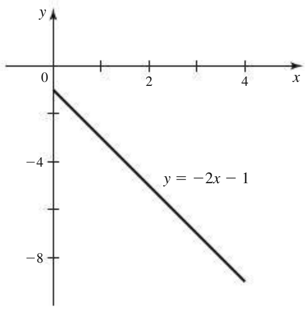
9.



$\int_{-1}^{2} 2x\,dx = 3$

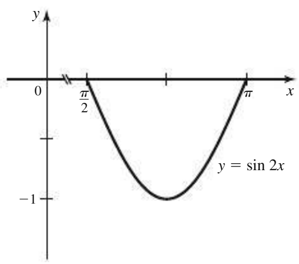
11. Both integrals equal 0. **13.** The length of the interval $[a, a]$ is $a - a = 0$, so the net area is 0. **15.** $\frac{a^2}{2}$

17. a.



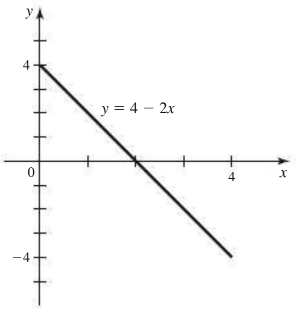
b. -16, -24, -20

19. a.



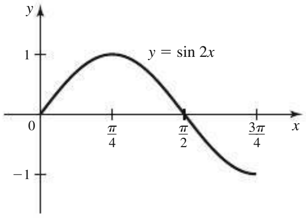
b. -0.948, -0.948, -1.026

21. a.



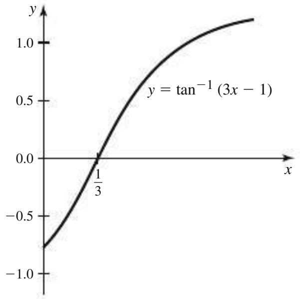
b. 4, −4, 0 **c.** Positive contributions on $[0, 2)$; negative contributions on $(2, 4]$

23. a.



b. 0.735, 0.146, 0.530 **c.** Positive contributions on $(0, \pi/2)$; negative contributions on $(\pi/2, 3\pi/4]$

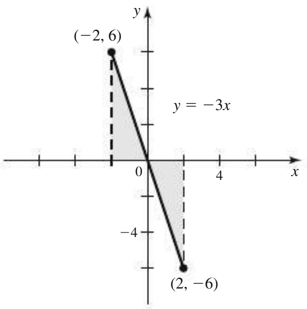
25. a.



b. 0.082; 0.555; 0.326

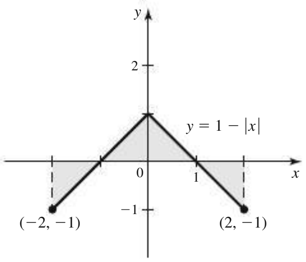
c. Positive contributions on $(\frac{1}{3}, 1]$; negative contributions on $[0, \frac{1}{3})$

27.



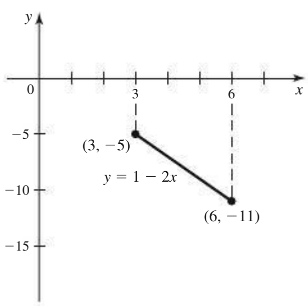
The area is 12; the net area is 0.

29.



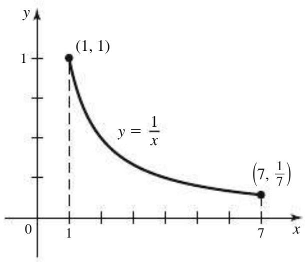
The area is 2; the net area is 0.

31. a.



b. $\Delta x = \frac{1}{2}$; 3, 3.5, 4, 4.5, 5, 5.5, 6 **c.** −22.5; −25.5 **d.** The left Riemann sum overestimates the integral; the right Riemann sum underestimates the integral.

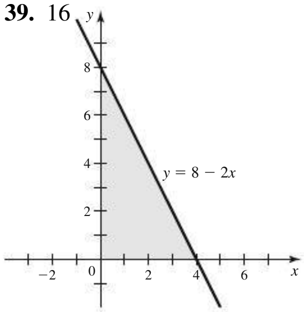
33. a.



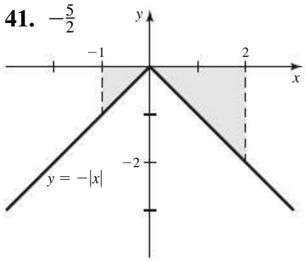
b. $\Delta x = 1$; 1, 2, 3, 4, 5, 6, 7 **c.** $\frac{49}{20}$; $\frac{223}{140}$ **d.** The left Riemann sum overestimates the integral; the right Riemann sum underestimates the integral.

35. $\int_0^2 (x^2 + 1)\, dx$ **37.** $\int_1^2 x \ln x\, dx$

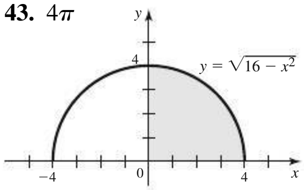
39. 16



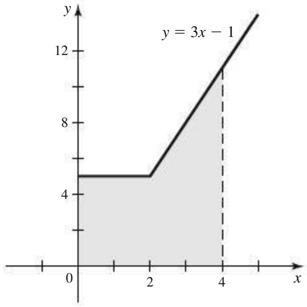
41. $-\frac{5}{2}$



43. 4π



45. 26



47. π **49.** -2π **51. a.** −32 **b.** $-\frac{32}{3}$ **c.** −64 **d.** Not possible **53. a.** 10 **b.** −3 **c.** −16 **d.** 3 **55. a.** 15 **b.** 5 **c.** 3 **d.** −2 **e.** 24 **f.** −10 **57. a.** $\frac{3}{2}$ **b.** $-\frac{3}{4}$ **59.** 16 **61.** 6 **63.** 32 **65.** −16 **67.** $\frac{\pi}{4} + 2$ **69. a.** True **b.** True **c.** True **d.** False **e.** False

71. a. Left: $\sum_{k=1}^{n} \left(\left(\frac{k-1}{n} \right)^2 + 1 \right) \cdot \frac{1}{n}$; right: $\sum_{k=1}^{n} \left(\left(\frac{k}{n} \right)^2 + 1 \right) \cdot \frac{1}{n}$

b.

n	Left Riemann sum	Right Riemann sum
20	1.30875	1.35875
50	1.3234	1.3434
100	1.32835	1.33835

Estimate: $\frac{4}{3}$

73. a. Left: $\sum_{k=1}^{n} \cos^{-1}\left(\frac{k-1}{n} \right) \frac{1}{n}$; right: $\sum_{k=1}^{n} \cos^{-1}\left(\frac{k}{n} \right) \frac{1}{n}$

b.

n	Left Riemann sum	Right Riemann sum
20	1.03619	0.95765
50	1.01491	0.983494
100	1.00757	0.99186

Estimate: 1

75. a. $\sum_{k=1}^{n} 2\sqrt{1 + \left(k - \frac{1}{2}\right)\frac{3}{n}} \cdot \frac{3}{n}$

b.

n	Midpoint Riemann sum
20	9.33380
50	9.33341
100	9.33335

Estimate: $\frac{28}{3}$

77. a. $\frac{4}{n}\sum_{k=1}^{n}\left(4\left(k - \frac{1}{2}\right)\frac{4}{n} - \left(\left(k - \frac{1}{2}\right)\frac{4}{n}\right)^2\right)$

b.

n	Midpoint Riemann sum
20	10.6800
50	10.6688
100	10.6672

Estimate: $\frac{32}{3}$

79. 6 **81.** 104 **83.** 18 **85.** 2 **87.** $25\pi/2$ **89.** 25 **91.** 35
95. For any such partition on $[0, 1]$, the grid points are $x_k = k/n$, for $k = 0, 1, \ldots, n$. That is, x_k is rational for each k so that $f(x_k) = 1$, for $k = 0, 1, \ldots, n$. Therefore, the left, right, and midpoint Riemann sums are $\sum_{k=1}^{n} 1 \cdot (1/n) = 1$.

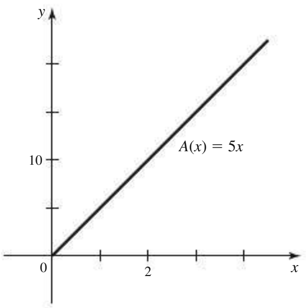
Section 5.3 Exercises, pp. 377–381

1. A is an antiderivative of f; $A'(x) = f(x)$.

3. $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f.

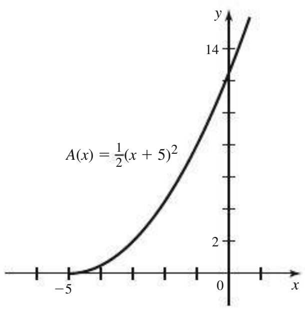
5. Increasing **7.** The derivative of the integral of f is f, or $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$. **9.** $f(x), 0$ **11.** 16 **13. a.** 0 **b.** -9 **c.** 25 **d.** 0 **e.** 16

15. a. $A(x) = 5x$ **b.** $A'(x) = 5$



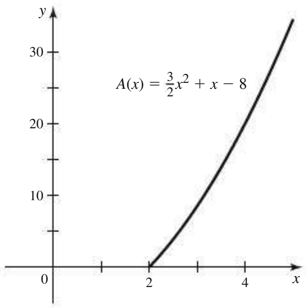
17. a. $A(2) = 2, A(4) = 8; A(x) = \frac{1}{2}x^2$ **b.** $F(4) = 6, F(6) = 16$; $F(x) = \frac{1}{2}x^2 - 2$ **c.** $A(x) - F(x) = \frac{1}{2}x^2 - \left(\frac{1}{2}x^2 - 2\right) = 2$

19. a.



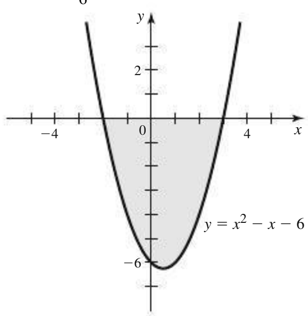
b. $A'(x) = \left(\frac{1}{2}(x + 5)^2\right)' = x + 5 = f(x)$

21. a.

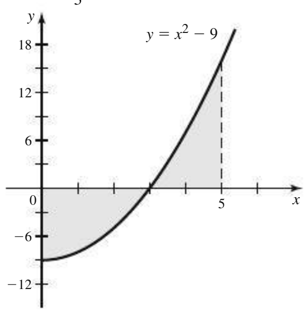


b. $A'(x) = \left(\frac{3}{2}x^2 + x - 8\right)' = 3x + 1 = f(x)$ **23.** $\frac{7}{3}$

25. $-\frac{125}{6}$



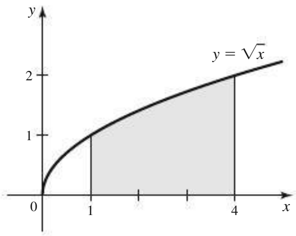
27. $-\frac{10}{3}$



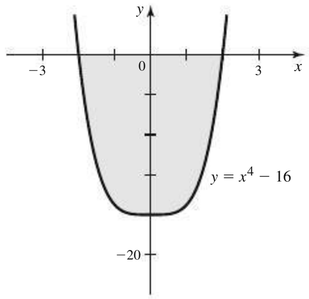
29. 16 **31.** 90 **33.** $\frac{7}{6}$ **35.** 8 **37.** $-\frac{32}{3}$ **39.** $-\frac{5}{2}$ **41.** $\frac{9}{2}$ **43.** $-\frac{3}{8}$
45. 1 **47.** $3 \ln 2$ **49.** $\frac{45}{4}$ **51.** $\frac{2}{3}$ **53.** 1 **55.** 2 **57.** $\frac{\pi}{12}$

59. $\frac{3}{2} + 4 \ln 2$ **61.** $\frac{3\pi}{2} - 1$

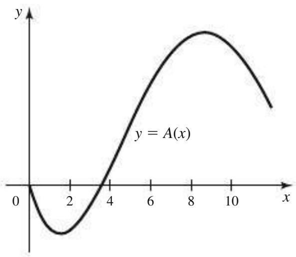
63. (i) $\frac{14}{3}$ **(ii)** $\frac{14}{3}$



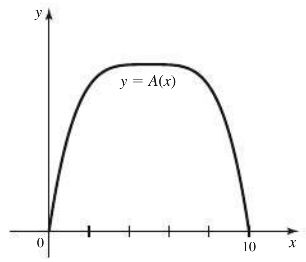
65. (i) -51.2 **(ii)** 51.2



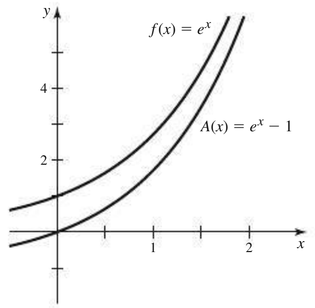
67. $\frac{94}{3}$ **69.** $\ln 2$ **71.** 2 **73.** $x^2 + x + 1$ **75.** $-\sqrt{x^4 + 1}$
77. $3/x^4$ **79.** $-(\cos^4 x + 6) \sin x$ **81.** $-\frac{\cos z}{\sin^4 z + 1}$
83. $\frac{9}{t}$ **85.** $2\sqrt{1 + x^2}$ **87.** a–C, b–B, c–D, d–A
89. a. $x = 0, x \approx 3.5$ **b.** Local min at $x \approx 1.5$; local max at $x \approx 8.5$ **c.**



91. a. $x = 0, 10$ **b.** Local max at $x = 5$
c.

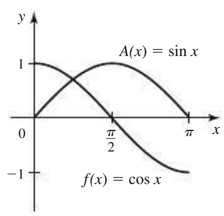


93. $-\pi, -\pi + \frac{9}{2}, -\pi + 9, 5 - \pi$
95. a. $A(x) = e^x - 1$
b.



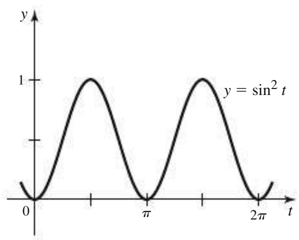
c. $A(\ln 2) = 1; A(\ln 4) = 3$

97. a. $A(x) = \sin x$
b.

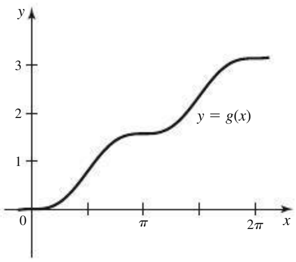


c. $A\left(\frac{\pi}{2}\right) = 1; A(\pi) = 0$

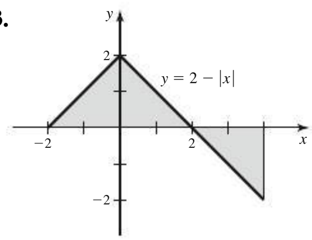
99. Critical pts. $x = 0, 3$, and 4; increasing on $(-\infty, 0)$, $(0, 3)$, and $(4, \infty)$; decreasing on $(3, 4)$
101. a.



b. $g'(x) = \sin^2 x$
c.

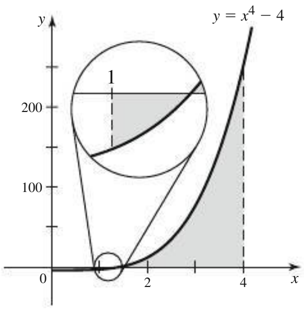


103.



Area $= 6$

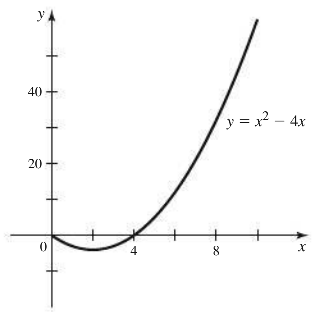
105.



Area ≈ 194.05

107. a. True **b.** True **c.** False **d.** True
109. 3

111. a.

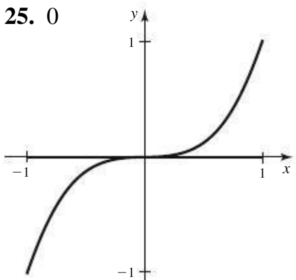


b. $b = 6$ **c.** $b = \frac{3a}{2}$

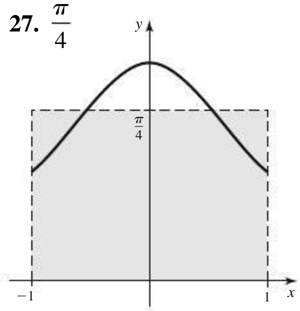
113. $f(x) = -2 \sin x + 3$ **115.** $\pi/2 \approx 1.57$
117. $(S'(x))^2 + \left(\frac{S''(x)}{2x}\right)^2 = (\sin x^2)^2 + \left(\frac{2x \cos x^2}{2x}\right)^2 = \sin^2 x^2 + \cos^2 x^2 = 1$
119. c. The summation relationship is a discrete analog of the Fundamental Theorem. Summing the difference quotient and integrating the derivative over the relevant interval give the difference of the function values at the endpoints.

Section 5.4 Exercises, pp. 385–387

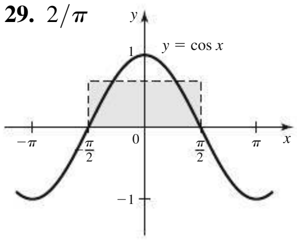
1. If f is odd, the regions between f and the positive x-axis and between f and the negative x-axis are reflections of each other through the origin. Therefore, on $[-a, a]$, the areas cancel each other.
3. a. 9 **b.** 0 **5.** $3x^3$ and x are odd functions. **7.** Even; even
9. If f is continuous on $[a, b]$, then there is a c in (a, b) such that $f(c) = \frac{1}{b - a}\int_a^b f(x)\,dx$. **11.** 0 **13.** $\frac{1000}{3}$ **15.** $\frac{16}{3}$ **17.** $-\frac{88}{3}$
19. 0 **21.** 2 **23.** 0
25. 0



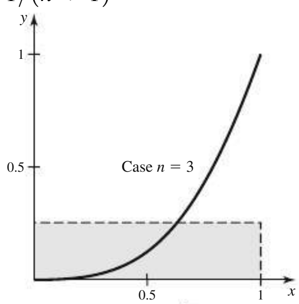
27. $\frac{\pi}{4}$



29. $2/\pi$



31. $1/(n + 1)$



33. 2000 **35.** 21 m/s **37.** $20/\pi$ **39.** 2 **41.** $a/\sqrt{3}$
43. $c = \pm\frac{1}{2}$ **45. a.** True **b.** True **c.** True **d.** False
47. 420 ft **49.** $f(g(-x)) = f(g(x)) \Rightarrow$ the integrand is even; $\int_{-a}^{a} f(g(x))\,dx = 2\int_0^a f(g(x))\,dx$ **51.** $p(g(-x)) = p(g(x)) \Rightarrow$ the integrand is even; $\int_{-a}^{a} p(g(x))\,dx = 2\int_0^a p(g(x))\,dx$
53. a. $a/6$ **b.** $(3 \pm \sqrt{3})/6$, independent of a
57.

Even	Even
Even	Odd

Section 5.5 Exercises, pp. 395–398

1. The Chain Rule **3.** $u = g(x)$ **5.** The lower bound a becomes $g(a)$ and the upper bound b becomes $g(b)$. **7.** $\frac{(x^2+1)^5}{5} + C$

9. $\frac{1}{4}\sin^4 x + C$ **11.** $\frac{(x+1)^{13}}{13} + C$ **13.** $\frac{(2x+1)^{3/2}}{3} + C$

15. a. $\frac{1}{10}e^{10x} + C$ **b.** $\frac{1}{5}\sec 5x + C$ **c.** $-\frac{1}{7}\cos 7x + C$

d. $7\sin\frac{x}{7} + C$ **e.** $\frac{1}{27}\tan^{-1}\frac{x}{3} + C$ **f.** $\sin^{-1}\frac{x}{6} + C$

17. $\frac{(x^2-1)^{100}}{100} + C$ **19.** $-\frac{(1-4x^3)^{1/2}}{3} + C$ **21.** $\frac{(x^2+x)^{11}}{11} + C$

23. $\frac{(x^4+16)^7}{28} + C$ **25.** $\frac{1}{2}\sin^{-1}\frac{x}{3} + C$ **27.** $\frac{4^{x^3}}{\ln 2} + C$

29. $\frac{(x^6-3x^2)^5}{30} + C$ **31.** $\frac{3}{5}\sin^{-1}5x + C$ **33.** $\frac{1}{6}\tan^{-1}\frac{e^w}{6} + C$

35. $-\frac{1}{2}\csc x^2 + C$ **37.** $\frac{1}{10}\tan(10x+7) + C$ **39.** $\frac{10^{4t+1}}{4\ln 10} + C$

41. $\frac{1}{2}\tan^2 x + C$ **43.** $\frac{1}{7}\sec^7 x + C$ **45.** $\frac{\sqrt{2}}{4}$ **47.** $\frac{7}{2}$ **49.** 1 **51.** $\frac{1}{3}$

53. $\frac{2-\sqrt{2}}{2}$ **55.** $(e^9-1)/3$ **57.** $\sqrt{2}-1$ **59.** $\frac{\pi}{6}$ **61.** $\frac{1}{2}\ln 17$

63. $\frac{\pi}{9}$ **65.** $\frac{1}{3}$ **67.** $\frac{3}{4}(4-3^{2/3})$ **69.** $\frac{32}{3}$ **71.** $-\ln 3$ **73.** $\frac{1}{7}$

75. 10 m/s **77. a.** 160 **b.** $\frac{4800}{49} \approx 98$ **c.** $\Delta p = \int_0^T \frac{200}{(t+1)^r}\,dt$; decreases as r increases **d.** $r \approx 1.28$ **e.** As $t \to \infty$, the population approaches 100. **79.** $\frac{2}{3}(x-4)^{1/2}(x+8) + C$

81. $\frac{3}{5}(x+4)^{2/3}(x-6) + C$ **83.** $\frac{3}{112}(2x+1)^{4/3}(8x-3) + C$

85. $\frac{(x+10)^{10}(x-1)}{11} + C$ **87.** π

89. $\frac{\theta}{2} - \frac{1}{4}\sin\left(\frac{6\theta+\pi}{3}\right) + C$ **91.** $\frac{\pi}{4}$ **93.** $\ln\frac{9}{8}$ **95. a.** True **b.** True **c.** False **d.** False **e.** False **97.** 1 **99.** $\frac{2}{3}$; constant

101. a. π/p **b.** 0 **103.** $2/\pi$ **105.** One area is $\int_4^9 \frac{(\sqrt{x}-1)^2}{2\sqrt{x}}\,dx$. Changing variables by letting $u = \sqrt{x} - 1$ yields $\int_1^2 u^2\,du$, which is the other area. **107.** $7297/12$ **109.** $\frac{2}{15}(3-2a)(1+a)^{3/2} + \frac{4}{15}a^{5/2}$

111. $\frac{1}{3}\sec^3\theta + C$ **113. a.** $I = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

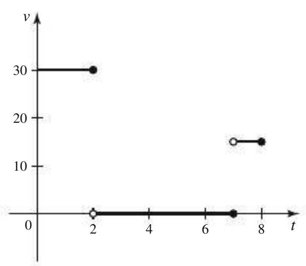
b. $I = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

117. $\frac{4}{3}(-2+\sqrt{1+x})\sqrt{1+\sqrt{1+x}} + C$ **119.** $-4+\sqrt{17}$

Chapter 5 Review Exercises, pp. 398–402

1. a. True **b.** False **c.** True **d.** True **e.** False **f.** True **g.** True

3. a.



b. 75 **c.** The area is the distance the diver ascends.

5. 9.34; 10.28; 9.82

7.

n	Midpoint Riemann sum
10	114.167
30	114.022
60	114.006

$\int_1^{25}\sqrt{2x-1}\,dx = 114$

9. a. $1((3\cdot 2-2)+(3\cdot 3-2)+(3\cdot 4-2)) = 21$

b. $\sum_{k=1}^{n}\frac{3}{n}\left(3\left(1+\frac{3k}{n}\right)-2\right)$ **c.** $\frac{33}{2}$ **11.** $-\frac{16}{3}$ **13.** 56

15. a. 20 **b.** 0 **c.** 80 **d.** 10 **e.** 0 **17.** 18 **19.** 10

21. Not enough information **23. a.** 8.5 **b.** -4.5 **c.** 0 **d.** 11.5

25. 4π **27.** A: $\int_0^x f(t)\,dt$; B: $f(x)$; C: $f'(x)$ **29.** $\sqrt{1+x^4+x^6}$

31. $-\sin x^6$ **33.** $\frac{2}{x^{10}+1}$ **35.** Increasing on $(3, 6)$; decreasing on $(-\infty, 3)$ and $(6, \infty)$ **39.** $\frac{212}{5}$ **41.** $x^9 - x^7 + C$

43. $\frac{7}{6}$ **45.** $\frac{4}{\sqrt{3}}$ **47.** $\frac{\pi}{12}$ **49.** $-\frac{4}{3\sin^{3/4}x} + C$

51. $\frac{1}{3}\sin x^3 + C$ **53.** $\frac{1}{28}\tan^{-1}\left(\frac{\sin 7w}{4}\right) + C$ **55.** $\frac{1}{\ln 2}$

57. 78 **59.** $\frac{5}{6}e^2(e^3-1)$ **61.** $e^{e^x} + C$ **63.** $\frac{1}{2}\sin^{-1}2x + C$

65. $\pi + \frac{3\sqrt{3}}{4}$ **67.** $\frac{\pi}{2}$ **69.** $\frac{1}{3}\ln\frac{9}{2}$ **71.** 0 **73.** $\cos\frac{1}{x} + C$

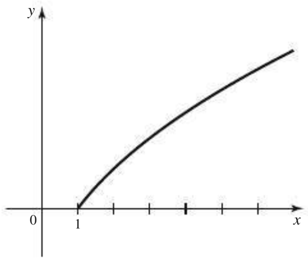
75. $\ln|\tan^{-1}x| + C$ **77.** $(x+3)^{11}\left(\frac{11x-3}{132}\right) + C$

79. 1 **81.** $\frac{\pi}{12}$ **83.** 0 **85.** 48 **87.** $\frac{256}{3}$ **89.** 8 **91.** $-\frac{4}{15}$; $\frac{4}{15}$

93. Approx. 431.5 ft **95.** Displacement = 0; distance = $20/\pi$

97. $\frac{3}{2\ln 2}$ **99. a.** $5/2$, $c = 3.5$ **b.** 3, $c = 3$ and $c = 5$ **101.** 24

103. $f(1) = 0$; $f'(x) > 0$ on $[1, \infty)$; $f''(x) < 0$ on $[1, \infty)$



105. a. $\frac{3}{2}, \frac{5}{6}$ **b.** x **c.** $\frac{1}{2}x^2$ **d.** $-1, \frac{1}{2}$ **e.** 1, 1 **f.** $\frac{3}{2}$ **107.** e^4

113. a. Increasing on $(-\infty, 1)$ and $(2, \infty)$; decreasing on $(1, 2)$

b. Concave up on $\left(\frac{13}{8}, \infty\right)$; concave down on $\left(-\infty, \frac{13}{8}\right)$

c. Local max at $x = 1$; local min at $x = 2$ **d.** Inflection point at $x = \frac{13}{8}$ **115.** Differentiating the first equation gives the second equation; no. **117.** $\sqrt[4]{12}$

CHAPTER 6

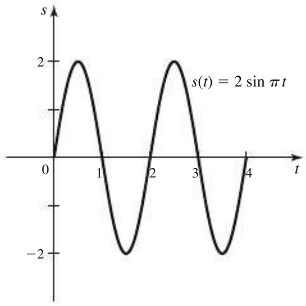
Section 6.1 Exercises, pp. 410–416

1. The position $s(t)$ is the location of the object relative to the origin. The displacement is the change in position between time $t = a$ and $t = b$. The distance traveled between $t = a$ and $t = b$ is $\int_a^b |v(t)|\, dt$, where $v(t)$ is the velocity at time t. **3.** The displacement between $t = a$ and $t = b$ is $\int_a^b v(t)\, dt$. **5.** $Q(t) = Q(0) + \int_0^t Q'(x)\, dx$
7. a. $[0, 1), (3, 5)$ **b.** -4 mi **c.** 26 mi **d.** 6 mi **e.** 6 mi on the positive side of the initial position **9. a.** 3 **b.** $\frac{13}{3}$ **c.** 3

d. $s(t) = \begin{cases} -\dfrac{t^2}{2} + 2t & \text{if } 0 \le t \le 3 \\ \dfrac{3t^2}{2} - 10t + 18 & \text{if } 3 < t \le 4 \\ -t^2 + 10t - 22 & \text{if } 4 < t \le 5 \end{cases}$

11. a. 3 m **b.** 3 m; 0 m; 3 m; 0 m **c.** 12 m
13. a. Positive direction for $2 < t \le 3$; negative direction for $0 < t < 2$ **b.** 0 m **c.** 8 m **15. a.** Positive direction for $0 \le t < 2$ and $4 < t \le 5$; negative direction for $2 < t < 4$
b. 20 m **c.** 28 m **17. a.** $s(t) = 2 - \cos t$ **19. a.** $s(t) = 6t - t^2$
21. a. $s(t) = 9t - \dfrac{t^3}{3} - 2$ **23. a.** $s(t) = 2 \sin \pi t$

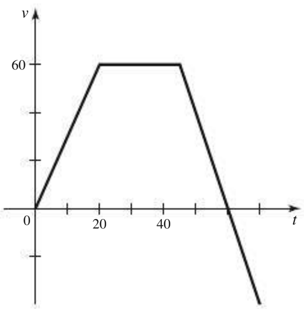
b.



c. $\frac{3}{2}, \frac{7}{2}, \frac{11}{2}$
d. $\frac{1}{2}, \frac{5}{2}, \frac{9}{2}$

25. a. $s(t) = 10t(48 - t^2)$ **b.** 880 mi **c.** $\dfrac{2720\sqrt{6}}{9} \approx 740.29$ mi

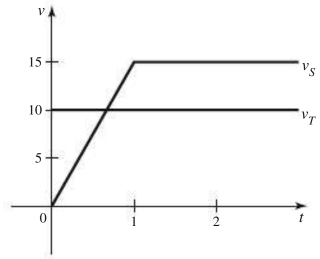
27.



a. Velocity is a maximum for $20 \le t \le 45$; $v = 0$ at $t = 0$ and $t = 60$ **b.** 1200 m **c.** 2550 m **d.** 2100 m

29. $v(t) = -32t + 70$; $s(t) = -16t^2 + 70t + 10$
31. $v(t) = -9.8t + 20$; $s(t) = -4.9t^2 + 20t$
33. $v(t) = -\frac{1}{200}t^2 + 10$; $s(t) = -\frac{1}{600}t^3 + 10t$
35. $v(t) = \frac{1}{2}\sin 2t + 5$; $s(t) = -\frac{1}{4}\cos 2t + 5t + \frac{29}{4}$
37. a. $s(t) = 44t^2$ **b.** 704 ft **c.** $\sqrt{30} \approx 5.477$ s

d. $\dfrac{5\sqrt{33}}{11} \approx 2.611$ s **e.** Approx. 180.023 ft
39. 6.154 mi; 1.465 mi **41. a.** 2639 people
b. $P(t) = 250 + 20t^{3/2} + 30t$ people **43. a.** 1897 cells; 1900 cells **b.** $N(t) = 1900 - 400e^{-0.25t}$ **45. a.** 27,250 barrels
b. 31,000 barrels **c.** 4000 barrels **47. a.** $\dfrac{10^7(1 - e^{-kt})}{k}$
b. $\dfrac{10^7}{k}$ = total number of barrels of oil extracted if the nation extracts the oil indefinitely, and it has at least $\dfrac{10^7}{k}$ barrels of oil in reserve.
c. $k = \dfrac{1}{200} = 0.005$ **d.** Approx. 138.6 yr
49. a. $\dfrac{120}{\pi} + 40 \approx 78.20$ m^3
b. $Q(t) = 20\left(t + \dfrac{12}{\pi}\sin\left(\dfrac{\pi}{12}t\right)\right)$ **c.** Approx. 122.6 hr
51. a. $V(t) = 5 + \cos\dfrac{\pi t}{2}$ **b.** 15 breaths/min **c.** 2 L, 6 L
53. a. 7200 MWh or 2.592×10^{13} J **b.** 16,000 kg; 5,840,000 kg
c. 450 g; 164,250 g **d.** About 1500 turbines **55. a.** \$96,875
b. \$86,875 **57. a.** \$69,583.33 **b.** \$139,583.33 **59. a.** False
b. True **c.** True **d.** True **61.** $\frac{2}{3}$ **63.** $\frac{25}{3}$
65. a.

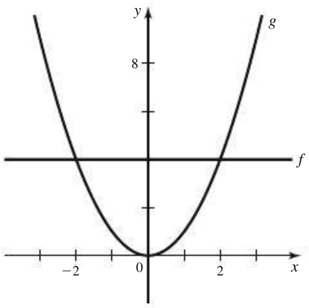


b. Theo **c.** Sasha **d.** Theo hits the 10-mi mark before Sasha; Sasha and Theo hit the 15-mi mark at the same time; Sasha hits the 20-mi mark before Theo. **e.** Sasha **f.** Theo

67. Approx. 11:23 A.M.
69. $\int_a^b f'(x)\, dx = f(b) - f(a) = g(b) - g(a) = \int_a^b g'(x)\, dx$

Section 6.2 Exercises, pp. 420–425

1. $\int_a^b (f(x) - g(x))\, dx + \int_b^c (g(x) - f(x))\, dx$
3.

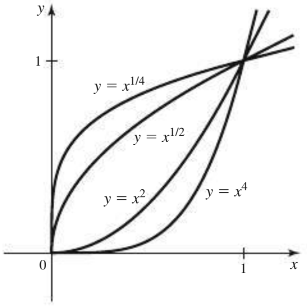


5. a. 1 **b.** 1
7. $\int_0^1 y\, dy + \int_1^2 (2 - y)\, dy$
9. $\dfrac{9}{2}$ **11.** $3\pi - 2$
13. $\dfrac{5}{2} - \dfrac{1}{\ln 2}$
15. $2 - \sqrt{2}$
17. $\dfrac{4}{3}$ **19.** $\dfrac{32}{3}$

21. 9 **23.** $\dfrac{81}{2}$ **25.** 2 **27.** $\dfrac{8\sqrt{2} - 7}{6}$ **29.** $\dfrac{125}{2}$
31. a. $\int_0^1 (\sqrt{x} - x^3)\, dx$ **b.** $\int_0^1 (\sqrt[3]{y} - y^2)\, dy$
33. $\frac{19}{6} \approx 3.17$ km; the faster runner jogged approximately 3.17 km farther than the slower runner. **35. a.** 7 **b.** 4 **37.** 25 **39.** $\frac{81}{32}$
41. $\pi - 2$ **43.** $\frac{1}{2} + \ln 2$ **45.** $\frac{7}{3}$ **47.** 3 **49.** $\frac{64}{5}$ **51.** $\ln 2$
53. $\frac{5}{24}$ **55.** $\frac{63}{4}$ **57.** $\frac{9}{2}$ **59.** $\frac{32}{3}$ **61.** $\frac{15}{8} - 2\ln 2$ **63.** $\frac{17}{3}$
65. a. False **b.** False **c.** True **67.** $\frac{4}{9}$ **69.** $\dfrac{n - 1}{2(n + 1)}$
71. $A_n = \dfrac{n - 1}{n + 1}$; $\lim_{n\to\infty} A_n = 1$; the region approximates a square with side length of 1. **73.** $k = 1 - \dfrac{1}{\sqrt{2}}$ **75. a.** The lowest p% of households own exactly p% of the wealth for $0 \le p \le 100$.

b. The function must be one-to-one and its graph must lie below $y = x$ because the poorest $p\%$ cannot own more than $p\%$ of the wealth. **c.** $p = 1.1$ is most equitable; $p = 4$ is least equitable.
e. $G(p) = \dfrac{p-1}{p+1}$ **f.** $0 \le G \le 1$ for $p \ge 1$ **g.** $\frac{5}{18}$

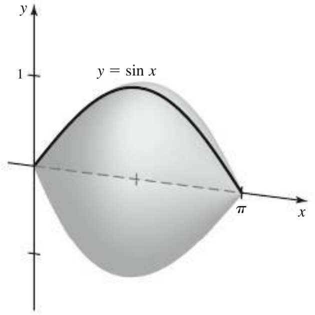
77. a.



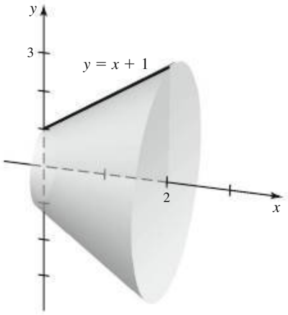
b. $A_n(x)$ is the net area of the region between the graphs of f and g from 0 to x. **c.** $x = n^{n/(n^2-1)}$; the root decreases with n.

Section 6.3 Exercises, pp. 434–439

1. $A(x)$ is the area of the cross section through the solid at the point x.
3. a. $3 - x$ **b.** $\int_0^2 (3 - x)\,dx$ **5. a.** $\sqrt{\cos x}$ **b.** $\pi \cos x$
c. $\int_0^{\pi/2} \pi \cos x\,dx$ **7. a.** $\sqrt{x} + 1$ **b.** 1 **c.** $\pi((\sqrt{x} + 1)^2 - 1)$
d. $\int_0^4 \pi((\sqrt{x} + 1)^2 - 1)\,dx$ **9. a.** $\sqrt{x}$ **b.** πx **c.** $\int_0^4 \pi x\,dx$
11. $\frac{4}{3}$ **13.** 1 **15.** $\frac{\pi}{3}$ **17.** 36π **19.** $\frac{15\pi}{32}$ **21.** $\frac{\pi^2}{2}$ **23.** $\frac{32\pi}{3}$
25. $\frac{5\pi}{6}$ **27.** $\frac{2\pi}{5}$ **29.** $\frac{\pi^2}{4}$ **31.** $\frac{\pi^2}{2}$ **33.** $\frac{\pi(\pi - 2)}{8}$
35. $\frac{4\pi - \pi^2}{4}$ **37.** $\frac{128\pi}{5}$ **39.** $\pi \ln 3$ **41.** $\frac{\pi}{2}(e^4 - 1)$
43. $\frac{49\pi}{2}$ **45.** Volumes are equal. **47.** x-axis **49.** $\frac{\pi}{2}$
51. $\pi\sqrt{3}$ **53.** $\frac{\pi}{6}$ **55.** $2\pi(8 + \pi)$ **57.** $(6\sqrt{3} - 2\pi)\pi$
59. 4π **61. a.** False **b.** True **c.** True
63. Volume $(S) = 8\pi a^{5/2}/15$; volume $(T) = \pi a^{5/2}/3$
65. a.



b.



67. Left: 166π; right: 309π; midpoint: 219π **69. a.** $\frac{1}{3}V_C$ **b.** $\frac{2}{3}V_C$
71. $24\pi^2$ **73. b.** $V = \pi r^2 h$

Section 6.4 Exercises, pp. 447–451

1. $\int_a^b 2\pi x(f(x) - g(x))\,dx$ **3.** x; y **5. a.** x **b.** $2 - x^2 - x$
c. $\int_0^1 2\pi x(2 - x^2 - x)\,dx$ **7. a.** $2 - y$ **b.** $4 - (2 - y)^2 = 4y - y^2$
c. $\int_0^2 2\pi(2 - y)(4y - y^2)\,dy$ **9.** $\frac{\pi}{6}$ **11.** π **13.** 8π **15.** $\frac{32\pi}{3}$
17. π **19.** $\frac{\pi}{2}$ **21.** $\frac{81\pi}{2}$ **23.** $\frac{2\pi}{3}$ **25.** $\frac{3\pi}{10}$ **27.** 90π
29. $2\pi e(e - 1)$ **31.** π **33.** $\frac{\pi}{5}$ **35.** $\frac{4\pi}{15}$ **37.** 500π
39. $\frac{11\pi}{6}$ **41.** $\frac{5\pi}{6}$ **43.** $\frac{23\pi}{15}$ **45.** $\frac{52\pi}{15}$ **47.** $\frac{36\pi}{5}$
51. a. $4\pi\int_1^5 x\sqrt{4 - (x - 3)^2}\,dx$ **b.** $12\pi\int_{-2}^2 \sqrt{4 - y^2}\,dy$
c. $24\pi^2$ **53.** $\frac{\pi}{9}$ **55.** $\frac{16\pi}{3}$ **57.** $\frac{608\pi}{3}$ **59.** $\pi(\sqrt{e} - 1)^2$
61. $\frac{5\pi}{6}$ **63. a.** True **b.** False **c.** True **65.** 24π **67.** 54π
69. a. $V_1 = \frac{\pi}{15}(3a^2 + 10a + 15)$; $V_2 = \frac{\pi}{2}(a + 2)$
b. $V(S_1) = V(S_2)$, for $a = 0$ and $a = -\frac{5}{6}$ **71.** 10π
73. a. $27\sqrt{3}\pi r^3/8$ **b.** $54\sqrt{2}/(3 + \sqrt{2})^3$ **c.** $500\pi/3$

Section 6.5 Exercises, pp. 455–457

1. Determine whether f has a continuous derivative on $[a, b]$. If so, calculate $f'(x)$ and evaluate the integral $\int_a^b \sqrt{1 + f'(x)^2}\,dx$.
3. $\int_{-2}^5 \sqrt{1 + 9x^4}\,dx$ **5.** $\int_0^2 \sqrt{1 + 4e^{-4x}}\,dx$ **7.** $4\sqrt{5}$
9. $8\sqrt{65}$ **11.** 168 **13.** $\frac{4}{3}$ **15.** $\frac{123}{32}$ **17.** $\frac{123}{32}$ **19.** $7\sqrt{5}$
21. a. $\int_{-1}^1 \sqrt{1 + 4x^2}\,dx$ **b.** 2.96 **23. a.** $\int_1^4 \sqrt{1 + \frac{1}{x^2}}\,dx$ **b.** 3.34
25. a. $\int_3^4 \sqrt{\frac{4y - 7}{4y - 8}}\,dy$ **b.** 1.08 **27. a.** $\int_0^\pi \sqrt{1 + 4\sin^2 2x}\,dx$
b. 5.27 **29. a.** $\int_1^{10} \sqrt{1 + 1/x^4}\,dx$ **b.** 9.15
31. Approx. 1326 m **33. a.** False **b.** True **c.** False
35. a. $f(x) = \pm 4x^3/3 + C$ **b.** $f(x) = \pm 3\sin 2x + C$
37. $y = 1 - x^2$ **39. a.** $L/2$ **b.** L/c

Section 6.6 Exercises, pp. 463–465

1. 15π **3.** Evaluate $\int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2}\,dx$ **5. a.** $4\sqrt{2}\pi$
7. $156\sqrt{10}\pi$ **9.** $\frac{2912\pi}{3}$ **11.** $\frac{\pi}{9}(17^{3/2} - 1)$ **13.** 2π
15. $15\sqrt{17}\,\pi$ **17.** $\frac{\pi}{8}(16 + e^8 - e^{-8})$ **19.** 96π
21. $\frac{9\pi}{125}\,\text{m}^3$ **23. a.** False **b.** False **c.** True **d.** False
25. a. $\int_0^{\pi/2} 2\pi(\cos x)\sqrt{1 + \sin^2 x}\,dx$ **b.** Approx. 7.21
27. a. $\int_0^{\pi/4} 2\pi(\tan x)\sqrt{1 + \sec^4 x}\,dx$ **b.** Approx. 3.84
29. $\frac{12\pi a^2}{5}$ **31.** $\frac{53\pi}{9}$ **33.** $\frac{275\pi}{32}$ **35.** $\frac{48{,}143\pi}{48}$ **39. a.** $\frac{6}{a}$ **b.** $\frac{3}{a}$
c. $\frac{3}{2a} + \frac{3a}{2\sqrt{a^2 - 1}}\sin^{-1}\left(\frac{\sqrt{a^2 - 1}}{a}\right)$ **d.** The sphere **e.** A sphere
41. a. $c^2 A$ **b.** A

Section 6.7 Exercises, pp. 473–477

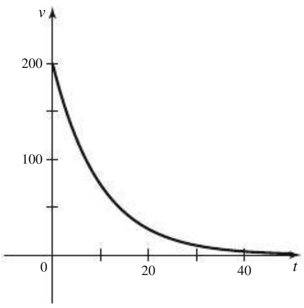
1. 150 g **3.** 25 J **5.** Horizontal cross sections of water at various locations in the tank are lifted different distances. **7.** $39{,}200\ \text{N/m}^2$
9. $\int_5^{10} 25\pi\rho g(15 - y)\,dy$ **11.** $\int_0^{10} 25\pi\rho g(10 - y)\,dy$ **13.** $\pi + 2$
15. 3 **17.** $(2\sqrt{2} - 1)/3$ **19.** 10 **21.** 9 J **23. a.** $k = 150$
b. 12 J **c.** 6.75 J **d.** 9 J **25. a.** 112.5 J **b.** 12.5 J

27. a. 31.25 J **b.** 312.5 J **29. a.** 625 J **b.** 391 J **31. a.** 22,050 J **b.** 36,750 J **33.** 3675 J **35.** 1.15×10^7 J **37** 3.94×10^6 J **39. a.** $66{,}150\pi$ J **b.** No **41. a.** 2.10×10^8 J **b.** 3.78×10^8 J **43. a.** 32,667 J **b.** Yes **45.** 7.70×10^3 J **47.** 1.47×10^7 N **49.** 2.94×10^7 N **51.** 6533 N **53.** 6737.5 N **55.** 8×10^5 N **57. a.** True **b.** True **c.** True **d.** False **59. a.** Compared to a linear spring, $F(x) = 16x$, the restoring force is less for large displacements. **b.** 17.87 J **c.** 31.6 J **61.** 1,381,800 J **63.** 0.28 J **65. a.** Yes **b.** 4.296 m **67.** Left: 16,730 N; right: 14,700 N **69. a.** 8.87×10^9 J **b.** $500\, GMx/(R(x+R)) = (2 \times 10^{17})x/(R(x+R))$ J **c.** GMm/R **d.** $v = \sqrt{2GM/R}$

Chapter 6 Review Exercises, pp. 478–482

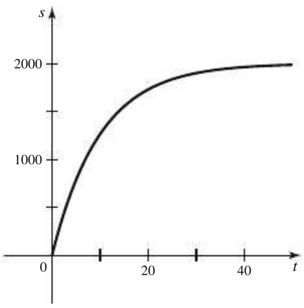
1. a. True **b.** True **c.** True **3. a.** Positive direction for $0 \le t < \frac{1}{2}$ and $2 < t \le 3$; negative direction for $\frac{1}{2} < t < 2$ **b.** 9 m **c.** 22.5 m **d.** $s(t) = 4t^3 - 15t^2 + 12t + 1$ **5.** $s(t) = 20t - 5t^2$; displacement $= 20t - 5t^2$; $D(t) = \begin{cases} 20t - 5t^2 & \text{if } 0 \le t < 2 \\ 5t^2 - 20t + 40 & \text{if } 2 \le t \le 4 \end{cases}$ **7. a.** $v(t) = -\frac{8}{\pi}\cos\frac{\pi t}{4}$; $s(t) = -\frac{32}{\pi^2}\sin\frac{\pi t}{4}$ **b.** Min value $= -\frac{32}{\pi^2}$; max value $= \frac{32}{\pi^2}$ **c.** 0; 0 **9. a.** $R(t) = 3t^{4/3}$ **b.** $R(t) = \begin{cases} 3t^{4/3} & \text{if } 0 \le t \le 8 \\ 2t + 32 & \text{if } t > 8 \end{cases}$ **c.** $t = 59$ min

11. a.

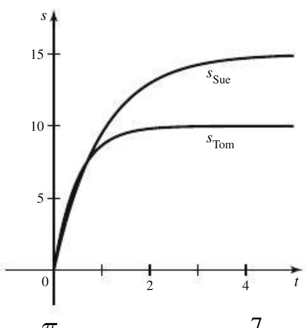


b. $10 \ln 4 \approx 13.86$ s

c. $s(t) = 2000(1 - e^{-t/10})$ **d.** No



13. a. $s_{\text{Tom}}(t) = -10e^{-2t} + 10$
$s_{\text{Sue}}(t) = -15e^{-t} + 15$



b. $t = 0$ and $t = \ln 2$ **c.** Sue **15.** $1 - \frac{\pi}{4}$ **17.** $e - 2$ **19.** $\frac{7}{3}$

21. 8 **23.** 1 **25.** $\frac{1}{3}$ **27.** R_1: $\frac{7}{6}$; R_2: $\frac{10}{3}$; R_3: $4\sqrt{3} - \frac{10}{3}$ **29.** $\frac{11\pi}{15}$ **31.** $\frac{14\pi}{3}$ **33.** $\int_1^3 2\pi(3 - x)(2\sqrt{x} - 3 + x)\,dx$ **35.** $\frac{7}{3}$ **37.** $\frac{31\pi}{5}$ **39.** R_1: $\sqrt{3}$; R_2: $\frac{4\pi}{3} - \sqrt{3}$ **41.** $\frac{1}{3}$ **43.** $\frac{5}{6}$ **45.** $\frac{8}{15}$ **47.** $\frac{8\pi}{5}$ **49.** $\pi(e-1)^2$ **51.** π **53.** $\frac{512\pi}{15}$ **55.** About $y = -2$: 80π; about $x = -2$: 112π **57.** $c = 5$ **59.** 1 **61.** $2\sqrt{3} - \frac{4}{3}$ **63.** $\int_2^4 \sqrt{4x^2 + 8x + 5}\,dx \approx 16.127$ **65.** $\sqrt{b^2+1} - \sqrt{2} + \ln\left(\frac{(\sqrt{b^2+1} - 1)(1 + \sqrt{2})}{b}\right)$; $b \approx 2.715$ **67. a.** 9π **b.** $\frac{9\pi}{2}$ **69. a.** $\frac{263{,}439\pi}{4096}$ **b.** $\frac{483}{64}$ **c.** $\frac{\pi}{8}(84 + \ln 2)$ **d.** $\frac{264{,}341\pi}{18{,}432}$ **71.** $\left(450 - \frac{450}{e}\right)$ g **73. a.** 562.5 J **b.** 56.25 J **75. a.** 980 J **b.** 627.2 J **77. a.** 1,411,200 J **b.** 940,800 J **79. a.** 1,477,805 J **b.** The work required to pump out the top 3 m of water is 1,015,991 J, and the work required to pump out the bottom 3 m of water is 461,814 J. More work is required to pump out the top 3 m of water. **81.** 4,987,592 J **83.** 5716.7 N **85.** 5.2×10^7 N

CHAPTER 7

Section 7.1 Exercises, pp. 490–492

1. $D = (0, \infty)$, $R = (-\infty, \infty)$ **3.** $\frac{4^x}{\ln 4} + C$ **5.** $e^{x \ln 3}$, $e^{\pi \ln x}$, $e^{(\sin x)(\ln x)}$ **7.** $3(\ln x + 1)$ **9.** $\frac{\cos(\ln x)}{x}$, $x > 0$ **11.** $-\frac{5}{x(\ln 2x)^6}$ **13.** $4^{2x+1}x^{4x}(1 + \ln 2x)$ **15.** $(\ln 2)\,2^{x^2+1}x$ **17.** $2(x+1)^{2x}\left(\frac{x}{x+1} + \ln(x+1)\right)$ **19.** $y^{\sin y}\left(\cos y \ln y + \frac{\sin y}{y}\right)$ **21.** $-20xe^{-10x^2}$ **23.** $x^{2x}(2\ln x + 2)$ **25.** $-(1/x)^x(1 + \ln x)$ **27.** $\left(-\frac{4}{x+4} + \ln\left(\frac{x+4}{x}\right)\right)\left(1 + \frac{4}{x}\right)^x$ **29.** $6(1 - \ln 2)$ **31.** $\frac{3}{8}$ **33.** $\frac{1}{2}\ln(4 + e^{2x}) + C$ **35.** $\frac{1}{\ln 2} - \frac{1}{\ln 3}$ **37.** $4 - \frac{4}{e^2}$ **39.** $2e^{\sqrt{x}} + C$ **41.** $\ln|e^x - e^{-x}| + C$ **43.** $\frac{99}{10 \ln 10}$ **45.** 3 **47.** $\frac{6^{x^3+8}}{3\ln 6} + C$ **49.** $\frac{1}{6}e^{3x^2+1} + C$ **51.** $-\frac{1}{9^x \ln 9} + C$ **53.** $\frac{10^{x^3}}{3\ln 10} + C$ **55.** $\frac{3 \cdot 3^{\ln 2} - 1}{\ln 3}$ **57.** $\frac{32}{3}$ **59.** $\frac{1}{3}\ln\frac{65}{16}$ **61.** $2e^{5+\sqrt{x}} + C$ **63.**

h	$(1 + 2h)^{1/h}$	h	$(1 + 2h)^{1/h}$
10^{-1}	6.1917	-10^{-1}	9.3132
10^{-2}	7.2446	-10^{-2}	7.5404
10^{-3}	7.3743	-10^{-3}	7.4039
10^{-4}	7.3876	-10^{-4}	7.3905
10^{-5}	7.3889	-10^{-5}	7.3892
10^{-6}	7.3890	-10^{-6}	7.3891

$\lim_{h \to 0}(1 + 2h)^{1/h} = e^2$

65.

x	$\frac{2^x - 1}{x}$	x	$\frac{2^x - 1}{x}$
10^{-1}	0.71773	-10^{-1}	0.66967
10^{-2}	0.69556	-10^{-2}	0.69075
10^{-3}	0.69339	-10^{-3}	0.69291
10^{-4}	0.69317	-10^{-4}	0.69312
10^{-5}	0.69315	-10^{-5}	0.69314
10^{-6}	0.69315	-10^{-6}	0.69315

$\lim_{x \to 0} \frac{2^x - 1}{x} = \ln 2$

67. a. True **b.** False **c.** False **d.** False **e.** True

69. $\frac{\ln p}{p - 1}, 0$ **71. a.** No **b.** No

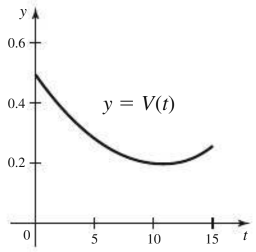
75. $\ln 2 = \int_1^2 \frac{dt}{t} < L_2 = \frac{5}{6} < 1$

$\ln 3 = \int_1^3 \frac{dt}{t} > R_7$

$= 2\left(\frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \frac{1}{15} + \frac{1}{17} + \frac{1}{19} + \frac{1}{21}\right) > 1$

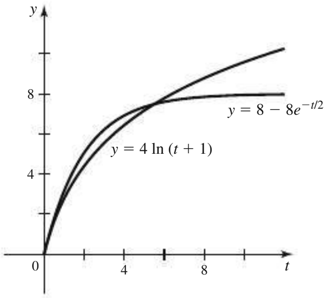
Section 7.2 Exercises, pp. 499–501

1. The relative growth is constant. **3.** The time it takes a function to double in value **5.** $T_2 = (\ln 2)/k$ **7.** $\frac{\ln 2}{20} \approx 0.03466$
9. Compound interest, world population **11.** $\ln 1.11 \approx 0.1044$.
13. $\frac{df}{dt} = 10.5; \frac{dg}{dt} \cdot \frac{1}{g} = \frac{1}{10}$
15. a. $\ln 1.024 \approx 0.02372; y(t) = 90{,}000\, e^{t \ln 1.024}$ **b.** 2028
17. a. $\frac{\ln 1.1}{10} \approx 0.009531; y(t) = 50{,}000\, e^{t \ln 1.1/10}$ **b.** 60,500
19. a. $\ln 1.016 \approx 0.01587; y(t) = 100\, e^{t \ln 1.016}$ **b.** \$126.88
21. 3.71% **23. a.** 88.1 years; 423.4 million
b. 99.4 years; 412.2 million **25.** 28.7 million **27.** 2026
29. $a(t) = 20e^{(t/36) \ln 0.5}$ mg with $t = 0$ at midnight; 15.87 mg; 119.6 hr $\approx$ 5 days **31.** 1.798 million; the downward turn in the population size may be temporary. **33.** 18,928 ft; 125,754 ft
35. 1.055 billion yr **37.** 6.2 hours **39.** 2 dollars **41.** 1044 days
43. a. False **b.** False **c.** True **d.** True **e.** True
45. a. $V_1(t) = 0.495e^{-0.1216t}$ **b.** $V_2(t) = 0.005e^{0.239t}$
c. $V(t) = 0.495e^{-0.1216t} + 0.005e^{0.239t}$
d.



The tumor initially shrinks significantly in size but eventually starts growing again. **e.** 10.9 days; give a second treatment just before the end of the 10th day after the first treatment.

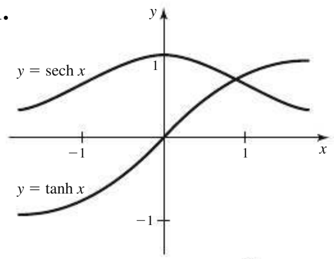
47. a. Bob; Abe **b.** $y = 4 \ln (t + 1)$ and $y = 8 - 8e^{-t/2}$; Bob



49. 10.034%; no **51.** 1.3 s
53. $k = \ln (1 + r); r = 2^{1/T_2} - 1; T_2 = (\ln 2)/k$

Section 7.3 Exercises, pp. 513–517

1. $\cosh x = \frac{e^x + e^{-x}}{2}; \sinh x = \frac{e^x - e^{-x}}{2}$ **3.** $\cosh^2 x - \sinh^2 x = 1$
5. $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ **7.** Evaluate $\sinh^{-1} \frac{1}{5}$.
9. $\int \frac{dx}{16 - x^2} = \frac{1}{4} \coth^{-1} \frac{x}{4} + C$ when $|x| > 4$; the values in the interval of integration $6 \le x \le 8$ satisfy $|x| > 4$.
23. $2 \cosh x \sinh x$ **25.** $2 \tanh x \operatorname{sech}^2 x$ **27.** $-2 \tanh 2x$
29. $2x\,(3x \sinh 3x + \cosh 3x)\cosh 3x$ **31.** $4/\sqrt{16x^2 - 1}$
33. $2v/\sqrt{v^4 + 1}$ **35.** $\sinh^{-1} x$ **37.** $(\sinh 2x)/2 + C$
39. $\ln (1 + \cosh x) + C$ **41.** $x - \tanh x + C$
43. $(\cosh^4 3 - 1)/12 \approx 856$ **45.** $\ln (5/4)$
47. $\frac{1}{2\sqrt{2}} \coth^{-1}\left(\frac{x}{2\sqrt{2}}\right) + C$ **49.** $\tanh^{-1}(e^x/6)/6 + C$
51. $-\operatorname{sech}^{-1}(x^4/2)/8 + C$ **53.** $-\operatorname{csch} z + C$
55. $\ln \sqrt{3} \cdot \ln (4/3) \approx 0.158$ **57.** $\frac{x^2 + 1}{2x} + C$
59. a. The values of $y = \coth x$ are close to 1 on $[5, 10]$.
b. $\ln(\sinh 10) - \ln(\sinh 5) \approx 5.0000454$; $|\text{error}| \approx 0.0000454$
61.



a. $x = \sinh^{-1} 1 = \ln (1 + \sqrt{2})$
b. $\pi/4 - \ln \sqrt{2} \approx 0.44$

63. $\sinh^{-1} 2 = \ln (2 + \sqrt{5})$ **65.** $-(\ln 5)/3 \approx -0.54$
67. $3 \ln\left(\frac{\sqrt{5} + 2}{\sqrt{2} + 1}\right) = 3(\sinh^{-1} 2 - \sinh^{-1} 1)$
69. $\frac{1}{15}\left(17 - \frac{8}{\ln (5/3)}\right) \approx 0.09$
71. a. Sag $= f(50) - f(0) = a(\cosh (50/a) - 1) = 10$; now divide by a. **b.** $t \approx 0.08$ **c.** $a = 10/t \approx 125$; $L = 250 \sinh (2/5) \approx 102.7$ ft **73.** $\lambda \approx 32.81$ m
75. b. When $d/\lambda < 0.05$, $2\pi d/\lambda$ is small. Because $\tanh x \approx x$ for small values of x, $\tanh (2\pi d/\lambda) \approx 2\pi d/\lambda$; therefore,

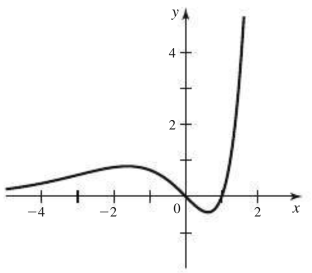
$$v = \sqrt{\frac{g\lambda}{2\pi} \tanh\left(\frac{2\pi d}{\lambda}\right)} \approx \sqrt{\frac{g\lambda}{2\pi} \cdot \frac{2\pi d}{\lambda}} = \sqrt{gd}.$$

c. $v = \sqrt{gd}$ is a function of depth alone; when depth d decreases, v also decreases. **77. a.** False **b.** False **c.** True **d.** False

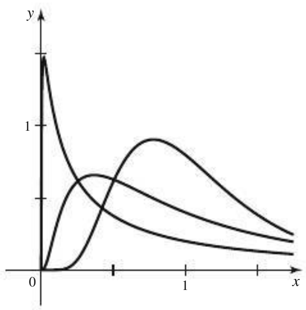
79. a. 1 **b.** 0 **c.** Undefined **d.** 1 **e.** $13/12$ **f.** $40/9$ **g.** $\left(\frac{e^2+1}{2e}\right)^2$ **h.** Undefined **i.** $\ln 4$ **j.** 1 **81.** $x = 0$
83. $x = \pm\tanh^{-1}(1/\sqrt{3}) = \pm\ln(2+\sqrt{3})/2 \approx \pm 0.658$
85. $\tan^{-1}(\sinh 1) - \pi/4 \approx 0.08$ **87.** Applying l'Hôpital's Rule twice brings you back to the initial limit; $\lim_{x\to\infty} \tanh x = 1$.
89. $2/\pi$ **91.** 1 **93.** $12(3\ln(3+\sqrt{8}) - \sqrt{8}) \approx 29.5$
95. a. Approx. 360.8 m **b.** First 100 m: $t \approx 4.72$ s, $v_{av} \approx 21.2$ m/s; second 100 m: $t \approx 2.25$ s, $v_{av} \approx 44.5$ m/s **97. a.** $\sqrt{mg/k}$ **b.** $35\sqrt{3} \approx 60.6$ m/s **c.** $t = \sqrt{\frac{m}{kg}}\tanh^{-1} 0.95 = \frac{\ln 39}{2}\sqrt{\frac{m}{kg}}$
d. Approx. 736.5 m **109.** $\ln(21/4) \approx 1.66$

Chapter 7 Review Exercises, pp. 518–519

1. a. False **b.** False **c.** False **d.** True **3.** $\ln 4$
5. $\frac{1}{2}\ln(x^2+8x+25) + C$
7. $\cosh^{-1}(x/3) + C = \ln(x+\sqrt{x^2-9}) + C$
9. $\tanh^{-1}(1/3)/9 = (\ln 2)/18 \approx 0.0385$
11. $x^{3x^2+1}\left(6x\ln x + 3x + \frac{1}{x}\right)$ **13.** $\sinh^2 t + \cosh^2 t$
15. $3\sinh(6x-2)$ **17.** $-\csc x$ **19.** $\frac{2x}{\sqrt{x^4-1}}$
21. Approx. 7.3 hours **23. a.** $y(t) = 29{,}000e^{(t\ln 2)/2}$
b. Approx. 41,996,486 transistors (which closely approximates the actual number of transistors) **25.** 48.37 yr
27. Local max at $x = -\frac{1}{2}(\sqrt{5}+1)$; local min at $x = \frac{1}{2}(\sqrt{5}-1)$; inflection points at $x = -3$ and $x = 0$; $\lim_{x\to-\infty} f(x) = 0$; $\lim_{x\to\infty} f(x) = \infty$



29. a.



b. $\lim_{x\to 0} f(x) = 0$ **d.** $f(x^*) = \frac{1}{\sqrt{2\pi}}\frac{e^{\sigma^2/2}}{\sigma}$ **e.** $\sigma = 1$

31. $L(x) = \frac{5}{3} + \frac{4}{3}(x - \ln 3)$; $\cosh 1 \approx 1.535$
33. a. $\cosh x$ **b.** $(1 - x\tanh x)\operatorname{sech} x$

CHAPTER 8

Section 8.1 Exercises, pp. 523–525

1. $u = 4 - 7x$ **3.** $\sin^2 x = \frac{1-\cos 2x}{2}$ **5.** Complete the square in $x^2 - 4x - 9$. **7.** $\frac{1}{15(3-5x)^3} + C$ **9.** $\frac{\sqrt{2}}{4}$ **11.** $\frac{1}{2}\ln^2 2x + C$
13. $\ln(e^x+1) + C$ **15.** $\frac{32}{3}$ **17.** $\frac{21}{110}$
19. $\frac{(\ln w - 1)^9}{9} + \frac{(\ln w - 1)^8}{8} + C$
21. $\frac{1}{2}\ln(x^2+4) + \tan^{-1}\frac{x}{2} + C$
23. $-\frac{1}{3}\ln|\csc(3e^x+4) + \cot(3e^x+4)| + C$ **25.** 1
27. $3\sqrt{1-x^2} + 2\sin^{-1} x + C$ **29.** $\ln(\sqrt{2}+1)$
31. $\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right) + C$ **33.** $\frac{x^2}{2} + x + \ln(x^2+x+2) + C$
35. $\frac{3\pi+10}{12}$ **37.** $\sin^{-1}\left(\frac{\theta+3}{6}\right) + C$ **39.** $\tan\theta - \sec\theta + C$
41. $-x - \cot x - \csc x + C$ **43.** $\frac{1}{3}\ln(1+\sinh 3x) + C$
45. $\frac{1}{2}\ln|e^{2x}-2| + C$ **47.** $x - \ln|x+1| + C$
49. $\frac{4}{5}(9+\sqrt{t+1})^{3/2}(\sqrt{t+1}-6) + C$ **51.** $\frac{\ln 4 - \pi}{4}$
53. $\ln|\sec(e^x+1) + \tan(e^x+1)| + C$
55. $\frac{2\sin^3 x}{3} + C$ **57.** $2\tan^{-1}\sqrt{x} + C$
59. $\frac{1}{2}\ln(x^2+6x+13) - \frac{5}{2}\tan^{-1}\left(\frac{x+3}{2}\right) + C$
61. $-\frac{1}{e^x+1} + C$ **63.** $\frac{1}{2}$ **65. a.** False **b.** False **c.** False **d.** False **69. a.** $\frac{\tan^2 x}{2} + C$ **b.** $\frac{\sec^2 x}{2} + C$ **c.** The antiderivatives differ by a constant. **71. a.** $\frac{1}{2}(x+1)^2 - 2(x+1) + \ln|x+1| + C$ **b.** $\frac{x^2}{2} - x + \ln|x+1| + C$ **c.** The antiderivatives differ by a constant. **73.** $\frac{\ln 26}{3}$ **75.** $\frac{2}{3}(5\sqrt{5}-1)\pi$
77. $\pi\left(\frac{9}{2} - \frac{5\sqrt{5}}{6}\right)$ **79.** $\frac{2048 + 1763\sqrt{41}}{9375}$

Section 8.2 Exercises, pp. 529–532

1. Product Rule **3.** $\frac{x^2(2\ln x - 1)}{4} + C$ **5.** Products for which the choice for dv is easily integrated and when the resulting new integral is no more difficult than the original integral
7. $(\tan x + 2)\ln(\tan x + 2) - \tan x + C$
9. $\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$ **11.** $\frac{e^{6t}}{36}(6t-1) + C$
13. $\frac{x^2}{4}(2\ln 10x - 1) + C$ **15.** $(w+2)\sin 2w + \frac{1}{2}\cos 2w + C$
17. $\frac{3^x}{\ln 3}\left(x - \frac{1}{\ln 3}\right) + C$ **19.** $-\frac{1}{9x^9}\left(\ln x + \frac{1}{9}\right) + C$
21. $\frac{1}{8}\sin 2x - \frac{x}{4}\cos 2x + C$ **23.** $\frac{1}{4}(1-2x^2)\cos 2x + \frac{x}{2}\sin 2x + C$
25. $-e^{-t}(t^2+2t+2) + C$ **27.** $\frac{e^x}{2}(\sin x + \cos x) + C$
29. $-\frac{e^{-x}}{17}(\sin 4x + 4\cos 4x) + C$
31. $-e^{2x}\cos e^x + 2e^x\sin e^x + 2\cos e^x + C$ **33.** π **35.** $-\frac{1}{2}$
37. $\frac{1}{9}(5e^6+1)$ **39.** $\frac{\pi-2}{2}$ **41. a.** $x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$

b. $\frac{1}{2}x^2\tan^{-1}x^2 - \frac{1}{4}\ln(1+x^4) + C$ **43.** $\pi(1-\ln 2)$ **45.** π

47. $\frac{2\pi}{27}(13e^6-1)$ **49. a.** False **b.** True **c.** True

51. Let $u = x^n$ and $dv = \cos ax\,dx$. **53.** Let $u = \ln^n x$ and $dv = dx$.

55. $\frac{x^2\sin 5x}{5} + \frac{2x\cos 5x}{25} - \frac{2\sin 5x}{125} + C$

57. $6-2e$ **59. a.** $\frac{2}{3}(x-2)\sqrt{x+1} + C$

61. $\int \log_b x\,dx = \int \frac{\ln x}{\ln b}\,dx = \frac{1}{\ln b}(x\ln x - x) + C$

63. $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

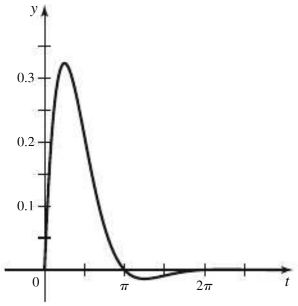
65. Let $u = x$ and $dv = f''(x)\,dx$.

67. $2e^3$ **69.** x-axis: $\pi^2/2$; y-axis: $2\pi^2$ **71.** $\pi(\pi-2)$

75. a. $t = k\pi$, for $k = 0, 1, 2, \ldots$ **b.** $\frac{e^{-\pi}+1}{2\pi}$

c. $(-1)^n\left(\frac{e^{\pi}+1}{2\pi e^{(n+1)\pi}}\right)$

d. $a_n = a_{n-1}\cdot\frac{1}{e^{\pi}}$



77. c. $\int f(x)g(x)dx = f(x)G_1(x) - f'(x)G_2(x) + f''(x)G_3(x) - \int f'''(x)G_3(x)dx$

f and its derivatives		*g* and its integrals
$f(x)$	+	$g(x)$
$f'(x)$	−	$G_1(x)$
$f''(x)$	+	$G_2(x)$
$f'''(x)$	−	$G_3(x)$

d. $\int x^2e^{x/2}dx = 2x^2e^{x/2} - 8xe^{x/2} + 16e^{x/2} + C$

f and its derivatives		*g* and its integrals
x^2	+	$e^{x/2}$
$2x$	−	$2e^{x/2}$
2	+	$4e^{x/2}$
0	−	$8e^{x/2}$

$\frac{d^n}{dx^n}(x^2) = 0$, for $n \ge 3$, so all entries in the left column of the table beyond row three are 0, which results in no additional contribution to the antiderivative. **e.** $x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C$; five rows are needed because $\frac{d^n}{dx^n}(x^3) = 0$, for $n \ge 4$.

f. $\frac{d^k}{dx^k}(p_n(x)) = 0$, for $k \ge n+1$

79. a. $\int e^x\cos x\,dx = e^x\sin x + e^x\cos x - \int e^x\cos x\,dx$

b. $\frac{1}{2}(e^x\sin x + e^x\cos x) + C$ **c.** $-\frac{3}{13}e^{-2x}\cos 3x - \frac{2}{13}e^{-2x}\sin 3x + C$

81. a. $I_1 = -\frac{1}{2}e^{-x^2} + C$ **b.** $I_3 = -\frac{1}{2}e^{-x^2}(x^2+1) + C$

c. $I_5 = -\frac{1}{2}e^{-x^2}(x^4 + 2x^2 + 2) + C$

Section 8.3 Exercises, pp. 536–538

1. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$; $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ **3.** Rewrite $\sin^3 x$ as $(1-\cos^2 x)\sin x$. **5.** A reduction formula expresses an integral with a power in the integrand in terms of another integral with a smaller power in the integrand. **7.** Let $u = \tan x$.

9. $\sin x - \frac{1}{3}\sin^3 x + C$ **11.** $\frac{x}{2} - \frac{1}{12}\sin 6x + C$

13. $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$ **15.** $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

17. $\frac{2}{3}\sin^{3/2}x - \frac{2}{7}\sin^{7/2}x + C$ **19.** $\frac{7}{24}$ **21.** $\frac{8}{45}$

23. $\frac{1}{8}x - \frac{1}{32}\sin 4x + C$ **25.** $\frac{1}{48}\sin^3 2x + \frac{1}{16}x - \frac{1}{64}\sin 4x + C$

27. $\tan x - x + C$ **29.** $-\frac{1}{3}\cot^3 x + \cot x + x + C$

31. $4\tan^5 x - \frac{20}{3}\tan^3 x + 20\tan x - 20x + C$ **33.** $\tan^{10} x + C$

35. $\frac{1}{3}\sec^3 x + C$ **37.** $\frac{1}{3}\tan^3(\ln\theta) + \tan(\ln\theta) + C$ **39.** $\ln 4$

41. $\frac{7}{6}$ **43.** $\frac{1}{8}\tan^2 4x + \frac{1}{4}\ln|\cos 4x| + C$ **45.** $\frac{2}{3}\tan^{3/2}x + C$

47. $\tan x - \cot x + C$ **49.** $\frac{1}{25}$ **51.** $-2\cot x - \frac{\cot^3 x}{3} + C$

53. $\frac{4}{3}$ **55.** $\frac{4}{3} - \ln\sqrt{3}$ **57.** $8\sqrt{2}/3$ **59.** $\sqrt{2}$ **61.** $2\sqrt{2}/3$

63. a. True **b.** False **65.** $\frac{2\pi}{35}$ **67.** $\frac{1}{8}\cos 4x - \frac{1}{20}\cos 10x + C$

69. $\frac{1}{2}\sin x - \frac{1}{10}\sin 5x + C$ **73.** $\frac{1}{2} - \ln\sqrt{2}$ **75. a.** $\frac{\pi}{2}; \frac{\pi}{2}$

b. $\frac{\pi}{2}$, for all n **d.** Yes **e.** $\frac{3\pi}{8}$, for all n

Section 8.4 Exercises, pp. 543–546

1. $x = 3\sec\theta$ **3.** $x = 10\sin\theta$ **5.** $\sqrt{4-x^2}/x$ **7.** $\pi/6$

9. $\frac{25\pi}{3}$ **11.** $\frac{\pi}{12}$ **13.** $\sin^{-1}\frac{x}{4} + C$ **15.** $-\frac{\sqrt{x^2+9}}{9x} + C$

17. $2 - \frac{\pi}{2}$ **19.** $\ln(\sqrt{x^2-81} + x) + C$

21. $\frac{x}{2}\sqrt{64-x^2} + 32\sin^{-1}\frac{x}{8} + C$ **23.** $\frac{x}{25\sqrt{25-x^2}} + C$

25. $-3\ln\left|\frac{\sqrt{9-x^2}+3}{x}\right| + \sqrt{9-x^2} + C$ **27.** $\sqrt{2}/6$

29. $\frac{1}{16}\left(\tan^{-1}\frac{x}{2} + \frac{2x}{x^2+4}\right) + C$

31. $8\sin^{-1}(x/4) - x\sqrt{16-x^2}/2 + C$

33. $\sqrt{x^2-9} - 3\sec^{-1}(x/3) + C$

35. $-1/\sqrt{x^2-1} - \sec^{-1}x + C$ **37.** $2 - \sqrt{2}$

39. $x/\sqrt{100-x^2} - \sin^{-1}(x/10) + C$ **41.** $x/\sqrt{1+4x^2} + C$

43. $\frac{\ln 3}{2}$ **45.** $81/(2(81-x^2)) + \ln\sqrt{81-x^2} + C$

47. $\frac{1}{16}(1 - \sqrt{3} - \ln(21 - 12\sqrt{3}))$ **49.** $\frac{1}{3} + \frac{\ln 3}{4}$

51. $\frac{x}{2}\sqrt{4+x^2} - 2\ln(x + \sqrt{4+x^2}) + C$

53. $\frac{9}{10}\cos^{-1}\frac{5}{3x} - \frac{\sqrt{9x^2-25}}{2x^2} + C$

55. $\frac{\sec^{-1}\frac{x}{10}}{2000} + \frac{\sqrt{x^2-100}}{200x^2} + C$

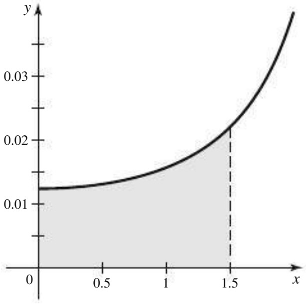
57. a. False **b.** True **c.** False **d.** False

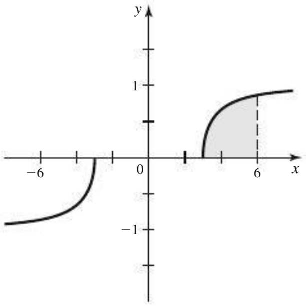
61. $\sin^{-1}\left(\frac{x+1}{2}\right) + C$ **63.** $\frac{1}{3}\tan^{-1}\left(\frac{x+3}{3}\right) + C$

65. $\frac{\pi\sqrt{2}}{48}$ **67.** $\frac{x-4}{\sqrt{9+8x-x^2}} - \sin^{-1}\left(\frac{x-4}{5}\right) + C$

69. $\ln\left((2+\sqrt{3})(\sqrt{2}-1)\right)$

71. $\dfrac{1}{81}+\dfrac{\ln 3}{108}$ **73.** $3\sqrt{3}-\pi$





75. $\dfrac{3}{80}$ **77.** $\dfrac{1}{4a}\left(20a\sqrt{1+400a^2}+\ln\left(20a+\sqrt{1+400a^2}\right)\right)$

81. b. $\displaystyle\lim_{L\to\infty}\frac{kQ}{a\sqrt{a^2+L^2}}=\lim_{L\to\infty}2\rho k\frac{1}{a\sqrt{\left(\frac{a}{L}\right)^2+1}}=\frac{2\rho k}{a}$

85. a. $\dfrac{1}{\sqrt{g}}\left(\dfrac{\pi}{2}-\sin^{-1}\left(\dfrac{2\cos b-\cos a+1}{\cos a+1}\right)\right)$

b. For $b=\pi$, the descent time is $\dfrac{\pi}{\sqrt{g}}$, a constant.

Section 8.5 Exercises, pp. 554–556

1. Rational functions **3. a.** $\dfrac{A}{x-3}$ **b.** $\dfrac{A}{x-4},\dfrac{B}{(x-4)^2},\dfrac{C}{(x-4)^3}$

c. $\dfrac{Ax+B}{x^2+2x+6}$ **5.** $\dfrac{A}{x-4}+\dfrac{B}{x-5}$ **7.** $\dfrac{A}{x-5}+\dfrac{B}{(x-5)^2}$

9. $\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{x-1}+\dfrac{D}{x+2}+\dfrac{E}{x-2}$ **11.** $\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$

13. $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{Cx+D}{x^2+x+2}+\dfrac{Ex+F}{(x^2+x+2)^2}$

15. $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x+2}+\dfrac{D}{(x+2)^2}+\dfrac{Ex+F}{x^2+4}+\dfrac{Gx+H}{(x^2+4)^2}$

17. $\dfrac{2}{x-1}+\dfrac{3}{x-2}$ **19.** $\dfrac{1}{x-4}-\dfrac{1}{x+2}$ **21.** $2+\dfrac{3}{x+1}-\dfrac{4}{x+2}$

23. $\ln\left|\dfrac{x-1}{x+2}\right|+C$ **25.** $3\ln\left|\dfrac{x-1}{x+1}\right|+C$

27. $3\ln|x-1|-\dfrac{1}{3}\ln|3x-2|+C$ **29.** $-\ln 4$

31. $\ln\left|\dfrac{(x-2)^2(x+1)}{(x+2)^2(x-1)}\right|+C$ **33.** $3x+\ln\dfrac{(x-2)^{14}}{|x-1|}+C$

35. $\ln\left|\dfrac{x(x-2)^3}{(x+2)^3}\right|+C$ **37.** $\ln\left|\dfrac{(x-3)^{1/3}(x+1)}{(x+3)^{1/3}(x-1)}\right|^{1/16}+C$

39. $\dfrac{9}{x}+\ln\left|\dfrac{x-9}{x}\right|+C$ **41.** $\ln 2-\dfrac{3}{4}$ **43.** $-\dfrac{2}{x}+\ln\left|\dfrac{x+1}{x}\right|^2+C$

45. $\dfrac{5}{x}+\ln\left|\dfrac{x}{x+1}\right|^6+C$ **47.** $\dfrac{x^2}{2}+2\ln|x-5|-\dfrac{10}{x-5}+C$

49. $\dfrac{3}{x-1}+\ln\left|\dfrac{(x-1)^5}{x^4}\right|+C$ **51.** $\ln|x+1|+\tan^{-1}x+C$

53. $\ln(x+1)^2+\tan^{-1}(x+1)+C$

55. $\ln\left|\dfrac{(x-1)^2}{x^2+4x+5}\right|+14\tan^{-1}(x+2)+C$

57. $\dfrac{1}{2}\ln|x^2+3|-\dfrac{1}{x^2+3}+C$

59. $\dfrac{1}{2}\ln(x^2+6x+10)-3\tan^{-1}(x+3)-\dfrac{1}{x^2+6x+10}+C$

61. $\ln\left(\dfrac{x^2}{x^2+1}\right)+\dfrac{1}{x^2+1}+C$

63. $\sqrt{\dfrac{3}{7}}\tan^{-1}\left(\sqrt{\dfrac{3}{7}}x\right)-\dfrac{1}{6(3x^2+7)}+C$

65. a. False **b.** False **c.** False **d.** True **67.** $\ln 6$

69. $\left(\dfrac{24}{5}-2\ln 5\right)\pi$ **71.** $\dfrac{2}{3}\pi\ln 2$ **73.** $\ln\sqrt{\left|\dfrac{x-1}{x+1}\right|}+C$

75. $\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}+\dfrac{Fx+G}{x^2+x+4}+\dfrac{Hx+I}{(x^2+x+4)^2}$; $\dfrac{1}{16x}-\dfrac{x+10}{100(x^2+1)}+\dfrac{4x+3}{50(x^2+1)^2}-\dfrac{21x-19}{400(x^2+x+4)}-\dfrac{4x+1}{20(x^2+x+4)^2}$ **77.** $\ln\left|\dfrac{e^x-1}{e^x+2}\right|^{1/3}+C$

79. $\dfrac{1}{4}\ln\left(\dfrac{1+\sin t}{1-\sin t}-\dfrac{2}{1+\sin t}\right)+C$

81. $\tan^{-1}e^x-\dfrac{1}{2(e^{2x}+1)}+C$ **83.** $x-\ln(1+e^x)+C$

89. $-\cot x-\csc x+C=-\cot(x/2)+C$

91. $\dfrac{1}{\sqrt{2}}\ln\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ **93. a.** Car A **b.** Car C

c. $S_A(t)=88t-88\ln|t+1|$;

$S_B(t)=88\left(t-\ln(t+1)^2-\dfrac{1}{t+1}+1\right)$;

$S_C(t)=88(t-\tan^{-1}t)$

d. Car C **95.** Because $\dfrac{x^4(1-x)^4}{1+x^2}>0$ on $(0,1)$, $\displaystyle\int_0^1\frac{x^4(1-x^4)}{1+x^2}\,dx>0$; therefore, $\dfrac{22}{7}>\pi$.

Section 8.6 Exercises, pp. 560–562

1. Integrate by parts. **3.** Let $x=8\sin\theta$. **5.** Use the method of partial fractions. **7.** $\dfrac{\pi}{4}$ **9.** $\dfrac{\pi}{6}$ **11.** $\dfrac{5}{4}-\dfrac{3\pi}{8}$ **13.** $-\dfrac{\sqrt{1-e^{2x}}}{e^x}+C$

15. $\dfrac{4}{\ln 2}$ **17.** $\dfrac{3e^4}{2}$ **19.** $\dfrac{16}{35}$ **21.** $\dfrac{x^{10}}{10}\ln 3x-\dfrac{x^{10}}{100}+C$

23. $\ln|\cos x+1|-\ln|\cos x|+C$

25. $\ln\left|\dfrac{x}{1+\sqrt{1-x^2}}\right|+C$ **27.** $\dfrac{3x}{8}-\dfrac{1}{2}\sin x+\dfrac{1}{16}\sin 2x+C$

29. $\ln|\sin x+\sin^2 x|+C$ **31.** $6\sin^{-1}\dfrac{x}{2}+\dfrac{3}{2}x\sqrt{4-x^2}+C$

33. $\dfrac{1}{a}\tan^{-1}\dfrac{e^x}{a}+C$ **35.** $\dfrac{11}{6}$ **37.** $\dfrac{\sqrt{3}+1}{2}$

39. $-\cos x\ln(\sin x)-\ln|\csc x+\cot x|+\cos x+C$

41. $-\dfrac{2}{5}\cot^{5/2}x-\dfrac{2}{9}\cot^{9/2}x+C$ **43.** $\dfrac{\sin^{-1}x^{10}}{10}+C$ **45.** $\ln\dfrac{4}{3}-\dfrac{1}{6}$

47. $x^2+3x+4\ln|x-2|+\ln|x+1|+C$

49. $\dfrac{\sec^{11}x}{11}-\dfrac{\sec^9 x}{9}+C$ **51.** $\dfrac{4}{7}(2^{7/4}-1)$

53. $-\dfrac{\cot^2 e^x}{2}-\ln|\sin e^x|+C$ **55.** $\ln|x^3+x|+3\tan^{-1}x+C$

57. $-2\sqrt{x}\cos\sqrt{x}+2\sin\sqrt{x}+C$ **59.** $-\dfrac{1}{x}-\tan^{-1}x+C$

61. $\frac{\sqrt{2}e^{\pi/4}-1}{2}$ **63.** $\frac{x^{a+1}}{a+1}\left(\ln x-\frac{1}{a+1}\right)+C$

65. $\frac{\pi}{18}$ **67.** $\frac{1}{54}(\sin^{-1}3x-3x\sqrt{1-9x^2})+C$

69. $\frac{1-\sqrt{1-x^2}}{x}+C$ **71.** $-2\cot x+2\csc x-x+C$

73. $\frac{40\sqrt{5}}{3}-\frac{224}{15}$ **75.** $\frac{7\pi^2}{144}$ **77.** $x\cos^{-1}x-\sqrt{1-x^2}+C$

79. $-\frac{\sin^{-1}x}{x}+\ln\left|\frac{x}{1+\sqrt{1-x^2}}\right|+C$

81. $\ln|x|+2\tan^{-1}x-\frac{3}{2(x^2+1)}+C$

83. $\frac{\sin^{999}e^x}{999}-\frac{\sin^{1001}e^x}{1001}+C$ **85. a.** True **b.** True **c.** False **d.** False **87.** $\pi(\sqrt{2}+\ln(1+\sqrt{2}))\approx 7.212$

89. $\frac{\pi(4\sqrt{2}+3)}{3}\approx 9.065$ **91.** $9800\pi\ln 2\approx 21{,}340.3$ J

93. $4x-2\ln(e^{2x}+2e^x+17)-\tan^{-1}\left(\frac{e^x+1}{4}\right)+C$

95. $\frac{1}{4}\ln|\tan x+1|-\frac{1}{4}\ln|\tan x-1|+\frac{x}{2}+C$

97. $x\tan^{-1}\sqrt[3]{x}-\frac{x^{2/3}}{2}+\frac{1}{2}\ln(1+x^{2/3})+C$

99. $\pi\left(\sqrt{5}-\sqrt{2}+\frac{1}{2}\ln\left(\frac{\sqrt{5}-1}{\sqrt{5}+1}\right)-\frac{1}{2}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)\approx 3.839$

Section 8.7 Exercises, pp. 565–567

1. Substitutions, integration by parts, partial fractions **3.** The CAS may not include the constant of integration, and it may use a trigonometric identity or other algebraic simplification.

5. $-\frac{1}{3}\sin^3 e^x+\sin e^x+C$ **7.** $x\cos^{-1}x-\sqrt{1-x^2}+C$

9. $\ln(x+\sqrt{16+x^2})+C$ **11.** $\frac{3}{4}(2u-7\ln|7+2u|)+C$

13. $-\frac{1}{4}\cot 2x+C$ **15.** $\frac{1}{12}(2x-1)\sqrt{4x+1}+C$

17. $\frac{1}{3}\ln\left|x+\sqrt{x^2-\left(\frac{10}{3}\right)^2}\right|+C$ **19.** $\ln(e^x+\sqrt{4+e^{2x}})+C$

21. $-\frac{1}{2}\ln\left|\frac{2+\sin x}{\sin x}\right|+C$

23. $\frac{2\ln^2 x-1}{4}\sin^{-1}(\ln x)+\frac{\ln x\sqrt{1-\ln^2 x}}{4}+C$

25. $\frac{x}{16\sqrt{16+9x^2}}+C$ **27.** $-\frac{1}{12}\ln\left|\frac{12+\sqrt{144-x^2}}{x}\right|+C$

29. $2x+x\ln^2 x-2x\ln x+C$

31. $\frac{x+5}{2}\sqrt{x^2+10x}-\frac{25}{2}\ln|x+5+\sqrt{x^2+10x}|+C$

33. $\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)+C$ **35.** $\ln x-\frac{1}{10}\ln(x^{10}+1)+C$

37. $2\ln(\sqrt{x-6}+\sqrt{x})+C$

39. $-\frac{\tan^{-1}x^3}{3x^3}+\ln\left|\frac{x}{(x^6+1)^{1/6}}\right|+C$ **41.** $4\sqrt{17}+\ln(4+\sqrt{17})$

43. $\sqrt{5}-\sqrt{2}+\ln\left(\frac{2+2\sqrt{2}}{1+\sqrt{5}}\right)$ **45.** $\frac{128\pi}{3}$ **47.** $\frac{\pi^2}{4}$

49. $\frac{(x-3)\sqrt{3+2x}}{3}+C$ **51.** $\frac{1}{3}\tan 3x-x+C$

53. $\frac{1540+243\ln 3}{8}$

55. $\frac{(x^2-a^2)^{3/2}}{3}-a^2\sqrt{x^2-a^2}+a^3\cos^{-1}\frac{a}{x}+C$ **57.** $\frac{\pi}{4}$

59. $-\frac{x}{8}(2x^2-5a^2)\sqrt{a^2-x^2}+\frac{3a^4}{8}\sin^{-1}\frac{x}{a}+C$

61. $2-\frac{\pi^2}{12}-\ln 4$ **63.** $\frac{27{,}456\sqrt{15}}{7}\approx 15{,}190.9$

65. $\frac{1}{8}e^{2x}(4x^3-6x^2+6x-3)+C$ **67.** $\frac{\tan^3 3y}{9}-\frac{\tan 3y}{3}+y+C$

69. $\frac{1}{24}(128-78\sqrt{2}-3\ln(3+2\sqrt{2}))$

71. $\frac{1}{a^2}(ax-b\ln|b+ax|)+C$

73. $\frac{1}{a^2}\left(\frac{(ax+b)^{n+2}}{n+2}-\frac{b(ax+b)^{n+1}}{n+1}\right)+C$

75. a. True **b.** True

79. $\frac{1}{16}((8x^2-1)\sin^{-1}2x+2x\sqrt{1-4x^2})+C$

81. $-\frac{\tan^{-1}x}{x}+\ln\left(\frac{|x|}{\sqrt{x^2+1}}\right)+C$ **83. b.** $\frac{\pi}{8}\ln 2$

85. a.

θ_0	T
0.10	6.27927
0.20	6.26762
0.30	6.24854
0.40	6.22253
0.50	6.19021
0.60	6.15236
0.70	6.10979
0.80	6.06338
0.90	6.01399
1.00	5.96247

b. All are within 10%.

87. b. $\frac{63\pi}{512}$ **c.** Decrease

Section 8.8 Exercises, pp. 578–582

1. $\frac{1}{2}$ **3.** The Trapezoid Rule approximates areas under curves using trapezoids. **5.** 42 **7.** $\frac{112}{3}$ **9.** $-1, 1, 3, 5, 7, 9$

11. 1.59×10^{-3}; 5.04×10^{-4} **13.** 1.72×10^{-3}; 6.32×10^{-4}

15. 576; 640; 656 **17.** 0.643950551 **19.** 704; 672; 664

21. 0.622 **23.** 2.28476811; 2.33512377 **25.** 1.76798499

27. $M(25)\approx 0.63703884$, $T(25)\approx 0.63578179$; 6.58×10^{-4}, 1.32×10^{-3}

29.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	99	102	1.00	2.00
8	99.75	100.5	0.250	0.500
16	99.9375	100.125	6.3×10^{-2}	0.125
32	99.984375	100.03125	1.6×10^{-2}	3.1×10^{-2}

31.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	1.50968181	1.48067370	9.7×10^{-3}	1.9×10^{-2}
8	1.50241228	1.49517776	2.4×10^{-3}	4.8×10^{-3}
16	1.50060256	1.49879502	6.0×10^{-4}	1.2×10^{-3}
32	1.50015061	1.49969879	1.5×10^{-4}	3.0×10^{-4}

33.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	-1.96×10^{-16}	0	2.0×10^{-16}	0
8	7.63×10^{-17}	-1.41×10^{-16}	7.6×10^{-17}	1.4×10^{-16}
16	1.61×10^{-16}	1.09×10^{-17}	1.6×10^{-16}	1.1×10^{-17}
32	6.27×10^{-17}	-4.77×10^{-17}	6.3×10^{-17}	4.8×10^{-17}

35. $T(4) \approx 690.3$ million ft^3; $S(4) \approx 692.2$ million ft^3 (answers may vary) **37.** 54.5°F, Trapezoid Rule **39.** 35.0°F, Trapezoid Rule **41. a.** Left sum: 204.917; right sum: 261.375; Trapezoid Rule: 233.146; the approximations measure the average temperature of the curling iron on $[0, 120]$. **b.** Left sum: underestimate; right sum: overestimate; Trapezoid Rule: underestimate **c.** 305°F is the change in temperature over $[0, 120]$. **43. a.** 5907.5 **b.** 5965 **c.** 5917

45. a. $T(25) \approx 3.19623162$
$T(50) \approx 3.19495398$
b. $S(50) \approx 3.19452809$
c. $e_T(50) \approx 4.3 \times 10^{-4}$
$e_S(50) \approx 4.5 \times 10^{-8}$

47. a. $T(50) \approx 1.00008509$
$T(100) \approx 1.00002127$
b. $S(100) \approx 1.00000000$
c. $e_T(100) \approx 2.1 \times 10^{-5}$
$e_S(100) \approx 4.6 \times 10^{-9}$

49.

n	$T(n)$	$S(n)$	Error in $T(n)$	Error in $S(n)$
4	1820.0000	—	284	—
8	1607.7500	1537.0000	71.8	1
16	1553.9844	1536.0625	18.0	6.3×10^{-2}
32	1540.4990	1536.0039	4.50	3.9×10^{-3}

51.

n	$T(n)$	$S(n)$	Error in $T(n)$	Error in $S(n)$
4	0.46911538	—	5.3×10^{-2}	—
8	0.50826998	0.52132152	1.3×10^{-2}	2.9×10^{-4}
16	0.51825968	0.52158957	3.4×10^{-3}	1.7×10^{-5}
32	0.52076933	0.52160588	8.4×10^{-4}	1.1×10^{-6}

53. a. True **b.** False **c.** True

55.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	0.40635058	0.40634782	1.4×10^{-6}	1.4×10^{-6}
8	0.40634920	0.40634920	7.6×10^{-10}	7.6×10^{-10}
16	0.40634920	0.40634920	6.6×10^{-13}	6.6×10^{-13}
32	0.40634920	0.40634920	8.9×10^{-16}	7.8×10^{-16}

57.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	4.72531819	4.72507878	1.2×10^{-4}	1.2×10^{-4}
8	4.72519850	4.72519849	9.1×10^{-9}	9.1×10^{-9}
16	4.72519850	4.72519850	0	8.9×10^{-16}
32	4.72519850	4.72519850	0	8.9×10^{-16}

63. Approximations will vary; exact value is 68.26894921 . . . . **65. a.** Approx. 1.6×10^{11} barrels **b.** Approx. 6.8×10^{10} barrels **67. a.** $M(50) \approx 34.4345566$
b. $f''(x) = \dfrac{3(x^4 + 4x)}{4(x^3 + 1)^{3/2}}$ **d.** $E_M \le 0.0028$
69. a. $T(40) = 0.874799972\ldots$ **b.** $f''(x) = e^x \cos e^x - e^{2x} \sin e^x$
d. $E_T \le \dfrac{1}{3200}$ **71. a.** $S(20) \approx 0.97774576$
b. $E_S \le 3.5 \times 10^{-8}$ **73.** Approximations will vary; exact value is 38.753792 . . . . **77.** Overestimate **79.** $S(20) \approx 1.00000175$

Section 8.9 Exercises, pp. 590–593

1. The interval of integration is infinite or the integrand is unbounded on the interval of integration. **3.** $\lim_{b\to\infty} \int_2^b \dfrac{dx}{x^{1/5}}$ **5.** $\int_{-\infty}^{\infty} f(x)\,dx$
7. $\dfrac{1}{3}$ **9.** Diverges **11.** $\dfrac{1}{a}$ **13.** Diverges **15.** $\dfrac{\pi}{10}$
17. Diverges **19.** Diverges **21.** $\dfrac{1}{\pi}$ **23.** $\dfrac{\pi}{4}$ **25.** $\dfrac{\pi}{6}$ **27.** 0
29. $\dfrac{\pi^3}{12}$ **31.** ln 2 **33.** Diverges **35.** Diverges **37.** 6
39. Diverges **41.** Diverges **43.** $2(e - 1)$ **45.** Diverges
47. $4 \cdot 10^{3/4}/3$ **49.** Diverges **51.** π **53.** -1
55. $\ln(2 + \sqrt{3})$ **57.** 2 **59.** \$41,666.67 **61.** 0.76 **63.** 20,000 hr
65. $\dfrac{\pi}{3}$ **67.** $3\pi/2$ **69.** $\pi/\ln 2$ **71.** 2π **73.** Does not exist
75. $\dfrac{72 \cdot 2^{1/3}\pi}{5}$ **77.** Converges **79.** Diverges **81.** Converges
83. Diverges **85.** Converges **87. a.** True **b.** False **c.** False
d. True **e.** True **89.** $1/b - 1/a$ **91. a.** $A(a, b) = \dfrac{e^{-ab}}{a}$, for $a > 0$
b. $b = g(a) = -\dfrac{1}{a}\ln 2a$ **c.** $b^* = -2/e$ **93.** π **107. a.** π
b. $\pi/(4e^2)$ **109. a.** $6.28 \times 10^7\, m$ J **b.** 11.2 km/s **c.** $\le$ 9 mm

Chapter 8 Review Exercises, pp. 593–596

1. a. True **b.** False **c.** False **d.** True **e.** False
3. $2(x - 8)\sqrt{x + 4} + C$ **5.** $\dfrac{1}{3}\sqrt{x + 2}(x - 4) + C$ **7.** $\dfrac{\pi}{4}$
9. $\dfrac{4}{105}$ **11.** $\sqrt{t - 1} - \tan^{-1}\sqrt{t - 1} + C$ **13.** $\dfrac{2}{15}(1 - e^{3\pi})$
15. $7 + \ln 40 - \ln 17$ **17.** $2 \ln|x| + 3\tan^{-1}(x + 1) + C$
19. $\dfrac{2}{x + 3} - \dfrac{2}{(x + 3)^2} + \ln|x + 3| + C$ **21.** $\sqrt{3} - 1 - \dfrac{\pi}{12}$
23. $\dfrac{1}{5}\tan^5 t + C$ **25.** $\dfrac{\pi}{8}$ **27.** $\dfrac{\sqrt{w^2 + 2w - 8}}{9(w + 1)} + C$ **29.** $-\dfrac{\cot^5 x}{5} + C$
31. $\dfrac{x \cosh 2x}{2} - \dfrac{\sinh 2x}{4} + C$ **33.** $\dfrac{1}{15}\sec^5 3\theta - \dfrac{1}{9}\sec^3 3\theta + C$

35. $\frac{1}{6}(x^2-8)\sqrt{x^2+4}+C$ **37.** $\frac{1}{x+1}+\ln|(x+1)(x^2+4)|+C$

39. $\frac{t-\ln(2+e^t)}{2}+C$ **41.** $\frac{1}{4}(\csc 4\theta-\cot 4\theta)+C$

43. $\frac{e^x}{2}(\sin x-\cos x)+C$

45. $\ln|x|-\frac{1}{x}+\frac{1}{2}\ln(x^2+4x+9)-\frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$

47. $\frac{\theta}{2}+\frac{1}{16}\sin 8\theta+C$ **49.** $\frac{\sec^{49} 2z}{98}+C$ **51.** $\frac{4}{15}$

53. $2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\ln(\sqrt[6]{x}+1)+C$

55. $-\frac{\sqrt{9-y^2}}{9\sqrt{2}\,y}+C$ **57.** $\frac{\pi}{9}$ **59.** $-\operatorname{sech} x+C$ **61.** $\frac{\pi}{3}$

63. $\frac{1}{8}\ln\left|\frac{x-5}{x+3}\right|+C$ **65.** $\frac{\ln 2}{4}+\frac{\pi}{8}$ **67.** 3 **69.** $\frac{1}{3}\ln\left|\frac{x-2}{x+1}\right|+C$

71. $2(x-2\ln|x+2|)+C$ **73.** $e^{2t}/(2\sqrt{1+e^{4t}})+C$

75. a. $\sec e^x+C$ **b.** $e^x\sec e^x-\ln|\sec e^x+\tan e^x|+C$

77. $\frac{\sqrt{6}}{3}\tan^{-1}\sqrt{\frac{2x-3}{3}}+C$

79. $\frac{1}{4}\sec^3 x\tan x+\frac{3}{8}\sec x\tan x+\frac{3}{8}\ln|\sec x+\tan x|+C$

81. $2(\ln^3 2-3\ln^2 2+\ln 64-3)$ **83.** 1 **85.** $\frac{\pi}{2}$

87. $\frac{2\pi}{\sqrt{3}}$ **89.** Converges **91.** Diverges **93.** 1.196288

95. $M(4)=44$; $T(4)=42$; $S(4)=\frac{124}{3}$

97. $M(40)\approx 0.398236$; $T(40)\approx 0.398771$; $S(40)\approx 0.398416$

99. 0.886227 **101.** y-axis **103.** $\pi(e-2)$ **105.** $\frac{\pi}{2}(e^2-3)$

107. a. 1.603 **b.** 1.870 **c.** $b\ln b-b=a\ln a-a$
d. Decreasing **109.** $20/(3\pi)$ **111.** 1901 cars

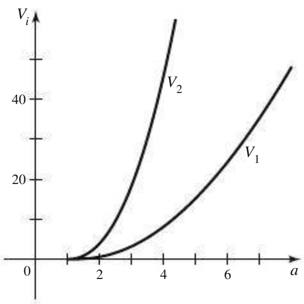
113. a. $I(p)=\frac{1}{(p-1)^2}(1-pe^{1-p})$ if $p\neq 1$, $I(1)=\frac{1}{2}$ **b.** $0, \infty$

c. $I(0)=1$ **115.** 0.4054651 **117.** $n=2$

119. a. $V_1(a)=\pi(a\ln^2 a-2a\ln a+2(a-1))$

b. $V_2(a)=\frac{\pi}{2}(2a^2\ln a-a^2+1)$

c. $V_2(a)>V_1(a)$ for all $a>1$



121. $a=\ln 2/(2b)$ **123.** $\ln(1+\sqrt{2}/2)$

CHAPTER 9

Section 9.1 Exercises, pp. 604–606

1. a. 1 **b.** Linear **3.** Yes **5.** $\frac{\pi}{2}<t<\frac{3\pi}{2}$

21. $y=3t-\frac{e^{-2t}}{2}+C$ **23.** $y=2\ln|\sec 2x|-3\sin x+C$

25. $y=2t^6+6t^{-1}-2t^2+C_1t+C_2$

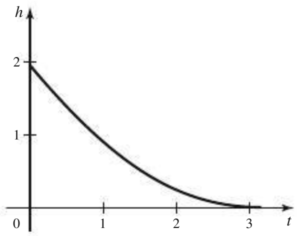
27. $u=\frac{x^{11}}{2}+\frac{x^9}{2}-\frac{x^7}{2}+\frac{5}{x}+C_1x+C_2$

29. $u=\ln(x^2+4)-\tan^{-1}\frac{x}{2}+C$ **31.** $y=\sin^{-1}x+C_1x+C_2$

33. $y=e^t+t+3$ **35.** $y=x^3+x^{-3}-2, x>0$

37. $y=-t^5+2t^3+1$ **39.** $y=e^t(t-2)+2(t+1)$

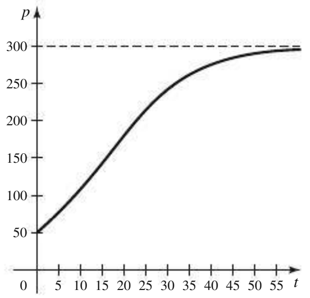
41. $u=\frac{1}{4}\tan^{-1}\frac{x}{4}-4x+2$ **43. a.** $v(t)=-9.8t+29.4$;
$s(t)=-4.9t^2+29.4t+30$; the object is above the ground for approximately $0\leq t\leq 6.89$. **b.** The highest point of 74.1 m is reached at $t=3$ s. **45.** The amount of resource is increasing for $H<75$ and is constant if $H=75$. If $H=100$, the resource vanishes at approximately 28 time units.
47. $h=(\sqrt{1.96}-0.1t\sqrt{2g})^2\approx(1.4-0.44t)^2, 0\leq t\leq 3.16$; the tank is empty after approximately 3.16 s.



49. a. False **b.** False **c.** True **51. c.** $y=C_1\sin kt+C_2\cos kt$

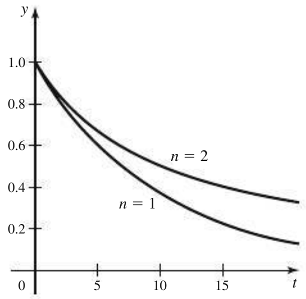
53. b. $C=\frac{K-50}{50}$

c.



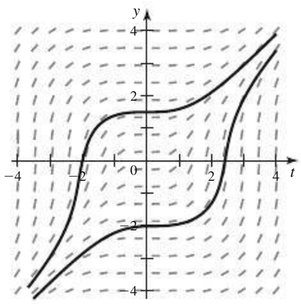
d. 300

55. c. The decay rate is greater for the $n=1$ model.

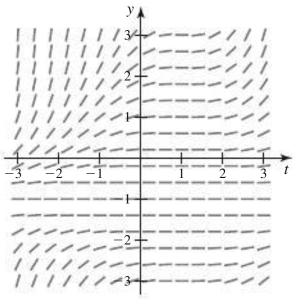


Section 9.2 Exercises, pp. 611–614

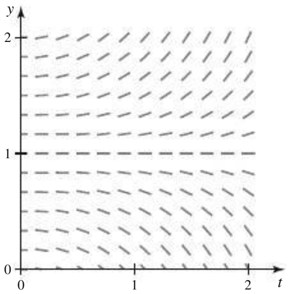
1. At selected points (t_0, y_0) in the region of interest draw a short line segment with slope $f(t_0, y_0)$. **3.** $y(3.1)\approx 1.6$
5. a. D **b.** B **c.** A **d.** C
7.



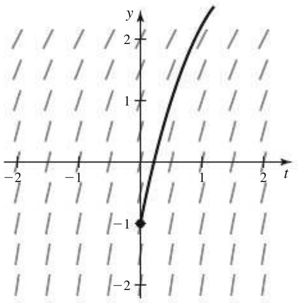
9. An initial condition of $y(0) = -1$ leads to a constant solution. For any other initial condition, the solutions are increasing over time.



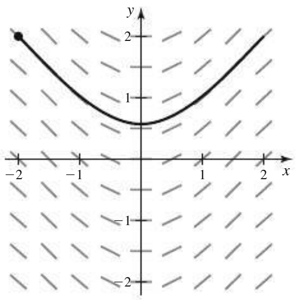
11. An initial condition of $y(0) = 1$ leads to a constant solution. Initial conditions $y(0) = A$ lead to solutions that are increasing over time if $A > 1$.



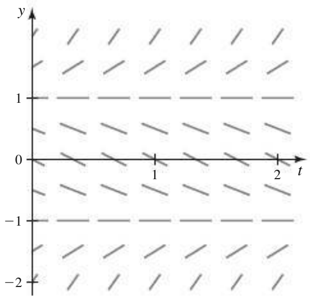
13.



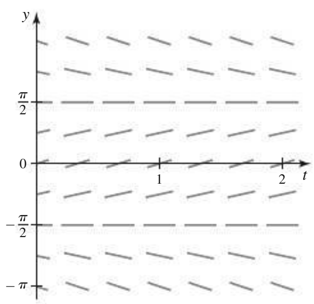
15.



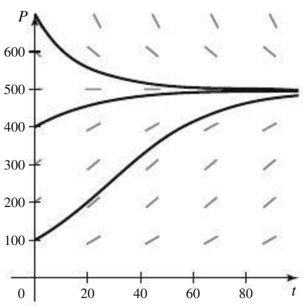
17. a. $y = 1, y = -1$ **b.** Solutions are increasing for $|y| > 1$ and decreasing for $|y| < 1$. **c.** Initial conditions $y(0) = A$ lead to increasing solutions if $|A| > 1$ and decreasing solutions if $|A| < 1$.
d.



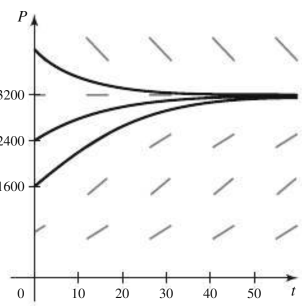
19. a. $y = \pi/2, y = -\pi/2$ **b.** Solutions are increasing for $|y| < \pi/2$ and decreasing for $|y| > \pi/2$. **c.** Initial conditions $y(0) = A$ lead to increasing solutions if $|A| < \pi/2$ and decreasing solutions if $\pi/2 < |A| < \pi$.
d.



21. The equilibrium solutions are $P = 0$ and $P = 500$.



23. The equilibrium solutions are $P = 0$ and $P = 3200$.



25. $y(0.5) \approx u_1 = 4; y(1) \approx u_2 = 8$
27. $y(0.1) \approx u_1 = 1.1; y(0.2) \approx u_2 = 1.19$

29. a.

Δt	approximation to $y(0.2)$	approximation to $y(0.4)$
0.20000	0.80000	0.64000
0.10000	0.81000	0.65610
0.05000	0.81451	0.66342
0.02500	0.81665	0.66692

b.

Δt	errors for $y(0.2)$	errors for $y(0.4)$
0.20000	0.01873	0.03032
0.10000	0.00873	0.01422
0.05000	0.00422	0.00690
0.02500	0.00208	0.00340

c. Time step $\Delta t = 0.025$; smaller time steps generally produce more accurate results. **d.** Halving the time steps results in approximately halving the error.

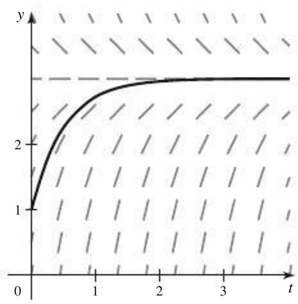
31. a.

Δt	approximation to $y(0.2)$	approximation to $y(0.4)$
0.20000	3.20000	3.36000
0.10000	3.19000	3.34390
0.05000	3.18549	3.33658
0.02500	3.18335	3.33308

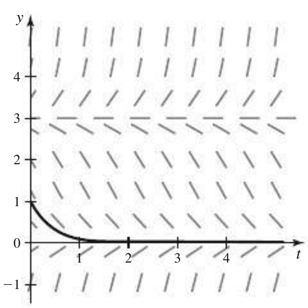
b.

Δt	errors for $y(0.2)$	errors for $y(0.4)$
0.20000	0.01873	0.03032
0.10000	0.00873	0.01422
0.05000	0.00422	0.00690
0.02500	0.00208	0.00340

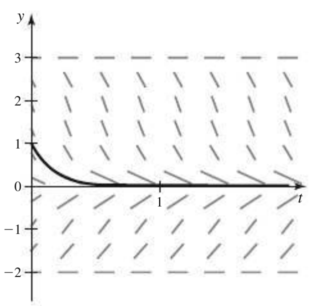
c. Time step $\Delta t = 0.025$; smaller time steps generally produce more accurate results. **d.** Halving the time steps results in approximately halving the error. **33. a.** $y(2) \approx 0.00604662$ **b.** 0.012269 **c.** $y(2) \approx 0.0115292$ **d.** Error in part (c) is approximately half of the error in part (b). **35. a.** $y(4) \approx 3.05765$ **b.** 0.0339321 **c.** $y(4) \approx 3.0739$ **d.** Error in part (c) is approximately half of the error in part (b). **37. a.** True **b.** False
39. a. $y = 3$ **b, c.**



41. a. $y = 0$ and $y = 3$
b, c.



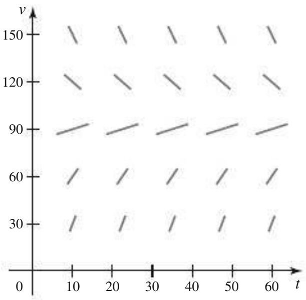
43. a. $y = -2, y = 0,$ and $y = 3$
b, c.



45. a. $\Delta t = \dfrac{b-a}{N}$ **b.** $u_1 = A + f(a, A)\dfrac{b-a}{N}$

c. $u_{k+1} = u_k + f(t_k, u_k)\dfrac{b-a}{N}$, where $u_0 = A$ and $t_k = a + k(b-a)/N$, for $k = 0, 1, 2, \ldots, N-1$.

47. a.



b. Increasing for $A < 98$ and decreasing for $A > 98$
c. $v(t) = 98$

Section 9.3 Exercises, pp. 618–620

1. A first-order separable differential equation has the form $g(y)\,y'(t) = h(t)$, where the factor $g(y)$ is a function of y and $h(t)$ is a function of t. **3.** No **5.** $y = \dfrac{t^4}{4} + C$

7. $y = \pm\sqrt{2t^3 + C}$ **9.** $y = -2\ln\left(\dfrac{1}{2}\cos t + C\right)$

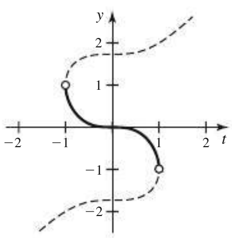
11. $y = \dfrac{x}{1 + Cx}$ **13.** $y = \pm\dfrac{1}{\sqrt{C - \cos t}}$ **15.** $u = \ln\left(\dfrac{e^{2x}}{2} + C\right)$

17. $y = \sqrt{t^3 + 81}$ **19.** Not separable **21.** $y(t) = -e^{e^t - 1}$

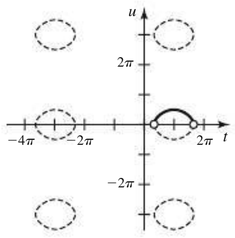
23. $y = \ln(e^x + 2)$ **25.** $y = \ln\left(\dfrac{\ln^4 t}{4} + 1\right)$

27. $y = \sqrt{\tan t}, 0 < t < \pi/2$ **29.** $y = \sqrt{t^2 + 3}$ **31.** $y = \ln t + 2$

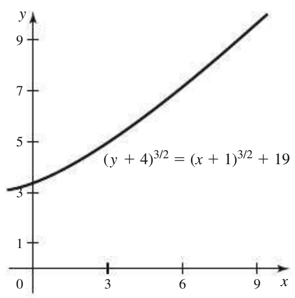
33. $y^3 - 3y = 2t^3$, $-1 < t < 1$



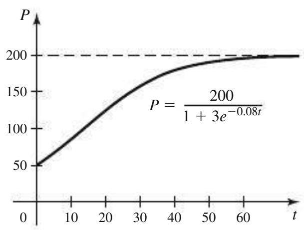
35. $\cos u = 2 - 2\sin\dfrac{x}{2}, \dfrac{\pi}{3} < x < \dfrac{5\pi}{3}$



37.



39. a.

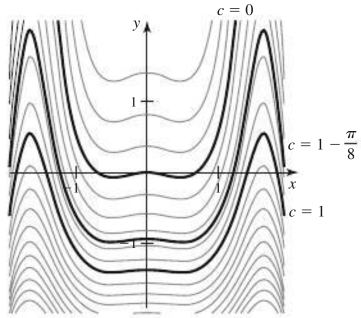


b. 200

41. a. True **b.** False **c.** True

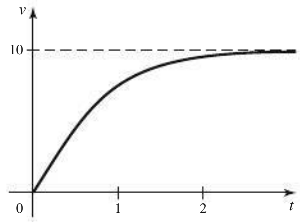
43. a. $y = -2\ln\left(\dfrac{x^2}{4} + \cos x^2 + C\right)$ **b.** $C = 0, 1, 1 - \dfrac{\pi}{8}$

c.



45. $y = kx$ **47. b.** $\sqrt{gm/k}$

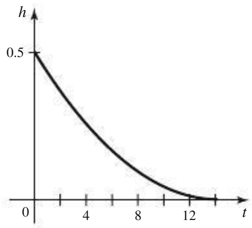
c. $v = \sqrt{\dfrac{g}{a}}\,\dfrac{Ce^{2\sqrt{ag}\,t} - 1}{Ce^{2\sqrt{ag}\,t} + 1}, t \geq 0$, where $a = \dfrac{k}{m}$



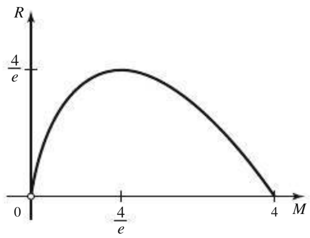
49. a. $h = \left(\sqrt{H} - \dfrac{kt}{2}\right)^2, 0 \leq t \leq \dfrac{2\sqrt{H}}{k}$

b. $h = (\sqrt{0.5} - 0.05t)^2, 0 \leq t \leq 14.1$

c. Approx. 14.1 s

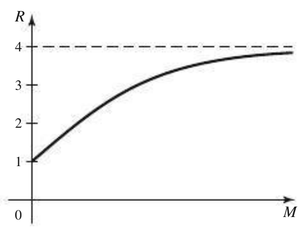


51. a.



R is positive if $0 < M < 4$; R has a maximum value when $M = \dfrac{4}{e}$; $\lim_{M \to 0} R(M) = 0$.

b. $M(t) = 4^{1 - e^{-t}}, t \geq 0$; the tumor grows quickly at first and then the rate of growth slows down; the limiting size of the tumor is 4.



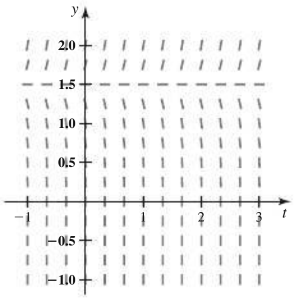
53. a. $y = \dfrac{1}{1 - t}, t < 1$ **b.** $y = \dfrac{1}{\sqrt{2}\sqrt{1 - t}}, t < 1$

c. $y = \dfrac{1}{(n(1 - t))^{1/n}}, t < 1$; as $t \to 1^-$, $y \to \infty$

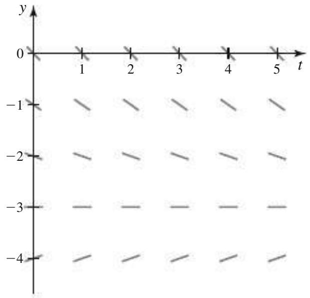
Section 9.4 Exercises, pp. 625–627

1. $y = 17e^{-10t} - 13$ **3.** $y = Ce^{-4t} + \frac{3}{2}$ **5.** $y = Ce^{3t} + \frac{4}{3}$
7. $y = Ce^{-2x} - 2$ **9.** $u = Ce^{-12t} + \frac{5}{4}$ **11.** $y = 7e^{3t} + 2$
13. $y = 4(e^{2t} - 1)$ **15.** $y = 4(2e^{3t-3} - 1)$

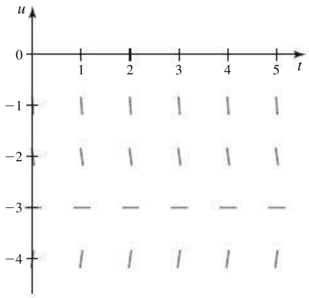
17. $y = \frac{3}{2}$; unstable



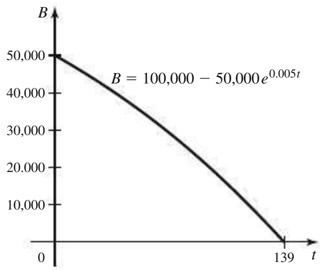
19. $y = -3$; stable



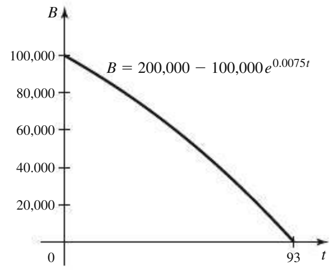
21. $u = -3$; stable



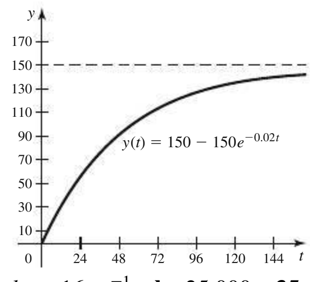
23. $B = 100{,}000 - 50{,}000e^{0.005t}$; reaches a balance of zero after approximately 139 months



25. $B = 200{,}000 - 100{,}000e^{0.0075t}$; reaches a balance of zero after approximately 93 months



27. Approx. 32 min **29.** Approx. 14 min
31. a.



b. 150 **c.** Approx. 115.1 hr

33. a. $h = 16\ \text{yr}^{-1}$ **b.** 25,000 **35. a.** False **b.** True **c.** False **d.** False **37. a.** $B = 20{,}000 + 20{,}000e^{0.03t}$; the unpaid balance is growing because the monthly payment of \$600 is less than the interest on the unpaid balance. **b.** \$20,000 **c.** $\dfrac{m}{r}$

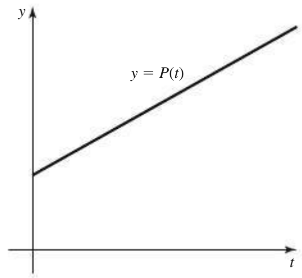
39. $y = 1 + \dfrac{t}{2} + \dfrac{5}{2t}, t > 0$ **41.** $y = \dfrac{1}{2}e^{3t} + \dfrac{7}{2}e^{t}$

45. $y(t) = \dfrac{6}{t}, t > 0$ **47.** $y = \dfrac{9t^5 + 20t^3 + 15t + 76}{15(t^2 + 1)}$

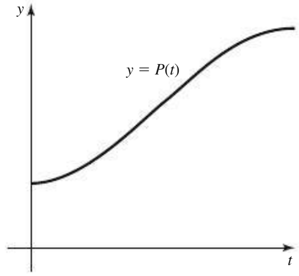
Section 9.5 Exercises, pp. 634–636

1. The growth rate function specifies the rate of growth of the population. The population is increasing when the growth rate function is positive, and the population is decreasing when the growth rate function is negative. **3.** If the growth rate function is positive (it does not matter whether it is increasing or decreasing), then the population is increasing. **5.** It is a linear, first-order differential equation. **7.** The solution curves in the FH-plane are closed curves that circulate around the equilibrium point.

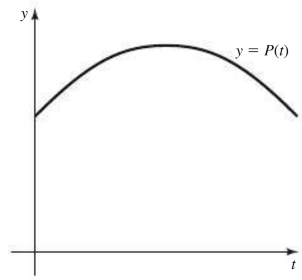
9.



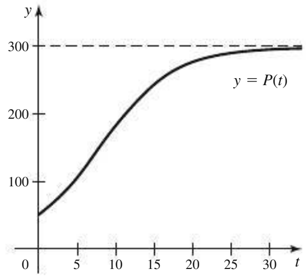
11.



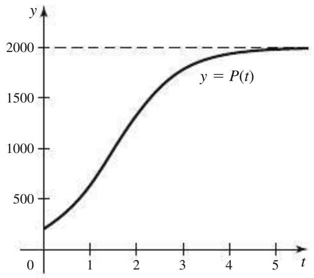
13.



15. $P' = 0.2\,P\left(1 - \dfrac{P}{300}\right)$; $P = \dfrac{300}{5e^{-0.2t} + 1}, t \geq 0$

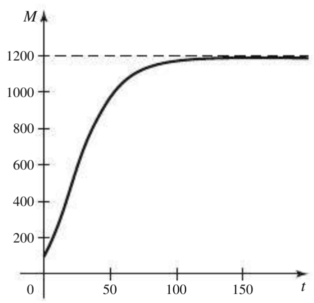


17. $P = \dfrac{2000}{9e^{-\ln(27/7)t} + 1}, t \geq 0$



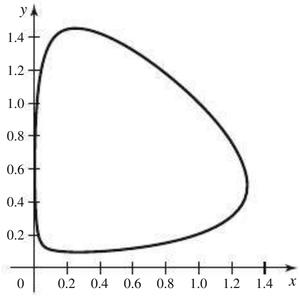
19. $M = K\left(\dfrac{M_0}{K}\right)^{e^{-rt}}, t \geq 0$

21. $M = 1200 \cdot 0.075^{\exp(-0.05t)}$, $t \geq 0$

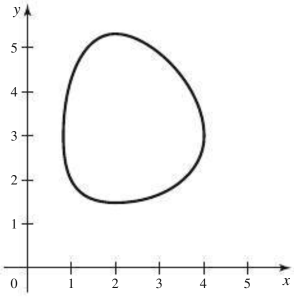


23. a. $m'(t) = -0.008t + 80, m(0) = 0$
b. $m = 10{,}000 - 10{,}000e^{-0.008t}, t \geq 0$
25. a. $m'(t) = -0.005\,t + 100, m(0) = 80{,}000$
b. $m = 60{,}000e^{-0.005t} + 20{,}000, t \geq 0$

27. a. x is the predator population; y is the prey population.
b. $x' = 0$ on the lines $x = 0$ and $y = \frac{1}{2}$; $y' = 0$ on the lines $y = 0$ and $x = \frac{1}{4}$. **c.** $(0, 0)$, $\left(\frac{1}{4}, \frac{1}{2}\right)$
d. $x' > 0$ and $y' > 0$ for $0 < x < \frac{1}{4}$, $y > \frac{1}{2}$
$x' > 0$ and $y' < 0$ for $x > \frac{1}{4}$, $y > \frac{1}{2}$
$x' < 0$ and $y' < 0$ for $x > \frac{1}{4}$, $0 < y < \frac{1}{2}$
$x' < 0$ and $y' > 0$ for $0 < x < \frac{1}{4}$, $0 < y < \frac{1}{2}$
e. The solution evolves in the clockwise direction.



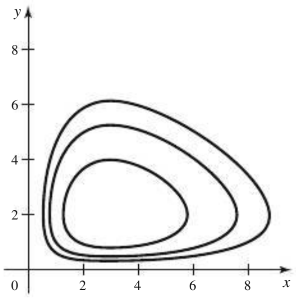
29. a. x is the predator population; y is the prey population.
b. $x' = 0$ on the lines $x = 0$ and $y = 3$; $y' = 0$ on the lines $y = 0$ and $x = 2$. **c.** $(0, 0)$, $(2, 3)$
d. $x' > 0$ and $y' > 0$ for $0 < x < 2$, $y > 3$
$x' > 0$ and $y' < 0$ for $x > 2$, $y > 3$
$x' < 0$ and $y' < 0$ for $x > 2$, $0 < y < 3$
$x' < 0$ and $y' > 0$ for $0 < x < 2$, $0 < y < 3$
e. The solution evolves in the clockwise direction.



31. a. True **b.** True **c.** True **35. c.** $\lim_{t\to\infty} m(t) = C_i V$, which is the amount of substance in the tank when the tank is filled with the inflow solution. **d.** Increasing R increases the rate at which the solution in the tank reaches the steady-state concentration.

37. a. $I = \frac{V}{R} e^{-t/(RC)}$ **b.** $Q = VC(1 - e^{-t/(RC)})$

39. a. $y'(x) = \frac{y(c - dx)}{x(-a + by)}$ **c.**

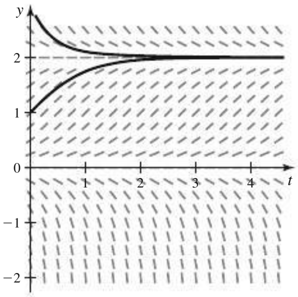


Chapter 9 Review Exercises, pp. 636–638

1. a. False **b.** False **c.** True **d.** True **e.** False
3. $y = Ce^{-2t} + 3$ **5.** $y = Ce^{t^2}$ **7.** $y = Ce^{\tan^{-1} t}$
9. $y = \tan(t^2 + t + C)$ **11.** $y = \sin t + t^2 + 1$
13. $Q = 8(1 - e^{t-1})$ **15.** $u = (3 + t^{2/3})^{3/2}$, $t > 0$

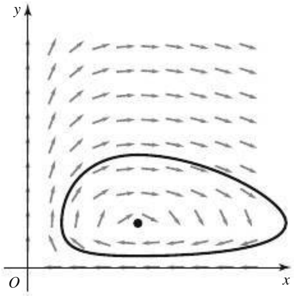
17. $s = \frac{t\sqrt{2}}{\sqrt{t^2 + 1}}$

19. a, b.



c. $0 < A < 2$
d. $A > 2$ or $A < 0$
e. $y = 0$ and $y = 2$

21. a. 1.05, 1.09762 **b.** 1.04939, 1.09651 **c.** 0.00217, 0.00106; the error in part (b) is smaller. **23.** $y = -3$ (unstable), $y = 0$ (stable), $y = 5$ (unstable) **25.** $y = -1$ (unstable), $y = 0$ (stable), $y = 2$ (unstable) **27. a.** 0.0713 **b.** $P = \frac{1600}{79e^{-0.0713t} + 1}$, $t \geq 0$
c. Approx. 61 hours **29. a.** $m = 2000(1 - e^{-0.005t})$
b. 2000 g **c.** Approx. 599 minutes **31. a.** x represents the predator. **b.** $x'(t) = 0$ when $x = 0$ and $y = 2$. $y'(t) = 0$ when $y = 0$ and $x = 5$. **c.** $(0, 0)$ and $(5, 2)$ **d.** $x' > 0$, $y' > 0$ when $0 < x < 5$ and $y > 2$; $x' > 0$, $y' < 0$ when $x > 5$ and $y > 2$; $x' < 0$, $y' < 0$ when $x > 5$ and $0 < y < 2$; $x' < 0$, $y' > 0$ when $0 < x < 5$ and $0 < y < 2$
e. Clockwise direction



33. a. $p_1 = 3$, $p_2 = -4$ **b.** $y(t) = t^3 - t^{-4}$, $t > 0$

CHAPTER 10

Section 10.1 Exercises, pp. 647–649

1. A sequence is an ordered list of numbers. Example: $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$
3. 1, 1, 2, 6, 24 **5.** $a_n = (-1)^{n+1} n$, for $n = 1, 2, 3, \ldots$; $a_n = (-1)^n (n + 1)$, for $n = 0, 1, 2, \ldots$ (Answers may vary.)
7. e **9.** 1, 5, 14, 30 **11.** $\sum_{k=1}^{\infty} 10$ (Answer is not unique.)
13. $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10{,}000}$ **15.** $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}$ **17.** $\frac{4}{3}, \frac{8}{5}, \frac{16}{9}, \frac{32}{17}$
19. 2, 1, 0, 1 **21.** 2, 4, 8, 16 **23.** 10, 18, 42, 114 **25.** $1, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}$
27. a. $\frac{1}{32}, \frac{1}{64}$ **b.** $a_1 = 1$, $a_{n+1} = \frac{1}{2} a_n$, for $n \geq 1$ **c.** $a_n = \frac{1}{2^{n-1}}$, for $n \geq 1$ **29. a.** 32, 64 **b.** $a_1 = 1$, $a_{n+1} = 2a_n$, for $n \geq 1$
c. $a_n = 2^{n-1}$, for $n \geq 1$ **31. a.** 243, 729 **b.** $a_1 = 1$, $a_{n+1} = 3a_n$, for $n \geq 1$ **c.** $a_n = 3^{n-1}$, for $n \geq 1$ **33. a.** $-5, 5$ **b.** $a_1 = -5$, $a_{n+1} = -a_n$, for $n \geq 1$ **c.** $a_n = (-1)^n \cdot 5$, for $n \geq 1$
35. 9, 99, 999, 9999; diverges **37.** $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10{,}000}$; converges to 0 **39.** 2, 4, 2, 4; diverges **41.** 2, 2, 2, 2; converges to 2
43. 54.545, 54.959, 54.996, 55.000; converges to 55

45.

n	a_n
1	0.83333333
2	0.96153846
3	0.99206349
4	0.99840256
5	0.99968010
6	0.99993600
7	0.99998720
8	0.99999744
9	0.99999949
10	0.99999990

The limit appears to be 1.

47.

n	a_n
1	2
2	6
3	12
4	20
5	30
6	42
7	56
8	72
9	90
10	110

The sequence appears to diverge.

49. a. $\frac{5}{2}, \frac{9}{4}, \frac{17}{8}, \frac{33}{16}$ **b.** 2

51.

n	a_n
1	3.00000000
2	3.50000000
3	3.75000000
4	3.87500000
5	3.93750000
6	3.96875000
7	3.98437500
8	3.99218750
9	3.99609375
10	3.99804688

The limit appears to be 4.

53.

n	a_n
0	1
1	5
2	21
3	85
4	341
5	1365
6	5461
7	21,845
8	87,381
9	349,525

The sequence appears to diverge.

55.

n	a_n
1	8.00000000
2	4.41421356
3	4.05050150
4	4.00629289
5	4.00078630
6	4.00009828
7	4.00001229
8	4.00000154
9	4.00000019
10	4.00000002

The limit appears to be 4.

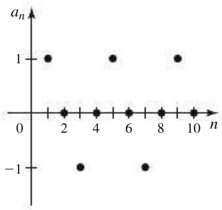
57. a. 20, 10, 5, $\frac{5}{2}$ **b.** $h_n = 20\left(\frac{1}{2}\right)^n$, for $n \geq 0$ **59. a.** 30, $\frac{15}{2}, \frac{15}{8}, \frac{15}{32}$ **b.** $h_n = 30\left(\frac{1}{4}\right)^n$, for $n \geq 0$ **61.** $S_1 = 0.3, S_2 = 0.33, S_3 = 0.333, S_4 = 0.3333; \frac{1}{3}$ **63.** $S_1 = 4, S_2 = 4.9, S_3 = 4.99, S_4 = 4.999$; 5 **65.** $S_1 = 0.6, S_2 = 0.66, S_3 = 0.666, S_4 = 0.6666; \frac{2}{3}$ **67. a.** $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}$ **b.** $S_n = \frac{2n}{2n+1}$; $\frac{10}{11}, \frac{12}{13}, \frac{14}{15}, \frac{16}{17}$ **c.** 1

69. a. 9, 9.9, 9.99, 9.999 **b.** $S_n = 10 - (0.1)^{n-1}$; 9.9999, 9.99999, 9.999999, 9.9999999 **c.** 10 **71. a.** True **b.** False **c.** True **73. a.** 20, 10, 5, $\frac{5}{2}, \frac{5}{4}$ **b.** $M_n = 20\left(\frac{1}{2}\right)^n$, for $n \geq 0$ **c.** $M_0 = 20$, $M_{n+1} = \frac{1}{2}M_n$, for $n \geq 0$ **d.** $\lim_{n\to\infty} M_n = 0$ **75. a.** 200, 190, 180.5, 171.475, 162.90125 **b.** $d_n = 200(0.95)^n$, for $n \geq 0$ **c.** $d_0 = 200$, $d_{n+1} = (0.95)d_n$, for $n \geq 0$ **d.** $\lim_{n\to\infty} d_n = 0$ **77. a.** 40, 70, 92.5, 109.375 **b.** 160 **79.** 0.739

Section 10.2 Exercises, pp. 659–662

1. $a_n = \frac{1}{n}, n \geq 1$ **3.** $a_n = \frac{n}{n+1}, n \geq 1$ **5.** Converges for $-1 < r \leq 1$, diverges otherwise **7.** Diverges monotonically **9.** Converges, oscillates; 0 **11.** $\{e^{n/100}\}$ grows faster then $\{n^{100}\}$. **13.** 0 **15.** $\frac{3}{2}$ **17.** $\frac{\pi}{4}$ **19.** 2 **21.** 0 **23.** $\frac{1}{4}$ **25.** 2 **27.** 0 **29.** 0 **31.** 3 **33.** Diverges **35.** $\frac{\pi}{2}$ **37.** 0 **39.** e^2 **41.** e^3 **43.** $e^{1/4}$ **45.** 0 **47.** 1 **49.** 0 **51.** 6

53.



Diverges

55. 0 **57.** Diverges **59.** Diverges **61.** 0 **63.** 0 **65.** 0 **67.** 0 **69.** 0 **71. a.** $d_{n+1} = \frac{1}{2}d_n + 80$, for $n \geq 1$ **b.** 160 mg

73. a. \$0, \$100, \$200.75, \$302.26, \$404.53 **b.** $B_{n+1} = 1.0075B_n + 100$, for $n \geq 0$ **c.** Approx. 43 months **75.** 0 **77.** Diverges **79.** 0 **81.** 1 **83. a.** True **b.** False **c.** True **d.** True **e.** False **f.** True **85. a.** Nondecreasing **b.** $\frac{1}{2}$ **87. a.** Nonincreasing **b.** 2 **89. a.** $d_{n+1} = 0.4d_n + 75$; $d_1 = 75$ **c.** 125; in the long run there will be approximately 125 mg of medication in the blood. **91.** 0.607 **93. b.** 9 **95. a.** $\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}$, $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$, or 1.41421, 1.84776, 1.96157, 1.99037 **c.** 2 **97. a.** 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 **b.** No **99. b.** 1, 2, 1.5, 1.6667, 1.6 **c.** Approx. 1.618 **e.** $\frac{a + \sqrt{a^2 + 4b}}{2}$ **101.** Given a tolerance $\varepsilon > 0$, look beyond a_N, where $N > 1/\varepsilon$. **103.** Given a tolerance $\varepsilon > 0$, look beyond a_N, where $N > \frac{1}{4}\sqrt{3/\varepsilon}$, provided $\varepsilon < \frac{3}{4}$. **105.** Given a tolerance $\varepsilon > 0$, look beyond a_N, where $N > c/(\varepsilon b^2)$. **107.** $a < 1$ **109.** $\{n^2 + 2n - 17\}_{n=3}^{\infty}$ **111.** $n = 4, n = 6, n = 25$

Section 10.3 Exercises, pp. 668–671

1. The constant r in the series $\sum_{k=0}^{\infty} ar^k$ **3.** No **5. a.** $a = \frac{2}{3}; r = \frac{1}{5}$ **b.** $a = \frac{1}{27}; r = -\frac{1}{3}$ **7.** $S_n = \frac{1}{4} - \frac{1}{n+4}$; $S_{36} = \frac{9}{40}$ **9.** 9841 **11.** Approx. 1.1905 **13.** Approx. 0.5392 **15.** $\frac{1093}{2916}$ **17.** \$15,920.22 **19. a.** $\frac{7}{9}$ **21.** $\frac{4}{3}$ **23.** $\frac{10}{19}$ **25.** 10 **27.** Diverges **29.** $\frac{1}{e^2 - 1}$ **31.** $\frac{1}{7}$ **33.** $\frac{1}{500}$ **35.** $\frac{3\pi}{\pi + 1}$ **37.** $\frac{\pi}{\pi - e}$ **39.** $\frac{9}{460}$ **41.** $\frac{4}{11}$ **43.** $A_5 = 266.406$; $A_{10} = 266.666$; $A_{30} = 266.667$; $\lim_{n\to\infty} A_n = 266\frac{2}{3}$ mg, which is the steady-state level. **45.** 400 mg **47.** $0.\overline{3} = \sum_{k=1}^{\infty} 3(0.1)^k = \frac{1}{3}$ **49.** $0.\overline{037} = \sum_{k=1}^{\infty} 37(0.001)^k = \frac{1}{27}$ **51.** $0.\overline{456} = \sum_{k=0}^{\infty} 0.456(0.001)^k = \frac{152}{333}$ **53.** $0.00\overline{952} = \sum_{k=0}^{\infty} 0.00952(0.001)^k = \frac{238}{24{,}975}$ **55.** $S_n = \frac{n}{2n+4}; \frac{1}{2}$ **57.** $S_n = \frac{1}{7} - \frac{1}{n+7}; \frac{1}{7}$ **59.** $S_n = \frac{1}{9} - \frac{1}{4n+9}; \frac{1}{9}$ **61.** $S_n = \ln(n+1)$; diverges

63. $S_n = \frac{1}{p+1} - \frac{1}{n+p+1}; \frac{1}{p+1}$

65. $S_n = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right) - \left(\frac{1}{\sqrt{n+2}} + \frac{1}{\sqrt{n+3}}\right); \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}$

67. $S_n = \frac{13}{12} - \frac{1}{n+2} - \frac{3}{n+3} - \frac{1}{n+4}; \frac{13}{12}$

69. $S_n = \tan^{-1}(n+1) - \tan^{-1}1; \frac{\pi}{4}$ **71. a, b.** $\frac{4}{3}$ **73.** $-\frac{1}{4}$

75. $\frac{2500}{19}$ **77.** $-\frac{2}{15}$ **79.** $\frac{1}{\ln 2}$ **81.** -2 **83.** $\frac{113}{30}$ **85.** $\frac{17}{10}$

87. a. True **b.** True **c.** False **d.** False **e.** True **f.** False **g.** True **89. a.** $\frac{1}{5}$ **b.** Approx. 0.19999695 **91.** Approx. 0.96

95. 462 months **99.** $\sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^k A_1 = \frac{A_1}{1 - 1/4} = \frac{4}{3}A_1$

101. a. $L_n = 3\left(\frac{4}{3}\right)^n$, so $\lim_{n\to\infty} L_n = \infty$ **b.** $\lim_{n\to\infty} A_n = \frac{2\sqrt{3}}{5}$

103. $R_n = S - S_n = \frac{1}{1-r} - \left(\frac{1-r^n}{1-r}\right) = \frac{r^n}{1-r}$ **105. a.** 60 **b.** 9 **107. a.** 13 **b.** 15 **109. a.** $1, \frac{5}{6}, \frac{2}{3}$, undefined, undefined **b.** $(-1, 1)$ **111.** Converges for x in $(-\infty, -2)$ or $(0, \infty)$; $x = \frac{1}{2}$

Section 10.4 Exercises, pp. 680–683

1. The series diverges. **3.** $\lim_{k\to\infty} a_k = 0$ **5.** Converges for $p > 1$ and diverges for $p \le 1$ **7.** $R_n = S - S_n$ **9.** Diverges
11. Inconclusive **13.** Diverges **15.** Diverges **17.** Diverges
19. Diverges **21.** Converges **23.** Diverges **25.** Converges
27. Diverges **29.** Converges **31.** Converges **33.** Converges
35. Diverges **37.** Diverges **39. a.** $S \approx S_2 = 1.0078125$
b. $R_2 < 0.0026042$ **c.** $L_2 = 1.0080411; U_2 = 1.0104167$

41. a. $\frac{1}{5n^5}$ **b.** 3 **c.** $L_n = S_n + \frac{1}{5(n+1)^5}; U_n = S_n + \frac{1}{5n^5}$

d. $(1.017342754, 1.017343512)$ **43. a.** $\frac{3^{-n}}{\ln 3}$ **b.** 7

c. $L_n = S_n + \frac{3^{-n-1}}{\ln 3}; U_n = S_n + \frac{3^{-n}}{\ln 3}$

d. $(0.499996671, 0.500006947)$ **45.** 1.0083 **47. a.** False
b. True **c.** False **d.** True **e.** False **f.** False **49.** Converges
51. Converges **53.** Diverges **55.** Diverges **57.** Diverges
59. Converges **61.** Converges **63.** Converges **65. a.** $p > 1$

b. $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ converges faster. **67.** $\zeta(3) \approx 1.202, \zeta(5) \approx 1.037$

69. $\frac{\pi^2}{8}$ **73. a.** $\frac{1}{2}, \frac{7}{12}, \frac{37}{60}$

Section 10.5 Exercises, pp. 687–688

1. Find an appropriate comparison series. Then take the limit of the ratio of the terms of the given series and the comparison series as $n \to \infty$. The value of the limit determines whether the series converges.

3. $\sum_{k=1}^{\infty}\frac{1}{k^2}$ **5.** $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^k$ **7.** $\sum_{k=1}^{\infty}\frac{1}{k}$ **9.** Converges
11. Diverges **13.** Converges **15.** Converges **17.** Converges
19. Diverges **21.** Diverges **23.** Converges **25.** Diverges
27. Converges **29.** Diverges **31.** Diverges **33.** Diverges
35. Converges **37. a.** False **b.** True **c.** True **d.** True
39. Converges **41.** Diverges **43.** Diverges **45.** Diverges
47. Converges **49.** Diverges **51.** Converges **53.** Converges
55. Diverges **57.** Converges **59.** Diverges **61.** Converges

Section 10.6 Exercises, pp. 694–696

1. Because $S_{n+1} - S_n = (-1)^n a_{n+1}$ alternates sign
3. Because the remainder $R_n = S - S_n$ alternates sign
5. $|R_n| = |S - S_n| \le |S_{n+1} - S_n| = a_{n+1}$ **7.** No; if a series of positive terms converges, it does so absolutely and not conditionally.

9. Yes, $\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$ has this property. **11.** Converges **13.** Diverges
15. Converges **17.** Converges **19.** Diverges **21.** Diverges
23. Diverges **25.** Diverges **27.** Converges **29.** $S_4 = -0.945939$; $|S - S_4| \le 0.0016$ **31.** $S_5 = 0.70696$; $|S - S_5| \le 0.001536$
33. 10,000 **35.** 5000 **37.** 10 **39.** -0.973 **41.** -0.269
43. -0.783 **45.** Converges conditionally **47.** Converges absolutely
49. Converges absolutely **51.** Converges absolutely **53.** Diverges
55. Converges conditionally **57.** Diverges **59.** Converges absolutely **61.** Converges conditionally **63.** Converges absolutely
65. a. False **b.** True **c.** True **d.** True **e.** False **f.** True
g. True **69.** x and y are divergent series.

71. b. $S_{2n} = \sum_{k=1}^{n}\left(\frac{1}{k^2} - \frac{1}{k}\right)$

Section 10.7 Exercises, pp. 699–700

1. Take the limit of the magnitude of the ratio of consecutive terms of the series as $k \to \infty$. The value of the limit determines whether the series converges absolutely or diverges. **3.** 999,000

5. $\frac{1}{(k+2)(k+1)}$ **7.** Ratio Test **9.** Converges absolutely
11. Diverges **13.** Converges absolutely **15.** Converges absolutely
17. Diverges **19.** Diverges **21.** Converges absolutely
23. Converges absolutely **25.** Diverges **27.** Converges absolutely
29. Diverges **31. a.** False **b.** True **c.** True **d.** True
33. Converges absolutely **35.** Diverges **37.** Converges absolutely
39. Converges conditionally **41.** Converges absolutely
43. Converges absolutely **45.** Converges conditionally
47. Converges absolutely **49.** Converges conditionally
51. $p > 1$ **53.** $p > 1$ **55.** $p < 1$ **57.** Diverges for all p
59. $-1 < x < 1$ **61.** $-1 \le x \le 1$ **63.** $-2 < x < 2$

Section 10.8 Exercises, pp. 703–704

1. Root Test **3.** Divergence Test **5.** p-series Test or Limit Comparison Test **7.** Comparison Test or Limit Comparison Test
9. Alternating Series Test **11.** Diverges **13.** Diverges
15. Converges **17.** Diverges **19.** Converges **21.** Converges
23. Converges **25.** Converges **27.** Converges **29.** Diverges
31. Converges **33.** Diverges **35.** Converges **37.** Diverges
39. Diverges **41.** Converges **43.** Diverges **45.** Converges
47. Diverges **49.** Converges **51.** Diverges **53.** Converges
55. Diverges **57.** Converges **59.** Converges **61.** Diverges
63. Diverges **65.** Converges **67.** Converges **69.** Diverges
71. Converges **73.** Converges **75.** Converges **77.** Diverges
79. Diverges **81.** Converges **83.** Converges **85.** Converges
87. a. False **b.** True **c.** True **d.** False **89.** Diverges
91. Diverges **93.** Diverges

Chapter 10 Review Exercises, pp. 704–707

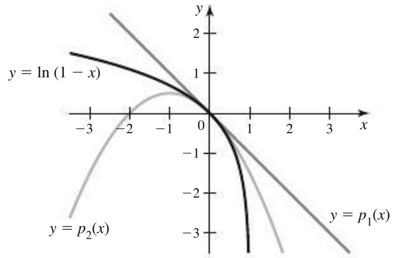
1. a. False **b.** False **c.** True **d.** False **e.** True **f.** False **g.** False **h.** True **3.** Approx. 1.25; approx. 0.05 **5.** $\lim_{k\to\infty} a_k = 0$, $\lim_{n\to\infty} S_n = 8$ **7.** $a_k = \frac{1}{k}$ **9. a.** 0 **b.** $\frac{5}{9}$ **11. a.** Yes; $\lim_{k\to\infty} a_k = 1$ **b.** No; $\lim_{k\to\infty} a_k \neq 0$ **13.** Diverges **15.** 5 **17.** 0 **19.** 0 **21.** $1/e$ **23.** Diverges **25. a.** 80, 48, 32, 24, 20 **b.** 16 **27.** Diverges **29.** Diverges **31.** Diverges **33.** $\frac{3\pi}{4}$ **35.** 3 **37.** 2/9 **39.** $\frac{311}{990}$ **41.** 200 mg **43.** Diverges **45.** Diverges **47.** Converges **49.** Converges **51.** Converges **53.** Converges **55.** Converges **57.** Diverges **59.** Converges **61.** Converges **63.** Converges **65.** Converges **67.** Converges **69.** Converges **71.** Converges **73.** Diverges **75.** Diverges **77.** Converges conditionally **79.** Converges absolutely **81.** Diverges **83.** Converges absolutely **85.** Converges absolutely **87.** Diverges **89. a.** Approx. 1.03666 **b.** 0.0004 **c.** $L_5 = 1.03685$; $U_5 = 1.03706$ **91.** 0.0067 **93.** 100 **95. a.** 803 m, 1283 m, $2000(1 - 0.95^N)$ m **b.** 2000 m **97. a.** $\frac{\pi}{2^{n-1}}$ **b.** 2π **99. a.** $T_1 = \frac{\sqrt{3}}{16}$, $T_2 = \frac{7\sqrt{3}}{64}$ **b.** $T_n = \frac{\sqrt{3}}{4}\left(1 - \left(\frac{3}{4}\right)^n\right)$ **c.** $\lim_{n\to\infty} T_n = \frac{\sqrt{3}}{4}$ **d.** 0 **101.** $\sqrt{\frac{20}{g}}\left(\frac{1 + \sqrt{p}}{1 - \sqrt{p}}\right)$ s

CHAPTER 11

Section 11.1 Exercises, pp. 718–721

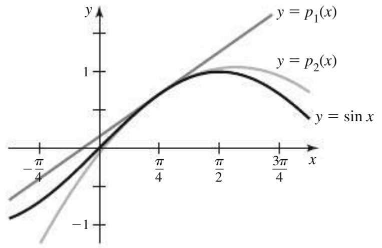
1. $f(0) = p_2(0)$, $f'(0) = p_2'(0)$, and $f''(0) = p_2''(0)$ **3.** 1, 1.05, 1.04875 **5.** $p_3(x) = 1 + x^2 + x^3$; 1.048 **7.** $p_3(x) = 1 + (x - 2) + 2(x - 2)^3$; 0.898 **9. a.** $p_1(x) = 8 + 12(x - 1)$ **b.** $p_2(x) = 8 + 12(x - 1) + 3(x - 1)^2$ **c.** 9.2; 9.23 **11. a.** $p_1(x) = 1 - 2x$ **b.** $p_2(x) = 1 - 2x + 2x^2$ **c.** 0.8, 0.82 **13. a.** $p_1(x) = 1 - x$ **b.** $p_2(x) = 1 - x + x^2$ **c.** 0.95, 0.9525 **15. a.** $p_1(x) = 2 + \frac{1}{12}(x - 8)$ **b.** $p_2(x) = 2 + \frac{1}{12}(x - 8) - \frac{1}{288}(x - 8)^2$ **c.** $1.958\overline{3}$, 1.95747 **17.** $p_1(x) = 1$, $p_2(x) = p_3(x) = 1 - 18x^2$, $p_4(x) = 1 - 18x^2 + 54x^4$ **19.** $p_3(x) = 1 - 3x + 6x^2 - 10x^3$, $p_4(x) = 1 - 3x + 6x^2 - 10x^3 + 15x^4$ **21.** $p_1(x) = 1 + 3(x - 1)$, $p_2(x) = 1 + 3(x - 1) + 3(x - 1)^2$, $p_3(x) = 1 + 3(x - 1) + 3(x - 1)^2 + (x - 1)^3$ **23.** $p_3(x) = 1 + \frac{1}{e}(x - e) - \frac{1}{2e^2}(x - e)^2 + \frac{1}{3e^3}(x - e)^3$ **25. a.** $p_1(x) = -x$, $p_2(x) = -x - \frac{x^2}{2}$

b.



27. a. $p_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$, $p_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$

b.



29. a. 1.0247 **b.** 7.6×10^{-6} **31. a.** 0.8613 **b.** 5.4×10^{-4} **33. a.** 1.1274988 **b.** Approx. 8.85×10^{-6} (Answers may vary if intermediate calculations are rounded.) **35. a.** Approx. −0.10033333 **b.** Approx. 1.34×10^{-6} (Answers may vary if intermediate calculations are rounded.) **37. a.** 1.0295635 **b.** Approx. 4.86×10^{-7} (Answers may vary if intermediate calculations are rounded.) **39. a.** Approx. 0.52083333 **b.** Approx. 2.62×10^{-4} (Answers may vary if intermediate calculations are rounded.) **41.** $R_n(x) = \frac{\sin^{(n+1)}(c)}{(n + 1)!}x^{n+1}$, for c between x and 0 **43.** $R_n(x) = \frac{(-1)^{n+1}e^{-c}}{(n + 1)!}x^{n+1}$, for c between x and 0 **45.** $R_n(x) = \frac{\sin^{(n+1)}(c)}{(n + 1)!}\left(x - \frac{\pi}{2}\right)^{n+1}$, for c between x and $\frac{\pi}{2}$ **47.** 2.0×10^{-5} **49.** 1.6×10^{-5} $(e^{0.25} < 2)$ **51.** 2.6×10^{-4} **53.** With $n = 4$, $|\text{error}| \leq 2.5 \times 10^{-3}$ **55.** With $n = 2$, $|\text{error}| \leq 4.2 \times 10^{-2}$ $(e^{0.5} < 2)$ **57.** With $n = 2$, $|\text{error}| \leq 5.4 \times 10^{-3}$ **59.** 4 **61.** 3 **63.** 1 **65. a.** False **b.** True **c.** True **d.** True **67. a.** C **b.** E **c.** A **d.** D **e.** B **f.** F **69. a.** 0.1; 1.7×10^{-4} **b.** 0.2; 1.3×10^{-3} **71. a.** 0.995; 4.2×10^{-6} **b.** 0.98; 6.7×10^{-5} **73. a.** 1.05; 1.3×10^{-3} **b.** 1.1; 5×10^{-3} **75. a.** 1.1; 10^{-2} **b.** 1.2; 4×10^{-2}

77. a.

x	$\lvert \sec x - p_2(x) \rvert$	$\lvert \sec x - p_4(x) \rvert$
−0.2	3.39×10^{-4}	5.51×10^{-6}
−0.1	2.09×10^{-5}	8.51×10^{-8}
0.0	0	0
0.1	2.09×10^{-5}	8.51×10^{-8}
0.2	3.39×10^{-4}	5.51×10^{-6}

b. The errors decrease as $|x|$ decreases.

79. a.

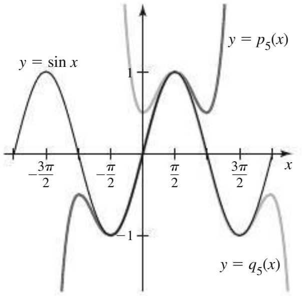
x	$\lvert e^{-x} - p_1(x) \rvert$	$\lvert e^{-x} - p_2(x) \rvert$
−0.2	2.14×10^{-2}	1.40×10^{-3}
−0.1	5.17×10^{-3}	1.71×10^{-4}
0.0	0	0
0.1	4.84×10^{-3}	1.63×10^{-4}
0.2	1.87×10^{-2}	1.27×10^{-3}

b. The errors decrease as $|x|$ decreases.

81. Centered at $x = 0$, for all n

83. a. $y = f(a) + f'(a)(x - a)$ **85. a.** $p_5(x) = x - \dfrac{x^3}{6} + \dfrac{x^5}{120}$; $q_5(x) = -(x - \pi) + \dfrac{1}{6}(x - \pi)^3 - \dfrac{1}{120}(x - \pi)^5$

b.



p_5 is a better approximation on $[-\pi, \pi/2)$; q_5 is a better approximation on $(\pi/2, 2\pi]$.

c.

x	$\lvert\sin x - p_5(x)\rvert$	$\lvert\sin x - q_5(x)\rvert$
$\pi/4$	3.6×10^{-5}	7.4×10^{-2}
$\pi/2$	4.5×10^{-3}	4.5×10^{-3}
$3\pi/4$	7.4×10^{-2}	3.6×10^{-5}
$5\pi/4$	2.3	3.6×10^{-5}
$7\pi/4$	20	7.4×10^{-2}

d. p_5 is a better approximation at $x = \pi/4$; at $x = \pi/2$ the errors are equal.

87. a. $p_1(x) = 6 + \dfrac{1}{12}(x - 36)$; $q_1(x) = 7 + \dfrac{1}{14}(x - 49)$

b.

x	$\lvert\sqrt{x} - p_1(x)\rvert$	$\lvert\sqrt{x} - q_1(x)\rvert$
37	5.7×10^{-4}	6.0×10^{-2}
39	5.0×10^{-3}	4.1×10^{-2}
41	1.4×10^{-2}	2.5×10^{-2}
43	2.6×10^{-2}	1.4×10^{-2}
45	4.2×10^{-2}	6.1×10^{-3}
47	6.1×10^{-2}	1.5×10^{-3}

c. p_1 is a better approximation at $x = 37$, 39, and 41.

Section 11.2 Exercises, pp. 729–730

1. $c_0 + c_1x + c_2x^2 + c_3x^3$ **3.** Ratio and Root Tests **5.** The radius of convergence does not change. The interval of convergence may change. **7.** $R = 10$; $[-8, 12]$ **9.** $R = \frac{1}{2}$; $\left(-\frac{1}{2}, \frac{1}{2}\right)$
11. $R = 0$; $\{x: x = 0\}$ **13.** $R = \infty$; $(-\infty, \infty)$ **15.** $R = 3$; $(-3, 3)$
17. $R = \infty$; $(-\infty, \infty)$ **19.** $R = 2$; $(-2, 2)$ **21.** $R = \infty$; $(-\infty, \infty)$
23. $R = 1$; $(0, 2]$ **25.** $R = \frac{1}{4}$; $\left[0, \frac{1}{2}\right]$ **27.** $R = 5$; $(-3, 7)$
29. $R = \infty$; $(-\infty, \infty)$ **31.** $R = \sqrt{3}$; $(-\sqrt{3}, \sqrt{3})$ **33.** $R = 1$; $(0, 2)$
35. $R = \infty$; $(-\infty, \infty)$ **37.** $R = e$ **39.** $R = e^4$
41. $\displaystyle\sum_{k=0}^{\infty}(3x)^k$; $\left(-\frac{1}{3}, \frac{1}{3}\right)$ **43.** $\displaystyle 2\sum_{k=0}^{\infty}x^{k+3}$; $(-1, 1)$
45. $\displaystyle 4\sum_{k=0}^{\infty}x^{k+12}$; $(-1, 1)$ **47.** $\displaystyle -\sum_{k=1}^{\infty}\frac{(3x)^k}{k}$; $\left[-\frac{1}{3}, \frac{1}{3}\right)$
49. $\displaystyle -2\sum_{k=1}^{\infty}\frac{x^{k+6}}{k}$; $[-1, 1)$ **51.** $\displaystyle g(x) = 2\sum_{k=1}^{\infty}k(2x)^{k-1}$; $\left(-\frac{1}{2}, \frac{1}{2}\right)$
53. $\displaystyle g(x) = \sum_{k=1}^{\infty}(-1)^k kx^{k-1}$; $(-1, 1)$
55. $\displaystyle g(x) = -\sum_{k=1}^{\infty}\frac{3^kx^k}{k}$; $\left[-\frac{1}{3}, \frac{1}{3}\right)$
57. $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}2kx^{2k-1}$; $(-1, 1)$ **59.** $\displaystyle\sum_{k=0}^{\infty}\left(-\frac{x}{3}\right)^k$; $(-3, 3)$
61. $\displaystyle \ln 2 - \frac{1}{2}\sum_{k=1}^{\infty}\frac{x^{2k}}{k\,4^k}$; $(-2, 2)$ **63. a.** True **b.** True **c.** True **d.** True **65.** $|x - a| < R$ **67.** $f(x) = \dfrac{1}{3 - \sqrt{x}}$; $1 < x < 9$
69. $f(x) = \dfrac{e^x}{e^x - 1}$; $0 < x < \infty$ **71.** $f(x) = \dfrac{3}{4 - x^2}$; $-2 < x < 2$
73. $\displaystyle\sum_{k=0}^{\infty}\frac{(-3x)^k}{k!}$; $-\infty < x < \infty$
75. $\displaystyle\lim_{k\to\infty}\left|\frac{c_{k+1}x^{k+1}}{c_kx^k}\right| = \lim_{k\to\infty}\left|\frac{c_{k+1}x^{k+m+1}}{c_kx^{k+m}}\right|$, so by the Ratio Test, the two series have the same radius of convergence.
77. a. $f(x)\,g(x) = c_0d_0 + (c_0d_1 + c_1d_0)x + (c_0d_2 + c_1d_1 + c_2d_0)x^2$ **b.** $\displaystyle\sum_{k=0}^{n}c_k d_{n-k}$

Section 11.3 Exercises, pp. 740–742

1. The nth Taylor polynomial is the nth partial sum of the corresponding Taylor series. **3.** $\displaystyle\sum_{k=0}^{\infty}\frac{(x - 2)^k}{k!}$ **5.** Replace x with x^2 in the Taylor series for $f(x)$; $|x| < 1$. **7.** The Taylor series for a function f converges to f on an interval if, for all x in the interval, $\lim_{n\to\infty}R_n(x) = 0$, where $R_n(x)$ is the remainder at x.
9. a. $1 - 2(x - 1) + 3(x - 1)^2 - 4(x - 1)^2$
b. $\displaystyle\sum_{k=0}^{\infty}(-1)^k(k + 1)(x - 1)^k$ **c.** $(0, 2)$ **11. a.** $1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!}$
b. $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^kx^k}{k!}$ **c.** $(-\infty, \infty)$ **13. a.** $2 + 6x + 12x^2 + 20x^3$
b. $\displaystyle\sum_{k=0}^{\infty}(k + 1)(k + 2)x^k$ **c.** $(-1, 1)$ **15. a.** $1 - x^2 + x^4 - x^6$
b. $\displaystyle\sum_{k=0}^{n}(-1)^kx^{2k}$ **c.** $(-1, 1)$ **17. a.** $1 + 2x + \dfrac{(2x)^2}{2!} + \dfrac{(2x)^3}{3!}$
b. $\displaystyle\sum_{k=0}^{\infty}\frac{(2x)^k}{k!}$ **c.** $(-\infty, \infty)$ **19. a.** $\dfrac{x}{2} - \dfrac{x^3}{3\cdot 2^3} + \dfrac{x^5}{5\cdot 2^5} - \dfrac{x^7}{7\cdot 2^7}$
b. $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^kx^{2k+1}}{(2k + 1)2^{2k+1}}$ **c.** $[-2, 2]$
21. a. $1 + (\ln 3)x + \dfrac{\ln^2 3}{2}x^2 + \dfrac{\ln^3 3}{6}x^3$ **b.** $\displaystyle\sum_{k=0}^{\infty}\frac{\ln^k 3}{k!}x^k$ **c.** $(-\infty, \infty)$
23. a. $1 + \dfrac{(3x)^2}{2} + \dfrac{(3x)^4}{24} + \dfrac{(3x)^6}{720}$ **b.** $\displaystyle\sum_{k=0}^{\infty}\frac{(3x)^{2k}}{(2k)!}$ **c.** $(-\infty, \infty)$
25. a. $(x - 3) - \dfrac{1}{2}(x - 3)^2 + \dfrac{1}{3}(x - 3)^3 - \dfrac{1}{4}(x - 3)^4$
b. $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(x - 3)^k}{k}$ **c.** $(2, 4]$
27. a. $1 - \dfrac{(x - \pi/2)^2}{2!} + \dfrac{(x - \pi/2)^4}{4!} - \dfrac{(x - \pi/2)^6}{6!}$
b. $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}(x - \pi/2)^{2k}$
29. a. $1 - (x - 1) + (x - 1)^2 - (x - 1)^3$ **b.** $\displaystyle\sum_{k=0}^{\infty}(-1)^k(x - 1)^k$
31. a. $\ln 3 + \dfrac{(x - 3)}{3} - \dfrac{(x - 3)^2}{3^2\cdot 2} + \dfrac{(x - 3)^3}{3^3\cdot 3}$
b. $\displaystyle\ln 3 + \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(x - 3)^k}{k3^k}$

33. a. $2 + 2(\ln 2)(x-1) + (\ln^2 2)(x-1)^2 + \frac{\ln^3 2}{3}(x-1)^3$
b. $\sum_{k=0}^{\infty} \frac{2(x-1)^k \ln^k 2}{k!}$ **35.** $x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4}$
37. $1 + 2x + 4x^2 + 8x^3$ **39.** $1 + \frac{x}{2} + \frac{x^2}{6} + \frac{x^3}{24}$
41. $1 - x^4 + x^8 - x^{12}$ **43.** $x^2 + \frac{x^6}{6} + \frac{x^{10}}{120} + \frac{x^{14}}{5040}$
45. a. $1 - 2x + 3x^2 - 4x^3$ **b.** 0.826
47. a. $1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3$ **b.** 1.029
49. a. $1 - \frac{2}{3}x + \frac{5}{9}x^2 - \frac{40}{81}x^3$ **b.** 0.895 **51.** $1 + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{16}$; $[-1, 1]$ **53.** $3 - \frac{3x}{2} - \frac{3x^2}{8} - \frac{3x^3}{16}$; $[-1, 1]$
55. $a + \frac{x^2}{2a} - \frac{x^4}{8a^3} + \frac{x^6}{16a^5}$; $|x| \le a$
57. $1 - 8x + 48x^2 - 256x^3$ **59.** $\frac{1}{16} - \frac{x^2}{32} + \frac{3x^4}{256} - \frac{x^6}{256}$
61. $\frac{1}{9} - \frac{2}{9}\left(\frac{4x}{3}\right) + \frac{3}{9}\left(\frac{4x}{3}\right)^2 - \frac{4}{9}\left(\frac{4x}{3}\right)^3$
63. $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$, where c is between 0 and x and $f^{(n+1)}(c) = \pm\sin c$ or $\pm\cos c$. Therefore, $|R_n(x)| \le \frac{|x|^{n+1}}{(n+1)!} \to 0$ as $n \to \infty$, for $-\infty < x < \infty$. **65.** $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$, where c is between 0 and x and $f^{(n+1)}(c) = (-1)^n e^{-c}$. Therefore, $|R_n(x)| \le \frac{|x|^{n+1}}{e^c(n+1)!} \to 0$ as $n \to \infty$, for $-\infty < x < \infty$.
67. a. False **b.** True **c.** False **d.** False **e.** True
69. a. $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!}$ **b.** $R = \infty$
71. a. $1 - \frac{2}{3}x^2 + \frac{5}{9}x^4 - \frac{40}{81}x^6$ **b.** $R = 1$
73. a. $1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6$ **b.** $R = 1$
75. a. $1 - 2x^2 + 3x^4 - 4x^6$ **b.** $R = 1$ **77.** Approx. 3.9149
79. Approx. 1.8989 **85.** $\sum_{k=0}^{\infty}\left(\frac{x-4}{2}\right)^k$ **87.** Use three terms of the Taylor series for cos x centered at $a = \pi/4$; 0.766 **89. a.** Use three terms of the Taylor series for $\sqrt[3]{125 + x}$ centered at $a = 0$; 5.03968 **b.** Use three terms of the Taylor series for $\sqrt[3]{x}$ centered at $a = 125$; 5.03968 **c.** Yes

Section 11.4 Exercises, pp. 748–750

1. Replace f and g with their Taylor series centered at a and evaluate the limit. **3.** Substitute $x = -0.6$ into the Taylor series for e^x centered at 0. Because the resulting series is an alternating series, the error can be estimated easily. **7.** 1 **9.** $\frac{1}{2}$ **11.** 2 **13.** $\frac{2}{3}$ **15.** $\frac{1}{3}$
17. $\frac{3}{5}$ **19.** $-\frac{8}{5}$ **21.** 1 **23.** $\frac{3}{4}$ **25. a.** $1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots$
b. e^x **c.** $-\infty < x < \infty$
27. a. $1 - x + x^2 - \cdots (-1)^{n-1}x^{n-1} + \cdots$ **b.** $\frac{1}{1+x}$ **c.** $|x| < 1$
29. a. $-2 + 4x - 8 \cdot \frac{x^2}{2!} + \cdots + (-2)^n \frac{x^{n-1}}{(n-1)!} + \cdots$
b. $-2e^{-2x}$ **c.** $-\infty < x < \infty$ **31. a.** $1 - x^2 + x^4 - \cdots$
b. $\frac{1}{1+x^2}$ **c.** $-1 < x < 1$
33. a. $2 + 2t + \frac{2t^2}{2!} + \cdots + \frac{2t^n}{n!} + \cdots$ **b.** $y(t) = 2e^t$
35. a. $2 + 16t + 24t^2 + 24t^3 + \cdots + \frac{3^{n-1} \cdot 16}{n!}t^n + \cdots$
b. $y(t) = \frac{16}{3}e^{3t} - \frac{10}{3}$ **37.** 0.2448 **39.** 0.6958
41. 0.0600 **43.** 0.4994 **45.** $1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!}$
47. $1 - 2 + \frac{2}{3} - \frac{4}{45}$ **49.** $\frac{1}{2} - \frac{1}{8} + \frac{1}{24} - \frac{1}{64}$ **51.** $e - 1$
53. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}x^k}{k}$, for $-1 < x \le 1$; ln 2 **55.** $\frac{2}{2-x}$ **57.** $\frac{4}{4+x^2}$
59. $-\ln(1-x)$ **61.** $-\frac{3x^2}{(3+x)^2}$ **63.** $\frac{6x^2}{(3-x)^3}$
65. a. False **b.** False **c.** True **67.** $\frac{a}{b}$ **69.** $e^{-1/6}$
71. $f^{(3)}(0) = 0$; $f^{(4)}(0) = 4e$ **73.** $f^{(3)}(0) = 2$; $f^{(4)}(0) = 0$
75. 2 **77.** 1.575 using four terms **79. a.** $S'(x) = \sin x^2$; $C'(x) = \cos x^2$ **b.** $\frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} - \frac{x^{15}}{15 \cdot 7!}$; $x - \frac{x^5}{5 \cdot 2!} + \frac{x^9}{9 \cdot 4!} - \frac{x^{13}}{13 \cdot 6!}$ **c.** $S(0.05) \approx 0.00004166664807$; $C(-0.25) \approx -0.2499023614$ **d.** 1 **e.** 2
81. a. $1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304}$ **b.** $R = \infty, -\infty < x < \infty$

Chapter 11 Review Exercises, pp. 750–752

1. a. True **b.** False **c.** True **d.** True **e.** True
3. $p_2(x) = 1 - \frac{3}{2}x^2$ **5.** $p_2(x) = 1 - (x-1) + \frac{3}{2}(x-1)^2$
7. $p_2(x) = 1 - \frac{1}{2}(x-1)^2$
9. $p_3(x) = \frac{5}{4} + \frac{3}{4}(x - \ln 2) + \frac{5}{8}(x - \ln 2)^2 + \frac{1}{8}(x - \ln 2)^3$
11. a. $p_1(x) = 1 + x$; $p_2(x) = 1 + x + \frac{x^2}{2}$
b.

n	$p_n(x)$	Error
1	0.92	3.1×10^{-3}
2	0.9232	8.4×10^{-5}

13. a. $p_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$; $p_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$
b.

n	$p_n(x)$	Error
1	0.5960	8.2×10^{-3}
2	0.5873	4.7×10^{-4}

15. $|R_3| < \frac{\pi^4}{4!}$ **17.** $R = \infty, (-\infty, \infty)$ **19.** $R = \infty, (-\infty, \infty)$
21. $R = 9, (-9, 9)$ **23.** $R = 2, [-4, 0)$ **25.** $R = \frac{3}{2}, [-2, 1]$
27. $R = \frac{1}{27}$ **29.** $\sum_{k=0}^{\infty} x^{2k}$; $(-1, 1)$ **31.** $\sum_{k=0}^{\infty}(-5x)^k$; $\left(-\frac{1}{5}, \frac{1}{5}\right)$

33. $\sum_{k=1}^{\infty} k(10x)^{k-1}$; $\left(-\frac{1}{10}, \frac{1}{10}\right)$ **35.** $1 + 3x + \frac{9x^2}{2!}$; $\sum_{k=0}^{\infty} \frac{(3x)^k}{k!}$

37. $-(x - \pi/2) + \frac{(x - \pi/2)^3}{3!} - \frac{(x - \pi/2)^5}{5!}$;

$\sum_{k=0}^{\infty} (-1)^{k+1} \frac{(x - \pi/2)^{2k+1}}{(2k + 1)!}$

39. $4x - \frac{(4x)^3}{3} + \frac{(4x)^5}{5}$; $\sum_{k=0}^{\infty} \frac{(-1)^k (4x)^{2k+1}}{2k + 1}$

41. $1 + 2(x - 1)^2 + \frac{2}{3}(x - 1)^4$; $\sum_{k=0}^{\infty} \frac{4^k (x - 1)^{2k}}{(2k)!}$

43. $1 + \frac{x}{3} - \frac{x^2}{9} + \cdots$ **45.** $1 - \frac{3}{2}x + \frac{3}{2}x^2 - \cdots$

47. $R_n(x) = \frac{(\sinh c + \cosh c)\, x^{n+1}}{(n + 1)!}$, where c is between 0 and x;

$\lim_{n\to\infty} |R_n(x)| = |\sinh c + \cosh c| \lim_{n\to\infty} \frac{|x|^{n+1}}{(n + 1)!} = 0$ because

$|x|^{n+1} \ll (n + 1)!$ for any fixed value of x.

49. $\frac{1}{24}$ **51.** $\frac{1}{8}$ **53.** $\frac{1}{6}$ **55.** Approx. 0.4615 **57.** Approx. 0.3819

59. $11 - \frac{1}{11} - \frac{1}{2 \cdot 11^3} - \frac{1}{2 \cdot 11^5}$ **61.** $-\frac{1}{3} + \frac{1}{3 \cdot 3^3} - \frac{1}{5 \cdot 3^5} + \frac{1}{7 \cdot 3^7}$

63. $y = 4 + 4x + \frac{4^2}{2!}x^2 + \frac{4^3}{3!}x^3 + \cdots + \frac{4^n}{n!}x^n + \cdots = 3 + e^{4x}$

65. a. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$ **b.** $\sum_{k=1}^{\infty} \frac{1}{k2^k}$ **c.** $2\sum_{k=0}^{\infty} \frac{x^{2k+1}}{2k + 1}$

d. $x = \frac{1}{3}$; $2\sum_{k=0}^{\infty} \frac{1}{3^{2k+1}(2k + 1)}$ **e.** Series in part (d)

CHAPTER 12

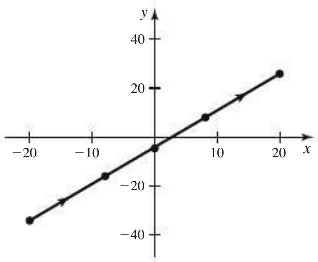
Section 12.1 Exercises, pp. 763–767

1. Plotting $\{(f(t), g(t)): a \le t \le b\}$ generates a curve in the xy-plane. **3.** $x = R \cos(\pi t/5)$, $y = -R \sin(\pi t/5)$ **5.** $x = t^2$, $y = t$, $-\infty < t < \infty$ **7.** $-\frac{1}{2}$ **9.** $x = t$, $y = t$, $0 \le t \le 6$; $x = 2t$, $y = 2t$, $0 \le t \le 3$; $x = 3t$, $y = 3t$, $0 \le t \le 2$ (answers will vary)

11. a.

t	−10	−4	0	4	10
x	−20	−8	0	8	20
y	−34	−16	−4	8	26

b.

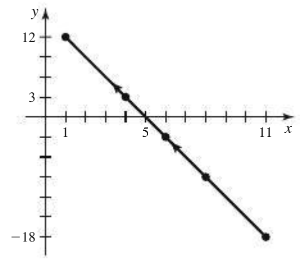


c. $y = \frac{3}{2}x - 4$
d. A line segment rising to the right as t increases

13. a.

t	−5	−2	0	2	5
x	11	8	6	4	1
y	−18	−9	−3	3	12

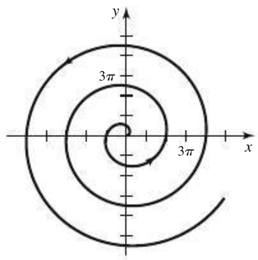
b.



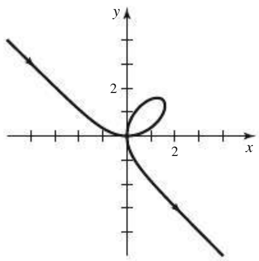
c. $y = -3x + 15$
d. A line segment rising to the left as t increases

15. a. $y = -x + 4$ **b.** A line segment starting at $(3, 1)$ and ending at $(4, 0)$ **17. a.** $y = 3x - 12$ **b.** A line segment starting at $(4, 0)$ and ending at $(8, 12)$ **19. a.** $x^2 + y^2 = 9$ **b.** Lower half of a circle of radius 3 centered at $(0, 0)$; starts at $(-3, 0)$ and ends at $(3, 0)$
21. a. $y = 1 - x^2$, $-1 \le x \le 1$ **b.** A parabola opening downward with a vertex at $(0, 1)$ starting at $(1, 0)$ and ending at $(-1, 0)$
23. a. $x^2 + (y - 1)^2 = 1$ **b.** A circle of radius 1 centered at $(0, 1)$; generated counterclockwise, starting and ending at $(1, 1)$
25. a. $y = (x + 1)^3$ **b.** A cubic curve rising to the right as r increases **27. a.** $x^2 + y^2 = 49$ **b.** A circle of radius 7 centered at $(0, 0)$; generated counterclockwise, starting and ending at $(-7, 0)$
29. a. $y = 1$, $-\infty < x < \infty$ **b.** A horizontal line with y-intercept 1, generated from left to right **31.** $x^2 + y^2 = 4$ **33.** $y = \sqrt{4 - x^2}$
35. $y = x^2$ **37.** $x = 4 \cos t$, $y = 4 \sin t$, $0 \le t \le 2\pi$
39. $x = \cos t + 2$, $y = \sin t + 3$, $0 \le t \le 2\pi$
41. $x = 2t$, $y = 8t$; $0 \le t \le 1$
43. $x = t$, $y = 2t^2 - 4$; $-1 \le t \le 5$ **45.** $x = 2$, $y = t$; $3 \le t \le 9$
47. $x = 4t - 2$, $y = -6t + 3$; $0 \le t \le 1$ and $x = t + 1$, $y = 8t - 11$; $1 \le t \le 2$
49. $x = 1 + 2t$, $y = 1 + 4t$; $-\infty < t < \infty$
51. $x = t^2$, $y = t$; $t \ge 0$
53. $x = 400 \cos\left(\frac{4\pi t}{3}\right)$, $y = 400 \sin\left(\frac{4\pi t}{3}\right)$; $0 \le t \le 1.5$
55. $x = 50 \cos\left(\frac{\pi t}{12}\right)$, $y(t) = 50 \sin\left(\frac{\pi t}{12}\right)$; $0 \le t \le 24$

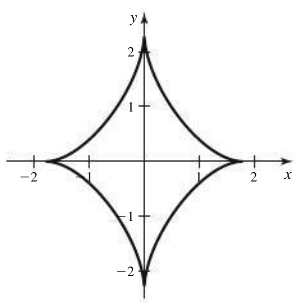
57.



59.



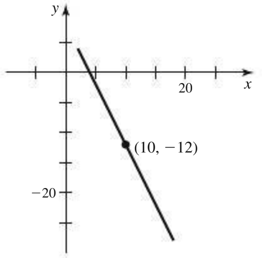
61.



63. Plot $x = 1 + \cos^2 t - \sin^2 t$, $y = t$.
65. Approx. 2857 m

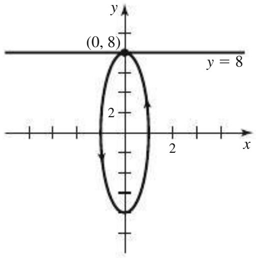
67. **a.** $\frac{dy}{dx} = -2; -2$

b.



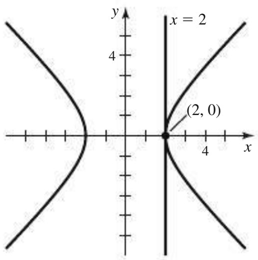
69. **a.** $\frac{dy}{dx} = -8 \cot t; 0$

b.



71. **a.** $\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}, t \neq 0;$ undefined

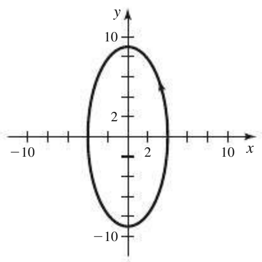
b.



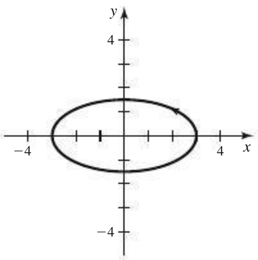
73. $y = \frac{13}{4}x + \frac{1}{4}$ **75.** $y = x - \frac{\pi\sqrt{2}}{4}$ **77.** $\left(-\frac{4}{\sqrt{5}}, \frac{8}{\sqrt{5}}\right)$ and $\left(\frac{4}{\sqrt{5}}, -\frac{8}{\sqrt{5}}\right)$ **79.** There is no such point. **81.** 10 **83.** $\pi\sqrt{2}$

85. $\frac{1}{3}(5\sqrt{5} - 8)$ **87.** $\frac{3}{2}$ **89.** **a.** False **b.** True **c.** False **d.** True **e.** True

91. $0 \leq t \leq 2\pi$

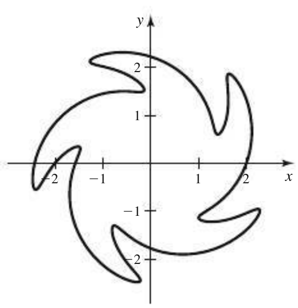


93. $x = 3 \cos t, y = \frac{3}{2} \sin t,$ $0 \leq t \leq 2\pi; \left(\frac{x}{3}\right)^2 + \left(\frac{2y}{3}\right)^2 = 1;$ in the counterclockwise direction

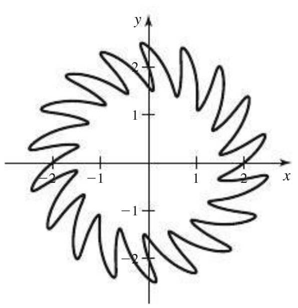


95. **a.** Lines intersect at $(1, 0)$. **b.** Lines are parallel. **c.** Lines intersect at $(4, 6)$.

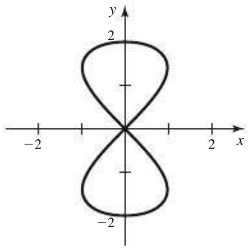
97.



99.



101.

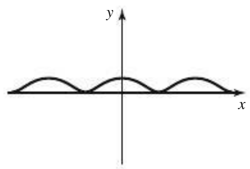


a. $(0, 2)$ and $(0, -2)$
b. $(1, \sqrt{2}), (1, -\sqrt{2}), (-1, \sqrt{2}), (-1, -\sqrt{2})$

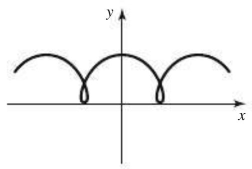
103. 27π **105.** $\frac{3\pi}{8}$ **107.** **a.** A circle of radius 3 centered at $(0, 4)$ **b.** A torus (doughnut); $48\pi^2$ **109.** $\frac{64\pi}{3}$

111. $\int_0^1 2\pi(e^{3t} + 1)\sqrt{4e^{4t} + 9e^{6t}}\, dt \approx 1445.9$

113. **a.**

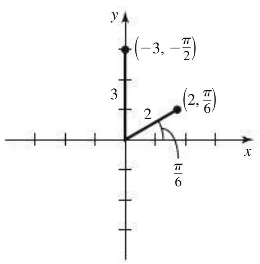


b.



Section 12.2 Exercises, pp. 775–779

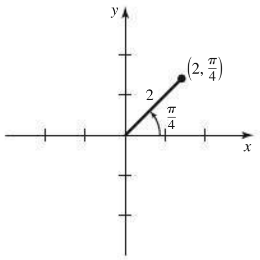
1.



$(-2, -5\pi/6), (2, 13\pi/6); (3, \pi/2), (3, 5\pi/2)$

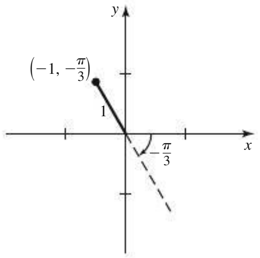
3. $r^2 = x^2 + y^2, \tan \theta = \frac{y}{x}$
5. $r \cos \theta = 5$ or $r = 5 \sec \theta$
7. x-axis symmetry occurs if (r, θ) on the graph implies $(r, -\theta)$ is on the graph. y-axis symmetry occurs if (r, θ) on the graph implies $(r, \pi - \theta) = (-r, -\theta)$ is on the graph. Symmetry about the origin occurs if (r, θ) on the graph implies $(-r, \theta) = (r, \theta + \pi)$ is on the graph.

9.



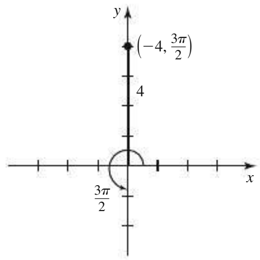
$(-2, -3\pi/4), (2, 9\pi/4)$

11.



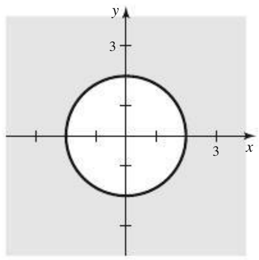
$(1, 2\pi/3), (1, 8\pi/3)$

13.



$(4, \pi/2), (4, 5\pi/2)$

15.



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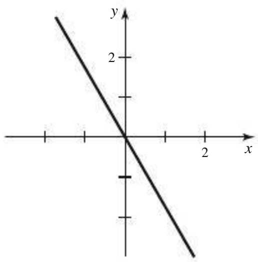

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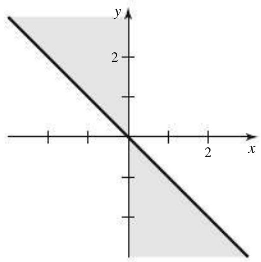

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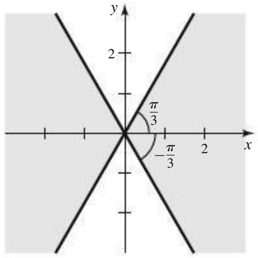
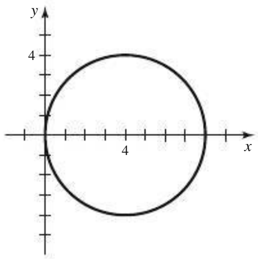
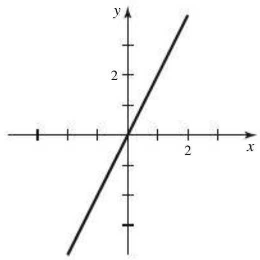
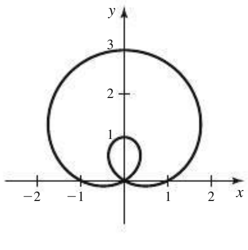


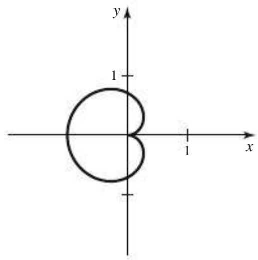
23. $\left(100, -\frac{\pi}{4}\right)$
25. $(3\sqrt{2}/2, 3\sqrt{2}/2)$
27. $(1/2, -\sqrt{3}/2)$
29. $(2\sqrt{2}, -2\sqrt{2})$
31. $(2\sqrt{2}, \pi/4)$, $(-2\sqrt{2}, 5\pi/4)$
33. $(2, \pi/3)$, $(-2, 4\pi/3)$
35. $(8, 2\pi/3)$, $(-8, -\pi/3)$

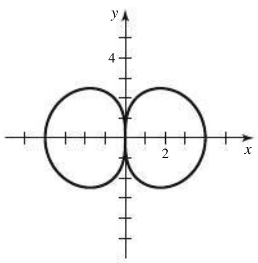
37. $x = -4$; vertical line passing through $(-4, 0)$
39. $x^2 + y^2 = 4$; circle of radius 2 centered at $(0, 0)$
41. $(x - 1)^2 + (y - 1)^2 = 2$; circle of radius $\sqrt{2}$ centered at $(1, 1)$
43. $(x - 3)^2 + (y - 4)^2 = 25$; circle of radius 5 centered at $(3, 4)$
45. $x^2 + (y - 1)^2 = 1$; circle of radius 1 centered at $(0, 1)$ and $x = 0$ **47.** $x^2 + (y - 4)^2 = 16$; circle of radius 4 centered at $(0, 4)$
49. $r = \tan\theta \sec\theta$ **51.** $r^2 = \sec\theta \csc\theta$ or $r^2 = 2 \csc 2\theta$

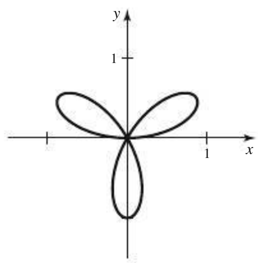
53.


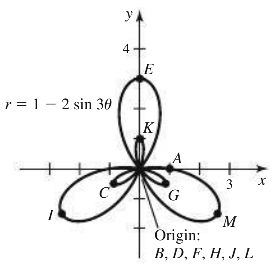
55.


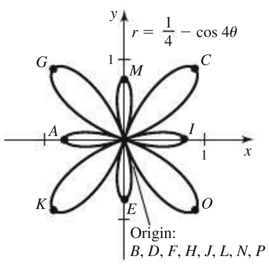
57.


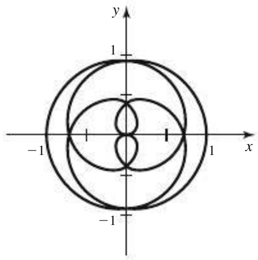
59.


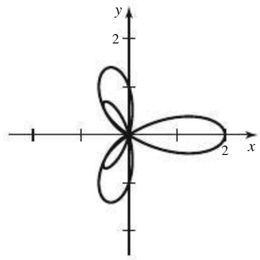
61.


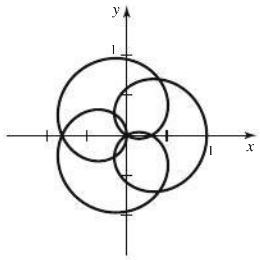
63.


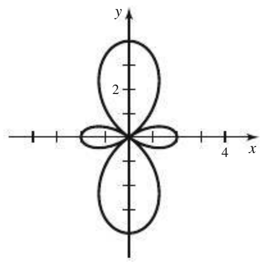
65.


67.


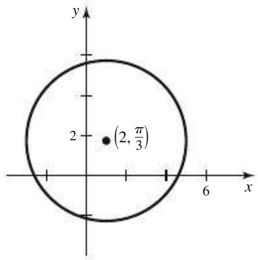
69. $[0, 8\pi]$


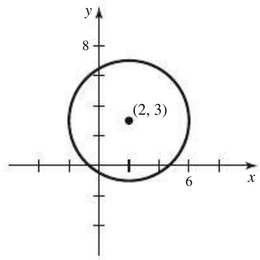
71. $[0, 2\pi]$


73. $[0, 5\pi]$


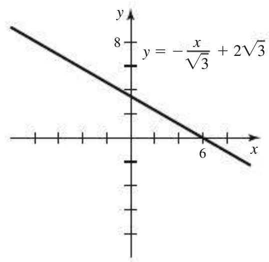
75. $[0, 2\pi]$


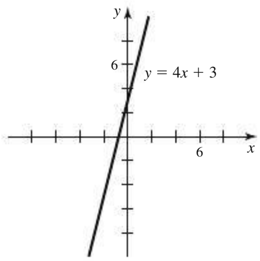
77. **a.** True **b.** True **c.** False **d.** True **e.** True

81. A circle of radius 4 and center $(2, \pi/3)$ (polar coordinates)


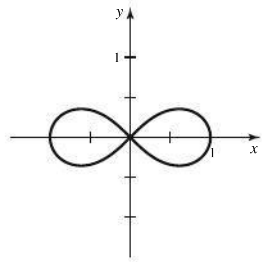
83. A circle of radius 4 centered at $(2, 3)$ (Cartesian coordinates)


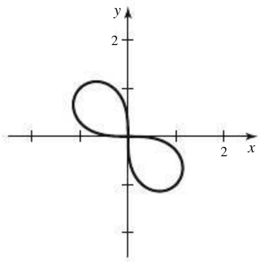
85. **a.** 132.3 miles **b.** 264.6 mi/hr

87.


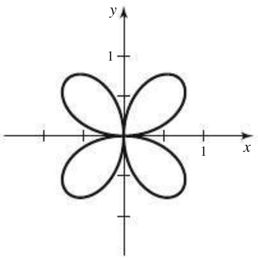
89.


91. **a.** A **b.** C **c.** B **d.** D **e.** E **f.** F

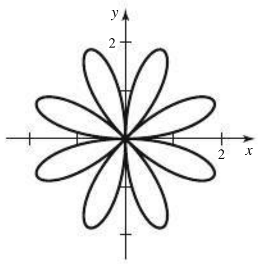
93.


95.


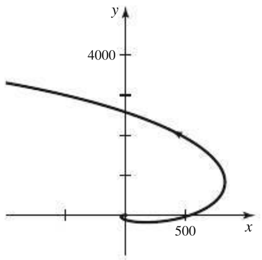
97.



99.

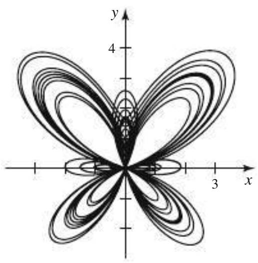


103.

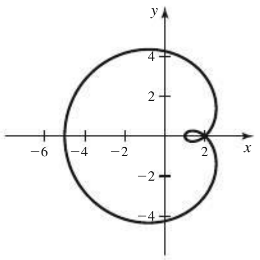


For $a = -1$, the spiral winds inward toward the origin.

105. a.



107. a.

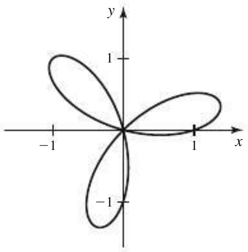


109. Symmetry about the x-axis **111.** $(x^2 + y^2)^2 = a^2(x^2 - y^2)$

Section 12.3 Exercises, pp. 786–788

1. $x = f(\theta)\cos\theta, y = f(\theta)\sin\theta$ **3.** The slope of the tangent line is the rate of change of the vertical coordinate with respect to the horizontal coordinate. **5.** $\sqrt{3}$ **7.** $\frac{\pi^2}{4}$ **9.** Both curves pass through the origin, but for different values of θ. **11.** 0 **13.** $-\sqrt{3}$ **15.** Undefined, undefined **17.** 0 at $(-4, \pi/2)$ and $(-4, 3\pi/2)$, undefined at $(4, 0)$ and $(4, \pi)$ **19.** ± 1 **21.** $\theta = \frac{3\pi}{4}$; $m = -1$

23. a. $[0, \pi]$

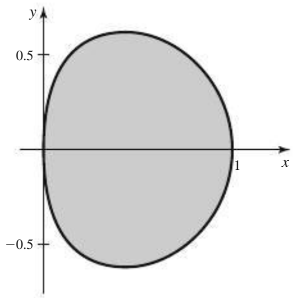


b. $\theta = \frac{\pi}{4}, m = 1; \theta = \frac{7\pi}{12}, m \approx -3.73;$ $\theta = \frac{11\pi}{12}, m \approx -0.27$

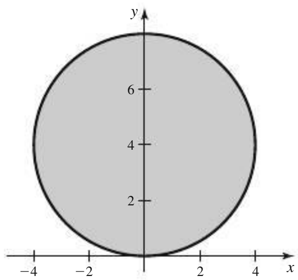
25. Horizontal: $(2\sqrt{2}, \pi/4), (-2\sqrt{2}, 3\pi/4)$; vertical: $(0, \pi/2), (4, 0)$ **27.** Horizontal: (0, 0) (0.943, 0.955), (−0.943, 2.186), (0.943, 4.097), (−0.943, 5.328); vertical: (0, 0), (0.943, 0.615), (−0.943, 2.526), (0.943, 3.757), (−0.943, 5.668) **29.** (2, 0) and (0, 0)

31. $\left(1, \frac{\pi}{12}\right), \left(1, \frac{5\pi}{12}\right), \left(1, \frac{7\pi}{12}\right), \left(1, \frac{11\pi}{12}\right),$ $\left(1, \frac{13\pi}{12}\right), \left(1, \frac{17\pi}{12}\right), \left(1, \frac{19\pi}{12}\right)$, and $\left(1, \frac{23\pi}{12}\right)$

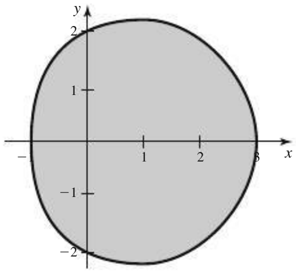
33. 1



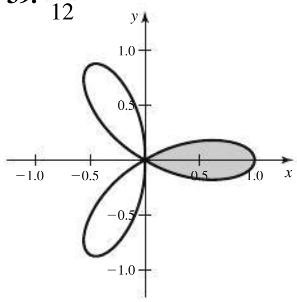
35. 16π



37. $9\pi/2$



39. $\frac{\pi}{12}$



41. a. $(0, 0), \left(\frac{3}{\sqrt{2}}, \frac{\pi}{4}\right)$ **b.** $\frac{9}{8}(\pi - 2)$

43. a. $\left(1 + \frac{1}{\sqrt{2}}, \frac{\pi}{4}\right), \left(1 - \frac{1}{\sqrt{2}}, \frac{5\pi}{4}\right), (0, 0)$ **b.** $\frac{3\pi}{2} - 2\sqrt{2}$

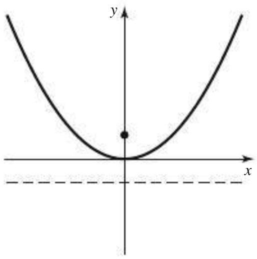
45. $\frac{1}{24}(3\sqrt{3} + 2\pi)$ **47.** $\frac{1}{4}(2 - \sqrt{3}) + \frac{\pi}{12}$ **49.** $\pi/20$ **51.** $4(4\pi/3 - \sqrt{3})$ **53.** $2\pi/3 - \sqrt{3}/2$ **55.** $9\pi + 27\sqrt{3}$ **57.** 6 **59.** 18π **61.** Intersection points: $\left(3, \pm\frac{\pi}{3}\right)$; area of region A $= 6\sqrt{3} - 2\pi$; area of region B $= 5\pi - 6\sqrt{3}$; area of region C $= 4\pi + 6\sqrt{3}$ **63.** πa **65.** $\frac{8}{3}\left((1 + \pi^2)^{3/2} - 1\right)$ **67.** 32 **69.** $63\sqrt{5}$ **71.** $\frac{2\pi - 3\sqrt{3}}{8}$ **73.** 26.73 **75. a.** False **b.** False **c.** True **77.** Horizontal: (0, 0), (4.05, 2.03), (9.83, 4.91); vertical: (1.72, 0.86), (6.85, 3.43), (12.87, 6.44) **79.** $\frac{\sqrt{1 + a^2}}{a}$

81. a. $A_n = \frac{1}{4e^{(4n+2)\pi}} - \frac{1}{4e^{4n\pi}} - \frac{1}{4e^{(4n-2)\pi}} + \frac{1}{4e^{(4n-4)\pi}}$ **b.** 0 **c.** $e^{-4\pi}$ **85.** $(a^2 - 2)\theta^* + \pi - \sin 2\theta^*$, where $\theta^* = \cos^{-1}(a/2)$ **87.** $a^2(\pi/2 + a/3)$

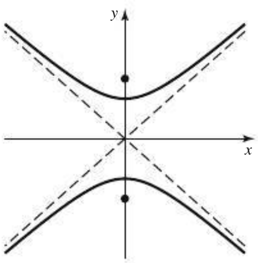
Section 12.4 Exercises, pp. 797–800

1. A parabola is the set of all points in a plane equidistant from a fixed point and a fixed line. **3.** A hyperbola is the set of all points in a plane whose distances from two fixed points have a constant difference.

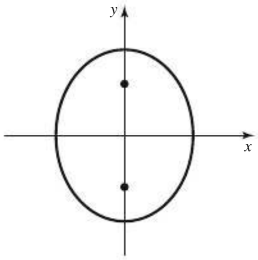
5. Parabola:



Hyperbola:

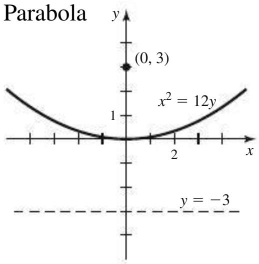


Ellipse:

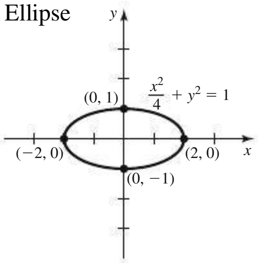


7. $\left(\frac{x}{a}\right)^2 + \frac{y^2}{a^2 - c^2} = 1$ **9.** $(\pm ae, 0)$ **11.** $y = \pm\frac{b}{a}x$

13. Parabola

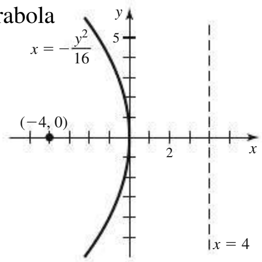


15. Ellipse

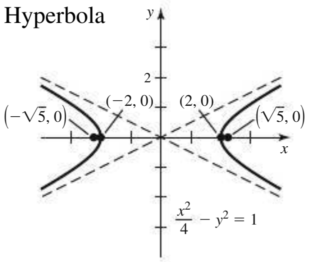


Vertices: $(\pm 2, 0)$; foci: $(\pm\sqrt{3}, 0)$; major axis has length 4; minor axis has length 2.

17. Parabola

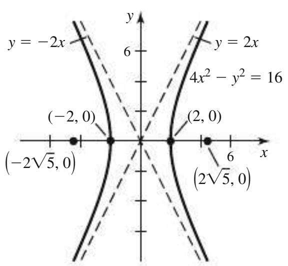


19. Hyperbola

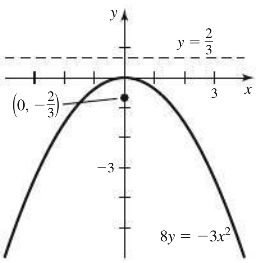


Vertices: $(\pm 2, 0)$;
foci: $(\pm\sqrt{5}, 0)$;
asymptotes: $y = \pm\frac{1}{2}x$

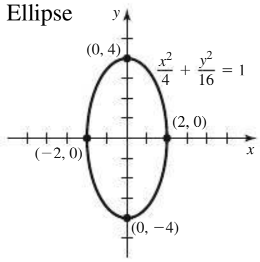
21. Hyperbola



23. Parabola

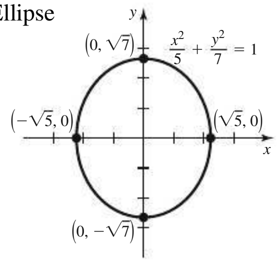


25. Ellipse



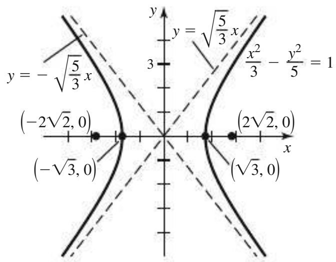
Vertices: $(0, \pm 4)$;
foci: $(0, \pm 2\sqrt{3})$; major axis has length 8; minor axis has length 4.

27. Ellipse



Vertices: $(0, \pm\sqrt{7})$; foci: $(0, \pm\sqrt{2})$; major axis has length $2\sqrt{7}$; minor axis has length $2\sqrt{5}$.

29. Hyperbola



31. $y^2 = 16x$ **33.** $y^2 = 12x$

35. $x^2 = -\frac{2}{3}y$

37. $y^2 = 4(x + 1)$

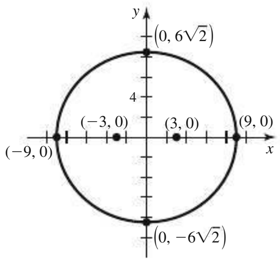
39. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

41. $\frac{x^2}{16} - \frac{y^2}{20} = 1$

43. $\frac{x^2}{25} + y^2 = 1$ **45.** $\frac{x^2}{4} - \frac{y^2}{9} = 1$ **47.** $\frac{x^2}{4} + \frac{y^2}{9} = 1$

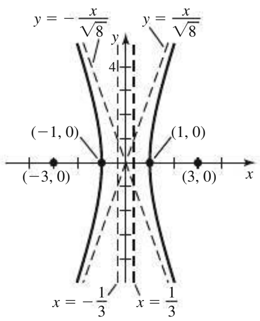
49. $\frac{x^2}{16} - \frac{y^2}{9} = 1$ **51. a.** True **b.** True **c.** True **d.** True

53. $\frac{x^2}{81} + \frac{y^2}{72} = 1$

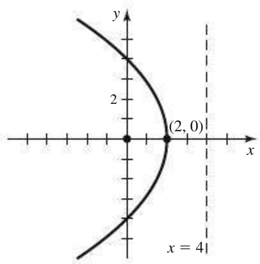


Directrices: $x = \pm 27$

55. $x^2 - \frac{y^2}{8} = 1$

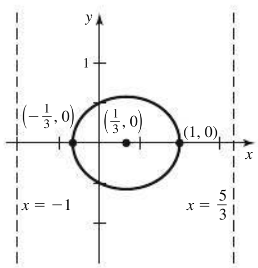


57.



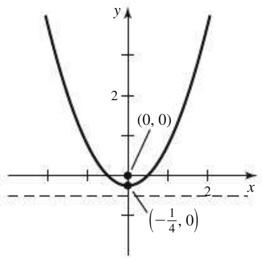
Vertex: $(2, 0)$; focus: $(0, 0)$; directrix: $x = 4$

59.



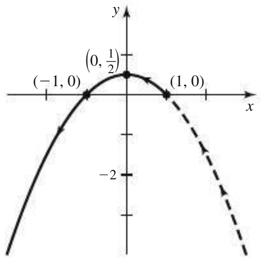
Vertices: $(1, 0)$, $\left(-\frac{1}{3}, 0\right)$; center: $\left(\frac{1}{3}, 0\right)$; foci: $(0, 0)$, $\left(\frac{2}{3}, 0\right)$; directrices: $x = -1$, $x = \frac{5}{3}$

61.



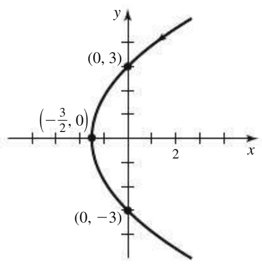
Vertex: $\left(0, -\frac{1}{4}\right)$; focus: $(0, 0)$; directrix: $y = -\frac{1}{2}$

63.



The parabola starts at $(1, 0)$ and goes through quadrants I, II, and III for θ in $[0, 3\pi/2]$; then it approaches $(1, 0)$ by traveling through quadrant IV on $(3\pi/2, 2\pi)$.

65.



The parabola begins in the first quadrant and passes through the points $(0, 3)$, $\left(-\frac{3}{2}, 0\right)$, and $(0, -3)$ as θ ranges from 0 to 2π.

67. The parabolas open to the left due to the presence of a positive $\cos\theta$ term in the denominator. As d increases, the directrix $x = d$ moves to the right, resulting in wider parabolas.

69. $y = 2x + 6$ **71.** $y = -\frac{3}{40}x - \frac{4}{5}$ **73.** $\frac{dy}{dx} = -\frac{b^2x}{a^2y}$, so $\frac{y - y_0}{x - x_0} = -\frac{b^2x_0}{a^2y_0}$, which is equivalent to the given equation.

75. $r = \frac{4}{1 - 2\sin\theta}$ **79.** $\frac{4\pi b^2 a}{3}$; $\frac{4\pi a^2 b}{3}$; yes, if $a \neq b$

81. a. $\frac{\pi b^2}{3a^2}(a - c)^2(2a + c)$ **b.** $\frac{4\pi b^4}{3a}$ **91.** $2p$

97. a. $u(m) = \frac{2m^2 - \sqrt{3m^2 + 1}}{m^2 - 1}$; $v(m) = \frac{2m^2 + \sqrt{3m^2 + 1}}{m^2 - 1}$; 2 intersection points for $|m| > 1$ **b.** $\frac{5}{4}, \infty$ **c.** 2, 2 **d.** $2\sqrt{3} - \ln(\sqrt{3} + 2)$

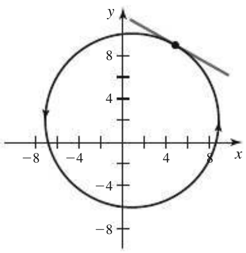
Chapter 12 Review Exercises, pp. 800–803

1. a. False **b.** False **c.** True **d.** False **e.** True **f.** True

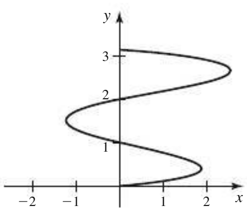
3. $\frac{x^2}{16} + \frac{y^2}{9} = 1$; ellipse generated counterclockwise

5. Segment of the parabola $y = \sqrt{x}$ starting at $(4, 2)$ and ending at $(9, 3)$

7. a. $(x - 1)^2 + (y - 2)^2 = 64$ **b.** $-\frac{1}{\sqrt{3}}$ **c.**



9. $x = 5(t - 1)(t - 2)\sin t$, $y = t$



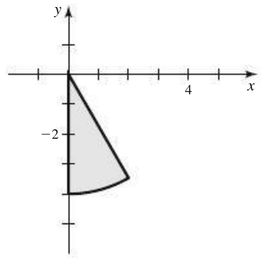
11. a. $x^2 + (y + 1)^2 = 9$ **b.** Lower half of a circle of radius 3 centered at $(0, -1)$, starting at $(3, -1)$ and ending at $(-3, -1)$ **c.** 0

13. At $t = \pi/6$: $y = (2 + \sqrt{3})x + \left(2 - \frac{\pi}{3} - \frac{\pi\sqrt{3}}{6}\right)$; at $t = \frac{2\pi}{3}$: $y = \frac{x}{\sqrt{3}} + 2 - \frac{2\pi}{3\sqrt{3}}$ **15.** $x = -1 + 2t, y = t$, for $0 \le t \le 1$; $x = 1 - 2t, y = 1 - t$, for $0 \le t \le 1$

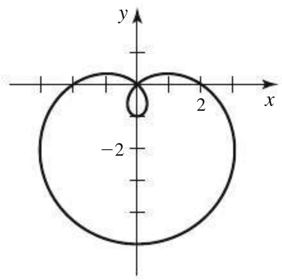
17. $x = 3\sin t, y = 3\cos t$, for $0 \le t \le 2\pi$

19. $\frac{4}{15}$ **21.** 9.1 **23.** $4 - 2\sqrt{2}$

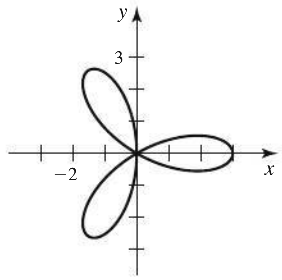
25.



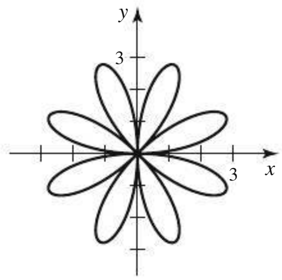
27.



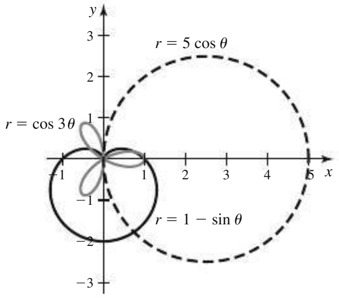
29.



31.



33. Liz should choose $r = 1 - \sin\theta$.



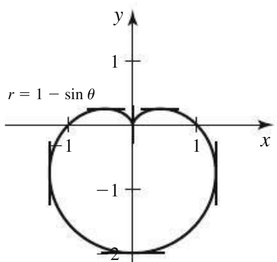
35. $(x - 3)^2 + (y + 1)^2 = 10$; a circle of radius $\sqrt{10}$ centered at $(3, -1)$

37. $r = 1 + \cos\theta$; a cardioid

39. $r = 8\cos\theta, 0 \le \theta \le \pi$

41. a. Horizontal: $\left(\frac{1}{2}, \frac{\pi}{6}\right), \left(\frac{1}{2}, \frac{5\pi}{6}\right), \left(2, \frac{3\pi}{2}\right)$; vertical: $\left(\frac{3}{2}, \frac{7\pi}{6}\right), \left(\frac{3}{2}, \frac{11\pi}{6}\right), \left(0, \frac{\pi}{2}\right)$ **b.** Undefined

c.

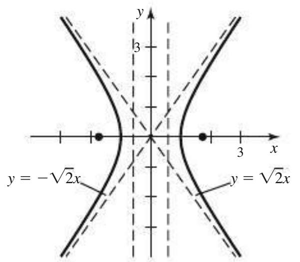


43. $\left(\frac{\pi}{12}, \frac{1}{2^{1/4}}\right), \left(\frac{3\pi}{4}, \frac{1}{2^{1/4}}\right), \left(\frac{17\pi}{12}, \frac{1}{2^{1/4}}\right), (0, 0)$

45. $\pi - \frac{3\sqrt{3}}{2}$ **47.** $2\sqrt{3} - \frac{2\pi}{3}$ **49.** 4 **51.** 40.09

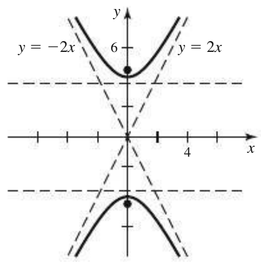
53. a. Hyperbola
b. Foci $(\pm\sqrt{3}, 0)$, vertices $(\pm 1, 0)$, directrices $x = \pm\dfrac{1}{\sqrt{3}}$
c. $e = \sqrt{3}$

d.



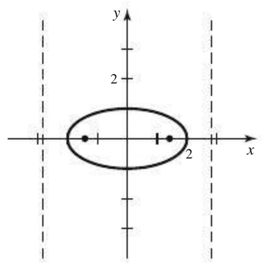
55. a. Hyperbola
b. Foci $(0, \pm 2\sqrt{5})$, vertices $(0, \pm 4)$, directrices $y = \pm\dfrac{8}{\sqrt{5}}$ **c.** $e = \dfrac{\sqrt{5}}{2}$

d.



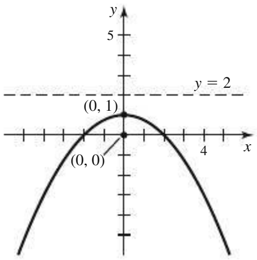
57. a. Ellipse
b. Foci $(\pm\sqrt{2}, 0)$, vertices $(\pm 2, 0)$, directrices $x = \pm 2\sqrt{2}$ **c.** $e = \dfrac{\sqrt{2}}{2}$

d.

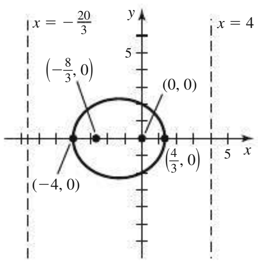


59. $y = \frac{3}{2}x - 2$

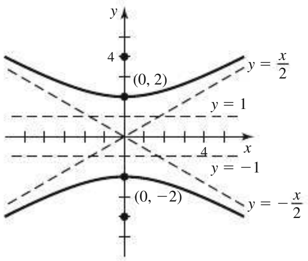
61. $e = 1$



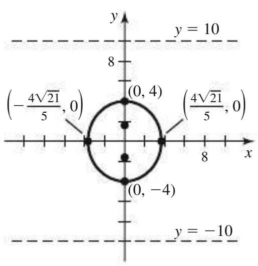
63. $e = \frac{1}{2}$



65. $\dfrac{y^2}{4} - \dfrac{x^2}{12} = 1$



67. $\dfrac{y^2}{16} + \dfrac{25x^2}{336} = 1$; foci: $\left(0, \pm\dfrac{8}{5}\right)$



69. $e = 2/3, y = \pm 9, (\pm 2\sqrt{5}, 0)$ **71.** $m = \dfrac{b}{a}$

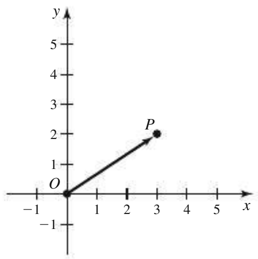
75. a. $x = \pm a\cos^{2/n} t, y = \pm b\sin^{2/n} t$
c. The curve becomes more rectangular as n increases.

CHAPTER 13

Section 13.1 Exercises, pp. 813–816

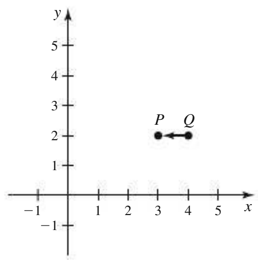
3. There are infinitely many vectors with the same direction and length as $\mathbf{v}$. **5.** $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$ **7.** No **9.** $|\langle v_1, v_2 \rangle| = \sqrt{v_1^2 + v_2^2}$ **11.** If P has coordinates (x_1, y_1) and Q has coordinates (x_2, y_2), then the magnitude of $\overrightarrow{PQ}$ is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. **13.** a, c, e **15. a.** $3\mathbf{v}$ **b.** $2\mathbf{u}$ **c.** $-3\mathbf{u}$ **d.** $-2\mathbf{u}$ **e.** $\mathbf{v}$ **17. a.** $3\mathbf{u} + 3\mathbf{v}$ **b.** $\mathbf{u} + 2\mathbf{v}$ **c.** $2\mathbf{u} + 5\mathbf{v}$ **d.** $-2\mathbf{u} + 3\mathbf{v}$ **e.** $3\mathbf{u} + 2\mathbf{v}$ **f.** $-3\mathbf{u} - 2\mathbf{v}$ **g.** $-2\mathbf{u} - 4\mathbf{v}$ **h.** $\mathbf{u} - 4\mathbf{v}$ **i.** $-\mathbf{u} - 6\mathbf{v}$

19. a.



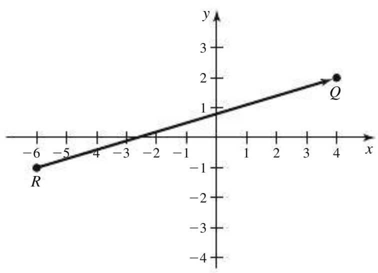
$\overrightarrow{OP} = \langle 3, 2 \rangle = 3\mathbf{i} + 2\mathbf{j}$
$|\overrightarrow{OP}| = \sqrt{13}$

b.



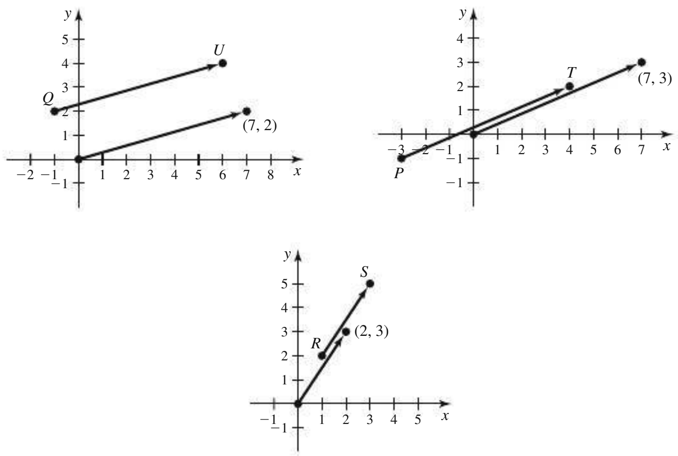
$\overrightarrow{QP} = \langle -1, 0 \rangle = -\mathbf{i}$
$|\overrightarrow{QP}| = 1$

c.



$\overrightarrow{RQ} = \langle 10, 3 \rangle = 10\mathbf{i} + 3\mathbf{j}$
$|\overrightarrow{RQ}| = \sqrt{109}$

21. $\overrightarrow{QU} = \langle 7, 2 \rangle, \overrightarrow{PT} = \langle 7, 3 \rangle, \overrightarrow{RS} = \langle 2, 3 \rangle$



23. $\overrightarrow{QT}$ **25.** $\langle -4, 10\rangle$ **27.** $\langle 52, -30\rangle$ **29.** $2\sqrt{2}$

31. $\mathbf{w} - \mathbf{u}$ **33.** $13\left\langle -\frac{5}{13}, \frac{12}{13}\right\rangle$ **35.** $\langle 3, 3\sqrt{3}\rangle$

37. $\left\langle \frac{15}{13}, -\frac{36}{13}\right\rangle$ **39.** $\left\langle \frac{30}{\sqrt{13}}, -\frac{20}{\sqrt{13}}\right\rangle$ **41.** $-\mathbf{i} + 10\mathbf{j}$

43. $\pm\frac{1}{\sqrt{61}}\langle 6, 5\rangle$ **45.** $\left\langle -\frac{28}{\sqrt{74}}, \frac{20}{\sqrt{74}}\right\rangle, \left\langle \frac{28}{\sqrt{74}}, -\frac{20}{\sqrt{74}}\right\rangle$

47. a. $\left\langle \frac{3}{5}, -\frac{4}{5}\right\rangle, \left\langle -\frac{3}{5}, \frac{4}{5}\right\rangle$ **b.** $b = \pm\frac{2\sqrt{2}}{3}$ **c.** $a = \pm\frac{3}{\sqrt{10}}$

49. $\langle -4\sqrt{3}, 4\rangle$ **51.** $\langle 15\sqrt{3}, -15\rangle$ **53. a.** $\mathbf{v}_a = \langle -320, 0\rangle$; $\mathbf{w} = \langle -20\sqrt{2}, -20\sqrt{2}\rangle$; $\mathbf{v}_g = \langle -320 - 20\sqrt{2}, -20\sqrt{2}\rangle$
b. Approx. 349.4 mi/hr; approx. 4.6° south of west
55. Approx. 490.3 mi/hr with a heading of about 1.2° west of north
57. $5\sqrt{65}$ km/hr $\approx$ 40.3 km/hr **59.** 1 m/s in the direction 30° east of north **61. a.** $\langle 20, 20\sqrt{3}\rangle$ **b.** Yes **c.** No **63.** $250\sqrt{2}$ lb
65. a. True **b.** True **c.** False **d.** False **e.** False **f.** False **g.** False **h.** True **67.** $\mathbf{x} = \left\langle \frac{1}{5}, -\frac{3}{10}\right\rangle$ **69.** $\mathbf{x} = \left\langle \frac{4}{3}, -\frac{11}{3}\right\rangle$

71. $4\mathbf{i} - 8\mathbf{j}$ **73.** $\langle a, b\rangle = \left(\frac{a+b}{2}\right)\mathbf{u} + \left(\frac{b-a}{2}\right)\mathbf{v}$

75. a. $\mathbf{0}$ **b.** The 6:00 vector **c.** Sum any six consecutive vectors. **d.** A vector pointing from 12:00 to 6:00 with a length 12 times the radius of the clock

77. $$\begin{aligned}\mathbf{u} + \mathbf{v} &= \langle u_1, u_2\rangle + \langle v_1, v_2\rangle = \langle u_1 + v_1, u_2 + v_2\rangle \\ &= \langle v_1 + u_1, v_2 + u_2\rangle = \langle v_1, v_2\rangle + \langle u_1, u_2\rangle \\ &= \mathbf{v} + \mathbf{u}\end{aligned}$$

79. $$\begin{aligned}a(c\mathbf{v}) &= a(c\langle v_1, v_2\rangle) = a\langle cv_1, cv_2\rangle \\ &= \langle acv_1, acv_2\rangle = \langle (ac)v_1, (ac)v_2\rangle \\ &= ac\langle v_1, v_2\rangle = (ac)\mathbf{v}\end{aligned}$$

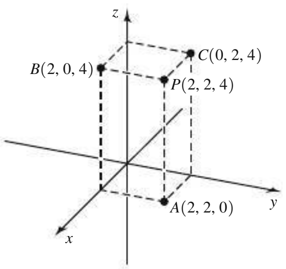
81. $$\begin{aligned}(a + c)\mathbf{v} &= (a + c)\langle v_1, v_2\rangle \\ &= \langle (a + c)v_1, (a + c)v_2\rangle \\ &= \langle av_1 + cv_1, av_2 + cv_2\rangle \\ &= \langle av_1, av_2\rangle + \langle cv_1, cv_2\rangle \\ &= a\langle v_1, v_2\rangle + c\langle v_1, v_2\rangle \\ &= a\mathbf{v} + c\mathbf{v}\end{aligned}$$

85. a. $\{\mathbf{u}, \mathbf{v}\}$ are linearly dependent. $\{\mathbf{u}, \mathbf{w}\}$ and $\{\mathbf{v}, \mathbf{w}\}$ are linearly independent. **b.** Two linearly dependent vectors are parallel. Two linearly independent vectors are not parallel. **87. a.** $\frac{5}{3}$ **b.** -15

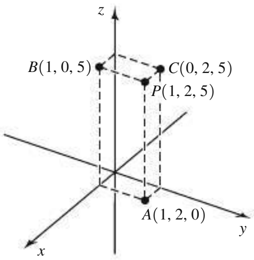
Section 13.2 Exercises, pp. 823–827

1. Move 3 units from the origin in the direction of the positive x-axis, then 2 units in the direction of the negative y-axis, and then 1 unit in the direction of the positive z-axis. **3.** It is parallel to the yz-plane and contains the point $(4, 0, 0)$. **5.** $\mathbf{u} + \mathbf{v} = \langle 9, 0, -6\rangle$; $3\mathbf{u} - \mathbf{v} = \langle 3, 20, -22\rangle$ **7.** $(0, 0, -4)$ **9.** $A(3, 0, 5), B(3, 4, 0), C(0, 4, 5)$ **11.** $A(3, -4, 5), B(0, -4, 0), C(0, -4, 5)$

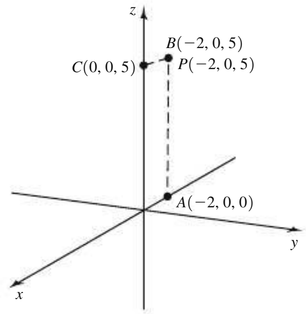
13. a.



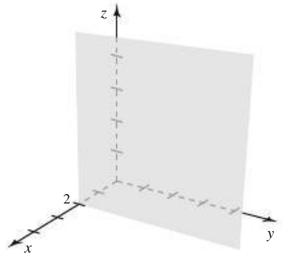
b.



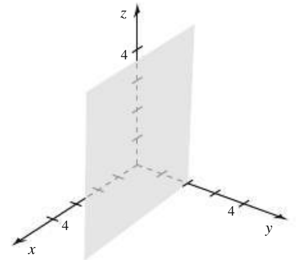
c.



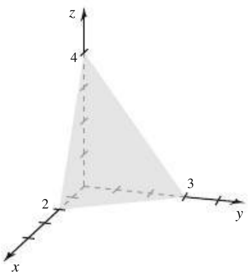
15.



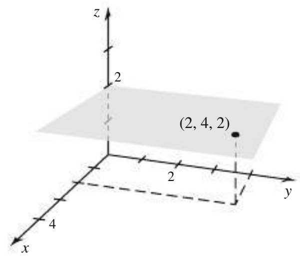
17.



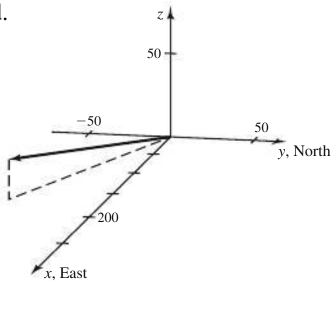
19.



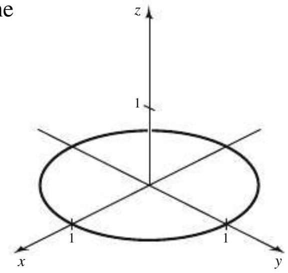
21.



23. $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 16$
25. $(x + 2)^2 + y^2 + (z - 4)^2 \le 1$
27. $\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 + (z - 7)^2 = \frac{13}{2}$ **29.** A sphere centered at $(1, 0, 0)$ with radius 3 **31.** A sphere centered at $(0, 1, 2)$ with radius 3 **33.** All points on or outside the sphere with center $(0, 7, 0)$ and radius 6 **35.** The ball centered at $(4, 7, 9)$ with radius 15
37. The single point $(1, -3, 0)$ **39. a.** $\langle 12, -7, 2\rangle$ **b.** $\langle 16, -13, -1\rangle$ **c.** 5 **41. a.** $\langle -4, 5, -4\rangle$ **b.** $\langle -9, 3, -9\rangle$ **c.** $3\sqrt{2}$ **43. a.** $\langle -15, 23, 22\rangle$ **b.** $\langle -31, 49, 33\rangle$ **c.** $3\sqrt{5}$
45. a. $\overrightarrow{PQ} = \langle 2, 6, 2\rangle = 2\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ **b.** $|\overrightarrow{PQ}| = 2\sqrt{11}$
c. $\left\langle \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right\rangle$ and $\left\langle -\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}}\right\rangle$
47. a. $\overrightarrow{PQ} = \langle 0, -5, 1\rangle = -5\mathbf{j} + \mathbf{k}$ **b.** $|\overrightarrow{PQ}| = \sqrt{26}$
c. $\left\langle 0, -\frac{5}{\sqrt{26}}, \frac{1}{\sqrt{26}}\right\rangle$ and $\left\langle 0, \frac{5}{\sqrt{26}}, -\frac{1}{\sqrt{26}}\right\rangle$
49. a. $\overrightarrow{PQ} = \langle -2, 4, -2\rangle = -2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ **b.** $|\overrightarrow{PQ}| = 2\sqrt{6}$
c. $\left\langle -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right\rangle$ and $\left\langle \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right\rangle$
51. a. $20\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}$; **b.** 30 mi/hr
53. The speed of the plane is approximately 220 mi/hr; the direction is slightly south of east and upward.



55. $5\sqrt{6}$ knots to the east, $5\sqrt{6}$ knots to the north, 10 knots upward **57. a.** False **b.** False **c.** False **d.** True **59.** All points in $\mathbb{R}^3$ except those on the coordinate axes **61.** A circle of radius 1 centered at $(0, 0, 0)$ in the xy-plane



63. A circle of radius 2 centered at $(0, 0, 1)$ in the horizontal plane $z = 1$ **65.** $(x - 2)^2 + (z - 1)^2 = 9, y = 4$ **67.** $6\left\langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \right\rangle$ **69.** $\left\langle -\frac{15}{4}, \frac{5}{2}, -\frac{5\sqrt{3}}{4} \right\rangle$ **71.** $\langle 12, -16, 0 \rangle, \langle -12, 16, 0 \rangle$ **73.** $\langle -\sqrt{3}, -\sqrt{3}, \sqrt{3} \rangle, \langle \sqrt{3}, \sqrt{3}, -\sqrt{3} \rangle$ **75. a.** Collinear; Q is between P and R. **b.** Collinear; P is between Q and R. **c.** Noncollinear **d.** Noncollinear **77.** $\left\langle \frac{500\sqrt{3}}{9}, 0, -\frac{500}{3} \right\rangle$, $\left\langle -\frac{250\sqrt{3}}{9}, -\frac{250}{3}, -\frac{500}{3} \right\rangle, \left\langle -\frac{250\sqrt{3}}{9}, \frac{250}{3}, -\frac{500}{3} \right\rangle$ **79.** $(3, 8, 9), (-1, 0, 3), (1, 0, -3)$

Section 13.3 Exercises, pp. 833–837

1. $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos\theta$ **3.** -40 **5.** $\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$, so $\theta = \cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$ **7.** $\left\langle -\frac{4}{3}, \frac{2}{3}, \frac{4}{3} \right\rangle$ **9.** -1 **11.** 2 **13.** $\frac{\pi}{2}; 0$ **15.** $100; \frac{\pi}{4}$ **17.** $\frac{1}{2}$ **19.** $0; \frac{\pi}{2}$ **21.** $1; \pi/3$ **23.** -2; 93.2° **25.** 2; 87.2° **27.** -4; 104° **29.** $\angle P = 78.8°$, $\angle Q = 47.2°$, $\angle R = 54.0°$ **31.** $\langle 3, 0 \rangle$; 3 **33.** $\langle 0, 3 \rangle$; 3 **35.** $\frac{6}{5}\langle -2, 1 \rangle; \frac{6}{\sqrt{5}}$ **37.** $\frac{14}{19}\langle -1, -3, 3 \rangle; -\frac{14}{\sqrt{19}}$ **39.** $-\mathbf{i} + \mathbf{j} - 2\mathbf{k}; \sqrt{6}$ **41.** $750\sqrt{3}$ ft-lb **43.** $25\sqrt{2}$ J **45.** 400 J **47.** $\frac{1}{2}\langle 5\sqrt{3}, -15 \rangle, \frac{1}{2}\langle -5\sqrt{3}, -5 \rangle$ **49.** $\langle 490, -490 \rangle$, $\langle -490, -490 \rangle$ **51. a.** False **b.** True **c.** True **d.** False **e.** False **f.** True **53.** $c = \frac{4}{9}$ **55.** $\langle 1, a, 4a - 2 \rangle$, a real **57. a.** $\text{proj}_{\mathbf{k}}\mathbf{u} = |\mathbf{u}| \cos 60°\left(\frac{\mathbf{k}}{|\mathbf{k}|}\right) = \frac{1}{2}\mathbf{k}$, for all such $\mathbf{u}$ **b.** Yes
59. The heads of the vectors lie on the line $y = 3 - x$.
61. The heads of the vectors lie on the plane $z = 3$.
63. $\mathbf{u} = \left\langle -\frac{4}{5}, -\frac{2}{5} \right\rangle + \left\langle -\frac{6}{5}, \frac{12}{5} \right\rangle$
65. $\mathbf{u} = \left\langle 1, \frac{1}{2}, \frac{1}{2} \right\rangle + \left\langle -2, \frac{3}{2}, \frac{5}{2} \right\rangle$ **67.** $3x - 7y = -36$
69. $-\frac{5}{3}$ **71.** $\mathbf{I} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}, \mathbf{J} = -\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$; $\mathbf{i} = \frac{1}{\sqrt{2}}(\mathbf{I} - \mathbf{J}), \mathbf{j} = \frac{1}{\sqrt{2}}(\mathbf{I} + \mathbf{J})$ **73. a.** $|\mathbf{I}| = |\mathbf{J}| = |\mathbf{K}| = 1$ **b.** $\mathbf{I} \cdot \mathbf{J} = 0, \mathbf{I} \cdot \mathbf{K} = 0, \mathbf{J} \cdot \mathbf{K} = 0$ **c.** $\langle 1, 0, 0 \rangle = \frac{1}{2}\mathbf{I} - \frac{1}{\sqrt{2}}\mathbf{J} + \frac{1}{2}\mathbf{K}$
75. a. The faces on $y = 0$ and $z = 0$ **b.** The faces on $y = 1$ and $z = 1$ **c.** The faces on $x = 0$ and $x = 1$ **d.** 0 **e.** 1 **f.** 2

77. a. $\left(\frac{2}{\sqrt{3}}, 0, \frac{2\sqrt{2}}{\sqrt{3}}\right)$ **b.** $\mathbf{r}_{OP} = \langle \sqrt{3}, -1, 0 \rangle, \mathbf{r}_{OQ} = \langle \sqrt{3}, 1, 0 \rangle$, $\mathbf{r}_{PQ} = \langle 0, 2, 0 \rangle, \mathbf{r}_{OR} = \left\langle \frac{2}{\sqrt{3}}, 0, \frac{2\sqrt{2}}{\sqrt{3}} \right\rangle, \mathbf{r}_{PR} = \left\langle -\frac{\sqrt{3}}{3}, 1, \frac{2\sqrt{2}}{\sqrt{3}} \right\rangle$
83. a. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$

$$= \left(\frac{\mathbf{v} \cdot \mathbf{i}}{|\mathbf{v}||\mathbf{i}|}\right)^2 + \left(\frac{\mathbf{v} \cdot \mathbf{j}}{|\mathbf{v}||\mathbf{j}|}\right)^2 + \left(\frac{\mathbf{v} \cdot \mathbf{k}}{|\mathbf{v}||\mathbf{k}|}\right)^2$$

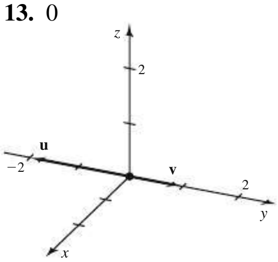
$$= \frac{a^2}{a^2 + b^2 + c^2} + \frac{b^2}{a^2 + b^2 + c^2} + \frac{c^2}{a^2 + b^2 + c^2} = 1$$

b. $\langle 1, 1, 0 \rangle$, 90° **c.** $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1 \right\rangle$, 45° **d.** No. If so, $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \cos^2\gamma = 1$, which has no solution. **e.** 54.7°
85. $|\mathbf{u} \cdot \mathbf{v}| = 33 = \sqrt{33} \cdot \sqrt{33} < \sqrt{70} \cdot \sqrt{74} = |\mathbf{u}||\mathbf{v}|$

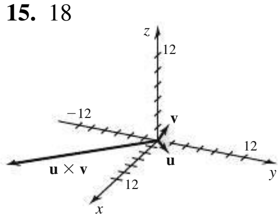
Section 13.4 Exercises, pp. 842–844

1. 0 **3. a.** $\mathbf{u}$ is orthogonal to $\mathbf{v}$. **b.** $\mathbf{u}$ is parallel to $\mathbf{v}$. **5.** $\sqrt{2}/2$
7. $-3\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ **9.** $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$ **11.** $15\mathbf{k}$

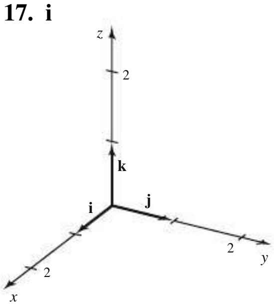
13. 0



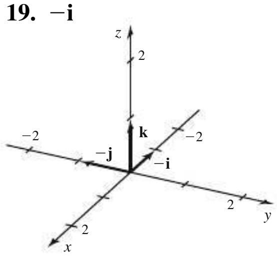
15. 18



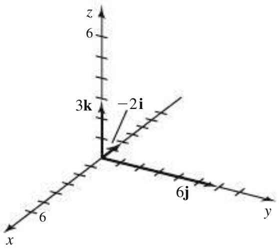
17. $\mathbf{i}$



19. $-\mathbf{i}$

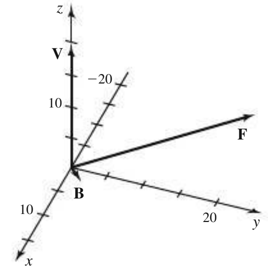


21. $6\mathbf{j}$



23. $\mathbf{u} \times \mathbf{v} = \langle -30, 18, 9 \rangle, \mathbf{v} \times \mathbf{u} = \langle 30, -18, -9 \rangle$
25. $\mathbf{u} \times \mathbf{v} = \langle 6, 11, 5 \rangle, \mathbf{v} \times \mathbf{u} = \langle -6, -11, -5 \rangle$
27. $\mathbf{u} \times \mathbf{v} = \langle 8, 4, 10 \rangle, \mathbf{v} \times \mathbf{u} = \langle -8, -4, -10 \rangle$ **29.** 11
31. $3\sqrt{10}$ **33.** $\sqrt{11}/2$ **35.** $4\sqrt{2}$ **37.** $9\sqrt{2}$ **41.** Not collinear
43. $\langle 3, -4, 2 \rangle$ **45.** $\langle 0, 20, -20 \rangle$ **47.** The force $\mathbf{F} = 5\mathbf{i} - 5\mathbf{k}$ produces the greater torque. **49.** $5/\sqrt{2}$ N-m **51.** $|\tau| = 13.2$ N-m; direction: into the page

53. The magnitude is $20\sqrt{2}$ at a 135° angle with the positive x-axis in the xy-plane.



55. 4.53×10^{-14} kg-m/s^2 **57. a.** False **b.** False **c.** False **d.** True **e.** False **59.** $\langle u_1, u_1 + 2, u_1 + 1\rangle$, u_1 real

61. $\dfrac{\sqrt{(ab)^2 + (ac)^2 + (bc)^2}}{2}$

63. $|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = |\mathbf{u}||\mathbf{v} \times \mathbf{w}||\cos\theta|$, where $|\mathbf{v} \times \mathbf{w}|$ is the area of the base of the parallelepiped and $|\mathbf{u}||\cos\theta|$ is its height.

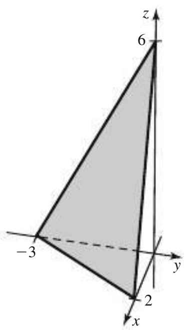
67. 1.76×10^7 m/s

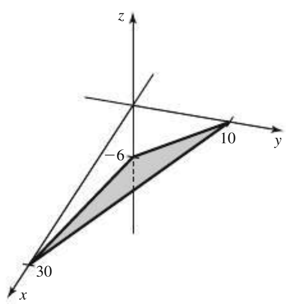
Section 13.5 Exercises, pp. 852–855

1. $\langle 4, -8, 9\rangle$ **3.** $\langle x_1 - x_0, y_1 - y_0, z_1 - z_0\rangle$ **5.** Perpendicular **7.** A point and a normal vector **9.** $(-6, 0, 0)$, $(0, -4, 0)$, $(0, 0, 3)$
11. $x = 4t, y = 7t, z = 1$; $\mathbf{r} = \langle 0, 0, 1\rangle + t\langle 4, 7, 0\rangle$
13. $x = 0, y = t, z = 1$; $\mathbf{r} = \langle 0, 0, 1\rangle + t\langle 0, 1, 0\rangle$
15. $x = t, y = 2t, z = 3t$; $\mathbf{r} = t\langle 1, 2, 3\rangle$
17. $x = -2t, y = 8t, z = -4t$; $\mathbf{r} = t\langle -2, 8, -4\rangle$
19. $x = -2t, y = -t, z = t$; $\mathbf{r} = t\langle -2, -1, 1\rangle$
21. $x = -2, y = 5 - 2t, z = 3 - t$; $\mathbf{r} = \langle -2, 5, 3\rangle + t\langle 0, -2, -1\rangle$ **23.** $x = 1 - 4t, y = 2 + 6t, z = 3 + 14t$; $\mathbf{r} = \langle 1, 2, 3\rangle + t\langle -4, 6, 14\rangle$ **25.** $x = 4, y = 3 - 9t, z = 3 + 6t$; $\mathbf{r} = \langle 4, 3, 3\rangle + t\langle 0, -9, 6\rangle$ **27.** $x = t, y = 2t, z = 3t, 0 \le t \le 1$ **29.** $x = 2 + 5t, y = 4 + t, z = 8 - 5t, 0 \le t \le 1$ **31.** Intersect at $(1, 3, 2)$ **33.** Skew **35.** Same line **37.** Parallel, distinct lines **39.** 13 **41. a.** Yes **b.** No **c.** $13.16° < \theta < 18.12°$
43. $x + y - z = 4$ **45.** $2x + y - 2z = -2$
47. $x + 4y + 7z = 0$ **49.** $7x + 2y + z = 10$
51. $-x + 2y - 4z = -17$ **53.** $3y - 2z = 0$
55. $8x - 7y + 2z = 0$ **57.** $x + 3y - z = -3$
59. Yes; $2x - y = -1$

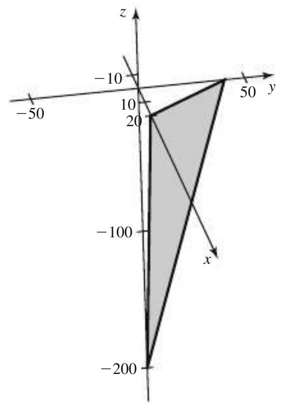
61. Intercepts $x = 2, y = -3, z = 6$; $3x - 2y = 6, z = 0$; $-2y + z = 6, x = 0$; and $3x + z = 6, y = 0$

63. Intercepts $x = 30, y = 10, z = -6$; $x + 3y = 30, z = 0$; $x - 5z = 30, y = 0$; and $3y - 5z = 30, x = 0$





65. Orthogonal **67.** Neither **69.** Q and T are identical; Q, R, and T are parallel; S is orthogonal to Q, R, and T.
71. $\mathbf{r} = \langle 2 + 2t, 1 - 4t, 3 + t\rangle$
73. $x = t, y = 1 + 2t, z = -1 - 3t$
75. $x = \frac{7}{5} + 2t, y = \frac{9}{5} + t, z = -t$ **77.** $(3, 3, 3)$ **79.** $(1, 1, 2)$
81. a. True **b.** False **c.** False **d.** True **e.** False **f.** False **g.** True **83.** 6 **85.** $\dfrac{x - 1}{4} = \dfrac{y - 2}{7} = \dfrac{z}{2}$ **87.** Approx. 43°
89. $6x - 4y + z = d$ **91.** The planes intersect in the point $(3, 6, 0)$.
93. a.

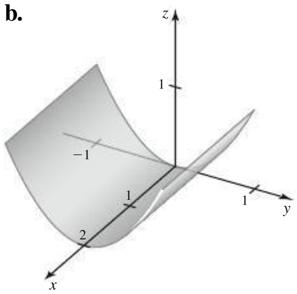


b. Positive
c. $2x + y = 40$, line in the xy-plane

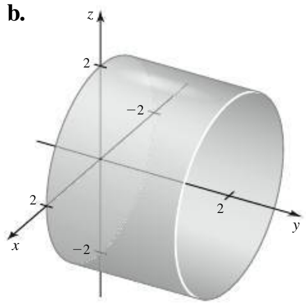
Section 13.6 Exercises, pp. 863–865

1. z-axis; x-axis; y-axis **3.** Intersection of the surface with a plane parallel to one of the coordinate planes **5.** Ellipsoid

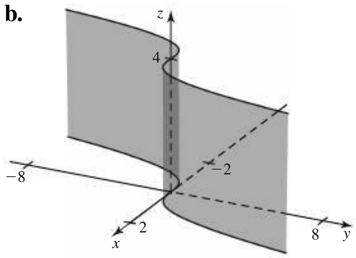
7. a. x-axis
b.



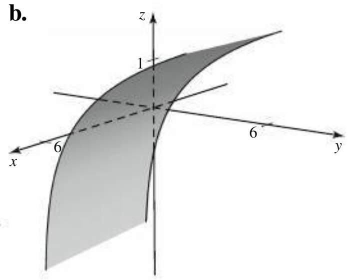
9. a. y-axis
b.



11. a. z-axis
b.



13. a. x-axis
b.

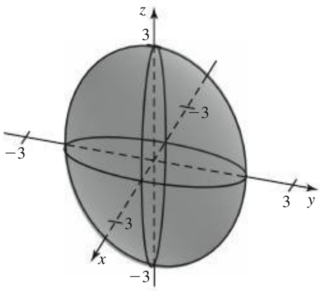


15. Ellipsoid; xy-trace: $x^2 + y^2 = 1$ (circle); xz-trace: $x^2 + \dfrac{z^2}{25} = 1$ (ellipse); yz-trace: $y^2 + \dfrac{z^2}{25} = 1$ (ellipse)
17. Paraboloid; xy-trace: $(0, 0, 0)$ (a single point); xz-trace: $z = 25x^2$ (parabola); yz-trace: $z = 25y^2$ (parabola) **19.** Hyperboloid of two sheets; xz-trace: $z^2 - 25x^2 = 25$ (hyperbola); yz-trace: $z^2 - 25y^2 = 25$ (hyperbola) **21.** Hyperbolic paraboloid
23. Elliptic paraboloid **25.** Hyperbolic cylinder
27. Elliptic paraboloid

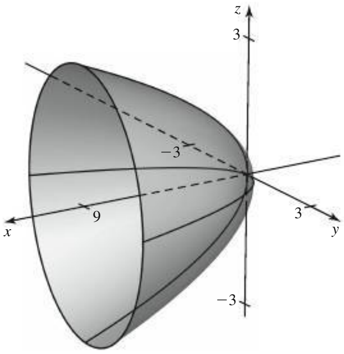
29. a. $x = \pm 1, y = \pm 2, z = \pm 3$

b. $x^2 + \dfrac{y^2}{4} = 1, x^2 + \dfrac{z^2}{9} = 1, \dfrac{y^2}{4} + \dfrac{z^2}{9} = 1$

c. Ellipsoid



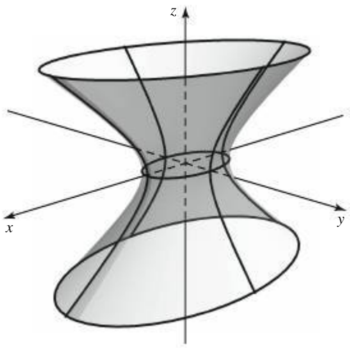
31. a. $x = y = z = 0$
b. $x = y^2, x = z^2$, origin
c. Elliptic paraboloid



33. a. $x = \pm 5, y = \pm 3$, no z-intercept

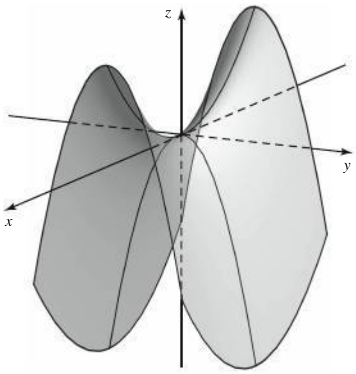
b. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1, \dfrac{x^2}{25} - z^2 = 1, \dfrac{y^2}{9} - z^2 = 1$

c. Hyperboloid of one sheet



35. a. $x = y = z = 0$ **b.** $\dfrac{x^2}{9} - y^2 = 0, z = \dfrac{x^2}{9}, z = -y^2$

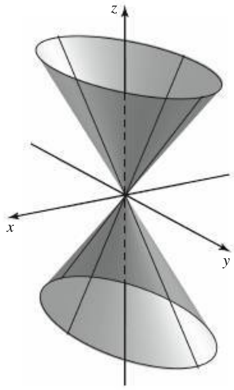
c. Hyperbolic paraboloid



37. a. $x = y = z = 0$

b. Origin, $\dfrac{y^2}{4} = z^2, x^2 = z^2$

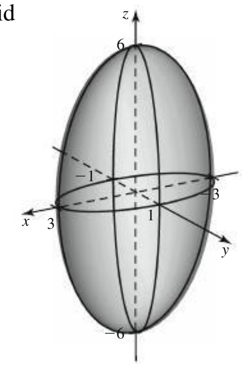
c. Elliptic cone



39. a. $x = \pm 3, y = \pm 1, z = \pm 6$

b. $\dfrac{x^2}{3} + 3y^2 = 3, \dfrac{x^2}{3} + \dfrac{z^2}{12} = 3, 3y^2 + \dfrac{z^2}{12} = 3$

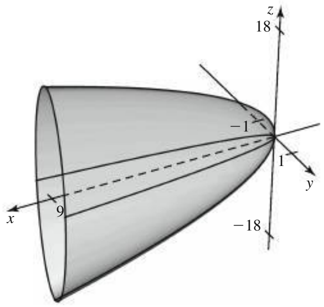
c. Ellipsoid



41. a. $x = y = z = 0$

b. Origin, $x - 9y^2 = 0, 9x - \dfrac{z^2}{4} = 0$

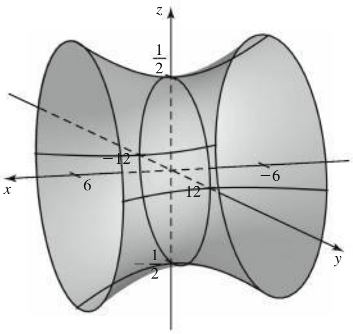
c. Elliptic paraboloid



43. a. No x-intercept, $y = \pm 12, z = \pm \dfrac{1}{2}$

b. $-\dfrac{x^2}{4} + \dfrac{y^2}{16} = 9, -\dfrac{x^2}{4} + 36z^2 = 9, \dfrac{y^2}{16} + 36z^2 = 9$

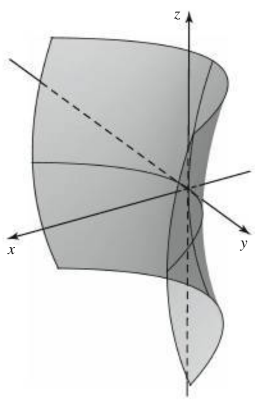
c. Hyperboloid of one sheet



45. a. $x = y = z = 0$

b. $5x - \dfrac{y^2}{5} = 0, 5x + \dfrac{z^2}{20} = 0, -\dfrac{y^2}{5} + \dfrac{z^2}{20} = 0$

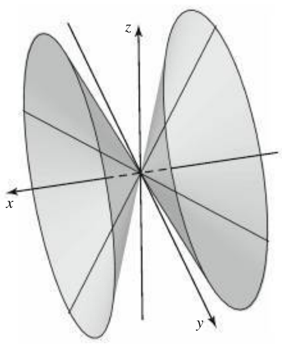
c. Hyperbolic paraboloid



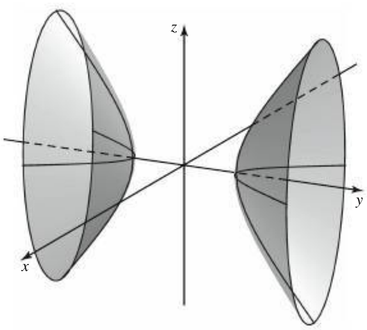
47. a. $x = y = z = 0$

b. $\dfrac{y^2}{18} = 2x^2, \dfrac{z^2}{32} = 2x^2$, origin

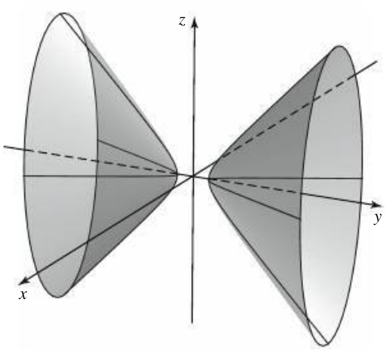
c. Elliptic cone



49. a. No x-intercept, $y = \pm 2$, no z-intercept
b. $-x^2 + \frac{y^2}{4} = 1$, no xz-trace, $\frac{y^2}{4} - \frac{z^2}{9} = 1$
c. Hyperboloid of two sheets



51. a. No x-intercept, $y = \pm\frac{\sqrt{3}}{3}$, no z-intercept
b. $-\frac{x^2}{3} + 3y^2 = 1$, no xz-trace, $3y^2 - \frac{z^2}{12} = 1$
c. Hyperboloid of two sheets

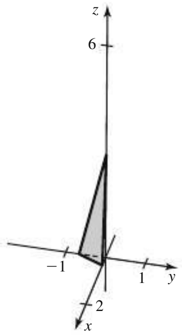


53. The graph of the ellipsoid $x^2 + 4y^2 + 9z^2 + 54z = 19$ is obtained by shifting the graph of the ellipsoid $x^2 + 4y^2 + 9z^2 = 100$ down 3 units. **55.** Hyperboloid of one sheet **57.** Hyperboloid of two sheets **59. a.** True **b.** True **c.** True **d.** False **e.** False **61.** All except the hyperbolic paraboloid **63.** 8 **65. b.** $\frac{x^2 + z^2}{(10.55/\pi)^2} + \frac{y^2}{(5.55)^2} = 1$
67. $4x^2 + 8y^2 + 4(z - 3)^2 = 9, 3 \le z \le 4.5$

Chapter 13 Review Exercises, pp. 865–867

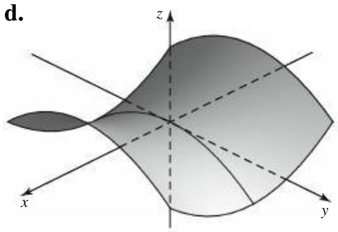
1. a. True **b.** False **c.** True **d.** False **e.** True **f.** True
3. $\langle 3, -6\rangle$ **5.** $\langle -5, 8\rangle$ **7.** $\sqrt{221}$ **9.** $12\left\langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right\rangle$
11. $\left\langle \frac{10}{3}, -\frac{20}{3}, \frac{20}{3}\right\rangle$ **13.** $\langle 58, 26, 44\rangle$ **15.** $a = -3$
17. a. $\mathbf{v} = -275\sqrt{2}\mathbf{i} + 275\sqrt{2}\mathbf{j}$ **b.** $-275\sqrt{2}\mathbf{i} + (275\sqrt{2} + 40)\mathbf{j}$
19. $\{(x, y, z): (x - 1)^2 + y^2 + (z + 1)^2 = 16\}$
21. $\{(x, y, z): x^2 + (y - 1)^2 + z^2 > 4\}$ **23.** A ball centered at $\left(\frac{1}{2}, -2, 3\right)$ of radius $\frac{3}{2}$ **25.** All points outside a sphere of radius 10 centered at (3, 0, 10) **27.** 50.15 m/s; 85.4° below the horizontal in the northerly horizontal direction **29.** 50 lb; 36.9° north of east
31. A circle of radius 1 centered at (0, 2, 0) in the vertical plane $y = 2$
33. a. 0.68 radian **b.** $\frac{7}{9}\langle 1, 2, 2\rangle; \frac{7}{3}$ **c.** $\frac{7}{3}\langle -1, 2, 2\rangle; 7$
35. $250\sqrt{2}$ ft-lb **37.** $90\sqrt{3}$ lb; 90 lb **39.** 11
41. $\pm\left\langle \frac{12}{\sqrt{197}}, \frac{7}{\sqrt{197}}, \frac{2}{\sqrt{197}}\right\rangle$ **43.** $\langle -10, 10, 10\rangle$

45. $|\tau|(\theta) = 39.2 \sin\theta$ has a maximum value of 39.2 N-m (when $\theta = \pi/2$) and a minimum value of 0 N-m (when $\theta = 0$). Direction does *not* change. **47.** $\mathbf{r} = \langle 0, -3, 9\rangle + t\langle 2, -5, -8\rangle, 0 \le t \le 1$
49. $\mathbf{r} = \langle t, 1 + 6t, 1 + 2t\rangle$
51. a. $18x - 9y + 2z = 6$ **b.** $x = \frac{1}{3}, y = -\frac{2}{3}, z = 3$
c.

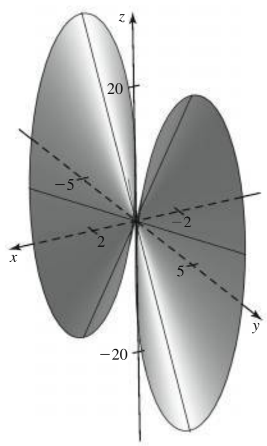


53. $x = t, y = 12 - 9t, z = -6 + 6t$ **55.** $4x + 2y + 13z = 39$
57. $3x + y + 7z = 4$ **59.** 3

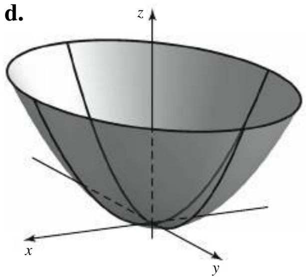
61. a. Hyperbolic paraboloid
b. $y^2 = 4x^2, z = \frac{x^2}{36}, z = -\frac{y^2}{144}$
c. $x = y = z = 0$
d.



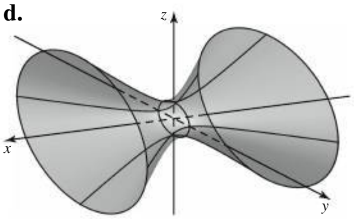
63. a. Elliptic cone
b. $y^2 = 4x^2$, origin, $y^2 = \frac{z^2}{25}$
c. Origin
d.



65. a. Elliptic paraboloid
b. Origin, $z = \frac{x^2}{16}, z = \frac{y^2}{36}$
c. Origin
d.

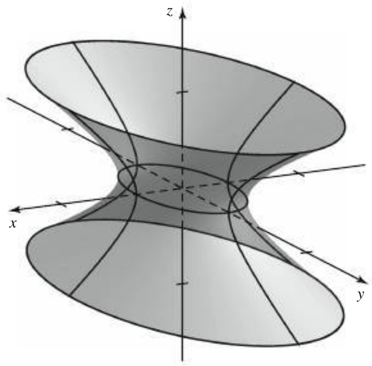


67. a. Hyperboloid of one sheet
b. $y^2 - 2x^2 = 1, 4z^2 - 2x^2 = 1, y^2 + 4z^2 = 1$ **c.** No x-intercept, $y = \pm 1, z = \pm\frac{1}{2}$
d.

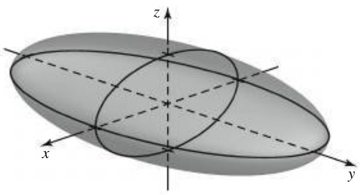


69. a. Hyperboloid of one sheet
b. $\frac{x^2}{4} + \frac{y^2}{16} = 4, \frac{x^2}{4} - z^2 = 4, \frac{y^2}{16} - z^2 = 4$
c. $x = \pm 4, y = \pm 8$, no z-intercept

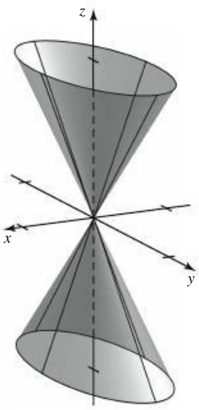
d.



71. a. Ellipsoid **b.** $\frac{x^2}{4} + \frac{y^2}{16} = 4, \frac{x^2}{4} + z^2 = 4, \frac{y^2}{16} + z^2 = 4$
c. $x = \pm 4, y = \pm 8, z = \pm 2$
d.



73. a. Elliptic cone **b.** Origin, $\frac{x^2}{9} = \frac{z^2}{64}, \frac{y^2}{49} = \frac{z^2}{64}$ **c.** Origin
d.

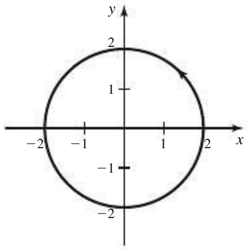


75. a. A **b.** D **c.** C **d.** B

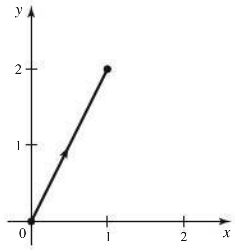
CHAPTER 14

Section 14.1 Exercises, pp. 873–875

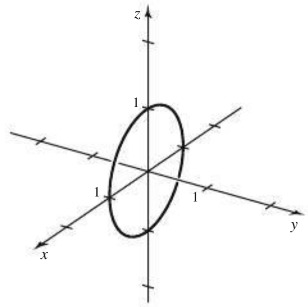
1. One **3.** Its output is a vector.
5. $\lim_{t \to a} \mathbf{r}(t) = \langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \rangle$
7. $\mathbf{r}(t) = t\,\mathbf{i} + 2t\,\mathbf{j} + 3t\,\mathbf{k}$
9. $\mathbf{r}(t) = \langle 2 + 2t, 3 + 3t, 7 - 4t \rangle$
11. $\mathbf{r}(t) = \langle 3 + 2t, 4, 5 - t \rangle$
13. $\mathbf{r}(t) = \langle 1 - t, 2, 1 + 2t \rangle$, for $0 \le t \le 1$
15.



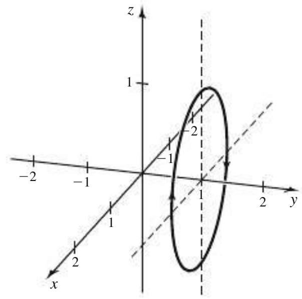
17.



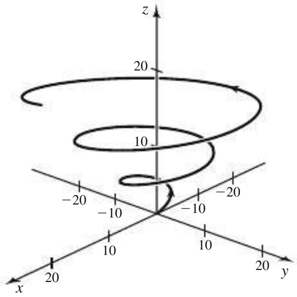
19.



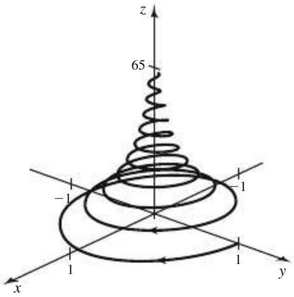
21.



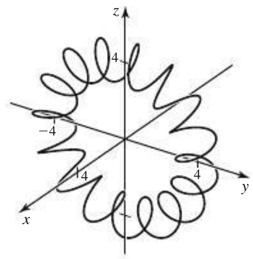
23.



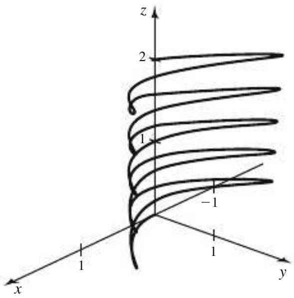
25.



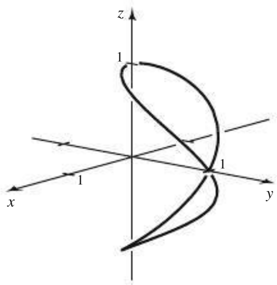
27.



29. When viewed from above, the curve is a portion of the parabola $y = x^2$.



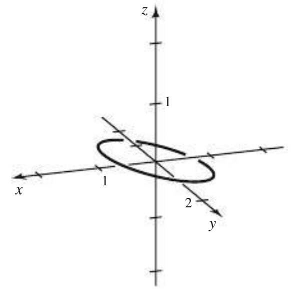
31. $-\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ **33.** $-2\mathbf{j} + \frac{\pi}{2}\mathbf{k}$ **35.** $\mathbf{i}$ **37. a.** True **b.** False
c. True **d.** True **39.** $\{t : |t| \le 2\}$ **41.** $\{t : 0 \le t \le 2\}$
43. $(4, 8, 16)$ **45. a.** E **b.** D **c.** F **d.** C **e.** A **f.** B
47. $\mathbf{r}(t) = \langle 2 \cos t, 2 \sin t, 4 \rangle$
49. $\mathbf{r}(t) = \langle 5 \cos t, 5 \sin t, 10 \cos t + 10 \sin t \rangle$
51. a. Ball has a parabolic trajectory in the yz-plane; 1200 ft
b. Approx. 1199.7 ft **c.** 1196 ft **53.** Hyperboloid of one sheet
55. Ellipsoid **57.** $(4, 2, 2)$; $\sqrt{179}$
59.



The curve lies on the sphere $x^2 + y^2 + z^2 = 1$.

61. $\frac{2\pi}{(m, n)}$, where (m, n) = greatest common factor of m and n

63. a.



b. Curve is a tilted circle of radius 1 centered at the origin.

65. $\langle cf - ed, be - af, ad - bc \rangle$ or any scalar multiple

Section 14.2 Exercises, pp. 881–883

1. $\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$ **3.** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$

5. $\int \mathbf{r}(t)\,dt = \left(\int f(t)\,dt\right)\mathbf{i} + \left(\int g(t)\,dt\right)\mathbf{j} + \left(\int h(t)\,dt\right)\mathbf{k}$

7. $\mathbf{C} = \langle -1, -3, -10 \rangle$ **9.** $\langle -\sin t, 2t, \cos t \rangle$

11. $\left\langle 6t^2, \dfrac{3}{\sqrt{t}}, -\dfrac{3}{t^2} \right\rangle$ **13.** $e^t\mathbf{i} - 2e^{-t}\mathbf{j} - 8e^{2t}\mathbf{k}$

15. $\langle e^{-t}(1 - t), 1 + \ln t, \cos t - t\sin t \rangle$

17. $\langle 1, 6, 3 \rangle$ **19.** $\langle 1, 0, 0 \rangle$ **21.** $8\mathbf{i} + 9\mathbf{j} - 10\mathbf{k}$

23. $\langle 2/3, 2/3, 1/3 \rangle$

25. $\dfrac{\langle 0, -\sin 2t, 2\cos 2t \rangle}{\sqrt{1 + 3\cos^2 2t}}$ **27.** $\dfrac{t^2}{\sqrt{t^4 + 4}}\left\langle 1, 0, -\dfrac{2}{t^2} \right\rangle$

29. $\langle 0, 0, -1 \rangle$ **31.** $\left\langle \dfrac{2}{\sqrt{5}}, 0, -\dfrac{1}{\sqrt{5}} \right\rangle$

33. $\langle 30t^{14} + 24t^3, 14t^{13} - 12t^{11} + 9t^2 - 3, -96t^{11} - 24 \rangle$

35. $4t(2t^3 - 1)(t^3 - 2)\langle 3t(t^3 - 2), 1, 0 \rangle$

37. $e^t(2t^3 + 6t^2) - 2e^{-t}(t^2 - 2t - 1) - 16e^{-2t}$

39. 11 **41.** $\langle 0, 7, 1 \rangle$ **43.** $\langle 2e^{2t}, -2e^t, 0 \rangle$ **45.** $\left\langle 4, -\dfrac{2}{\sqrt{t}}, 0 \right\rangle$

47. $\langle 1 + 6t^2, 4t^3, -2 - 3t^2 \rangle$ **49.** $5te^t(t + 2) - 6t^2e^{-t}(t - 3)$

51. $-3t^2\sin t + 6t\cos t + 2\sqrt{t}\cos 2t + \dfrac{1}{2\sqrt{t}}\sin 2t$

53. $\langle 2, 0, 0 \rangle, \langle 0, 0, 0 \rangle$ **55.** $\langle -9\cos 3t, -16\sin 4t, -36\cos 6t \rangle$, $\langle 27\sin 3t, -64\cos 4t, 216\sin 6t \rangle$

57. $\left\langle -\dfrac{1}{4}(t + 4)^{-3/2}, -2(t + 1)^{-3}, 2e^{-t^2}(1 - 2t^2) \right\rangle$, $\left\langle \dfrac{3}{8}(t + 4)^{-5/2}, 6(t + 1)^{-4}, -4te^{-t^2}(3 - 2t^2) \right\rangle$

59. $\left\langle \dfrac{t^5}{5} - \dfrac{3t^2}{2}, t^2 - t, 10t \right\rangle + \mathbf{C}$

61. $\left\langle 2\sin t, -\dfrac{2}{3}\cos 3t, \dfrac{1}{2}\sin 8t \right\rangle + \mathbf{C}$

63. $\frac{1}{3}e^{3t}\mathbf{i} + \tan^{-1}t\,\mathbf{j} - \sqrt{2t}\,\mathbf{k} + \mathbf{C}$

65. $\mathbf{r}(t) = \langle e^t + 1, 3 - \cos t, \tan t + 2 \rangle$

67. $\mathbf{r}(t) = \langle t + 3, t^2 + 2, t^3 - 6 \rangle$

69. $\mathbf{r}(t) = \langle \frac{1}{2}e^{2t} + \frac{1}{2}, 2e^{-t} + t - 1, t - 2e^t + 3 \rangle$

71. $\langle 2, 0, 2 \rangle$ **73.** $\mathbf{i}$ **75.** $\langle 0, 0, 0 \rangle$

77. $(e^2 + 1)\langle 1, 2, -1 \rangle$ **79. a.** False **b.** True **c.** True

81. $\langle 2 - t, 3 - 2t, \pi/2 + t \rangle$ **83.** $\langle 2 + 3t, 9 + 7t, 1 + 2t \rangle$

85. $(1, 0)$ **87.** $(1, 0, 0)$ **89.** $\mathbf{r}(t) = \langle a_1t, a_2t, a_3t \rangle$ or $\mathbf{r}(t) = \langle a_1e^{kt}, a_2e^{kt}, a_3e^{kt} \rangle$, where a_i and k are real numbers

Section 14.3 Exercises, pp. 892–896

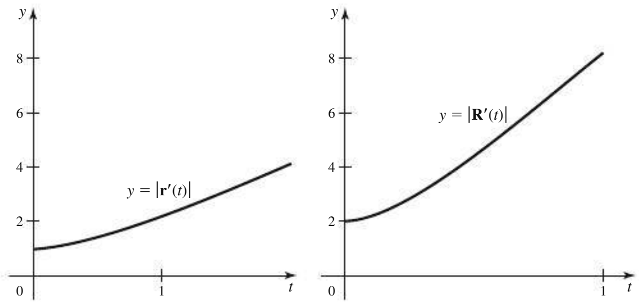
1. $\mathbf{v}(t) = \mathbf{r}'(t)$, speed $= |\mathbf{r}'(t)|$, $\mathbf{a}(t) = \mathbf{r}''(t)$ **3.** $m\mathbf{a}(t) = \mathbf{F}$

5. $\mathbf{v}(t) = \int \mathbf{a}(t)\,dt = \langle v_1(t), v_2(t) \rangle + \mathbf{C}$. Use initial conditions to find $\mathbf{C}$. **7. a.** $t = 3$ s **b.** $\mathbf{r}(t) = \langle 60t, -16t^2 + 96t + 3 \rangle$

9. a. $\langle 6t, 8t \rangle, 10t$ **b.** $\langle 6, 8 \rangle$ **11. a.** $\mathbf{v}(t) = \langle 2, -4 \rangle$, $|\mathbf{v}(t)| = 2\sqrt{5}$ **b.** $\mathbf{a}(t) = \langle 0, 0 \rangle$ **13. a.** $\mathbf{v}(t) = \langle 8\cos t, -8\sin t \rangle$, $|\mathbf{v}(t)| = 8$ **b.** $\mathbf{a}(t) = \langle -8\sin t, -8\cos t \rangle$ **15. a.** $\langle 2t, 2t, t \rangle, 3t$ **b.** $\langle 2, 2, 1 \rangle$ **17. a.** $\mathbf{v}(t) = \langle 1, -4, 6 \rangle$, $|\mathbf{v}(t)| = \sqrt{53}$ **b.** $\mathbf{a}(t) = \langle 0, 0, 0 \rangle$ **19. a.** $\mathbf{v}(t) = \langle 0, 2t, -e^{-t} \rangle$, $|\mathbf{v}(t)| = \sqrt{4t^2 + e^{-2t}}$ **b.** $\mathbf{a}(t) = \langle 0, 2, e^{-t} \rangle$

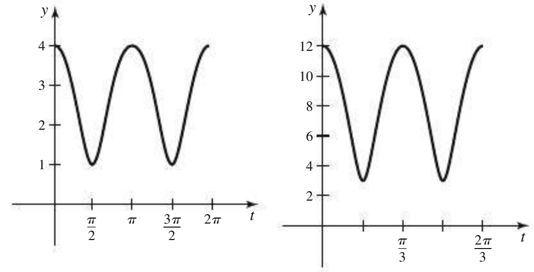
21. a. $[c, d] = [0, 1]$ **b.** $\langle 1, 2t \rangle, \langle 2, 8t \rangle$

c.



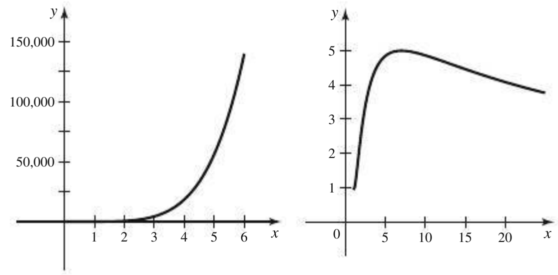
23. a. $\left[0, \frac{2\pi}{3}\right]$ **b.** $\mathbf{V_r}(t) = \langle -\sin t, 4\cos t \rangle$, $\mathbf{V_R}(t) = \langle -3\sin 3t, 12\cos 3t \rangle$

c.

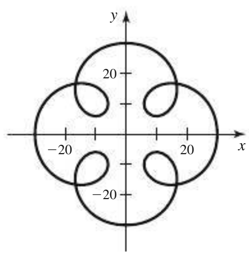


25. a. $[1, e^{36}]$ **b.** $\mathbf{V_r}(t) = \langle 2t, -8t^3, 18t^5 \rangle$, $\mathbf{V_R}(t) = \left\langle \dfrac{1}{t}, -\dfrac{4}{t}\ln t, \dfrac{9}{t}\ln^2 t \right\rangle$

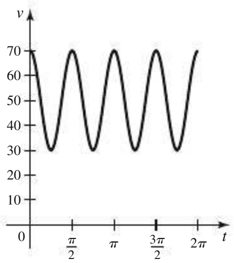
c.



27. a.



b. $\langle -20\sin t - 50\sin 5t, 20\cos t + 50\cos 5t \rangle$

c. 

d. 70 ft/s; 30 ft/s

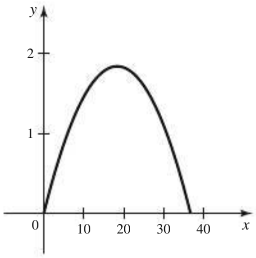
29. $\mathbf{r}(t)$ lies on a circle of radius 8; $\langle -16 \sin 2t, 16 \cos 2t \rangle \cdot \langle 8 \cos 2t, 8 \sin 2t \rangle = 0$. **31.** $\mathbf{r}(t)$ lies on a sphere of radius 2; $\langle \cos t - \sqrt{3} \sin t, \sqrt{3} \cos t + \sin t \rangle \cdot \langle \sin t + \sqrt{3} \cos t, \sqrt{3} \sin t - \cos t \rangle = 0$. **33.** 5

35. $\mathbf{v}(t) = \langle 2, t + 3 \rangle, \mathbf{r}(t) = \left\langle 2t, \frac{t^2}{2} + 3t \right\rangle$

37. $\mathbf{v}(t) = \langle 0, 10t + 5 \rangle, \mathbf{r}(t) = \langle 1, 5t^2 + 5t - 1 \rangle$

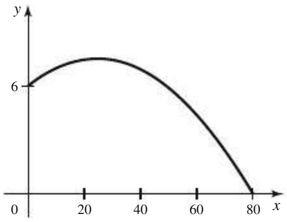
39. $\mathbf{v}(t) = \langle \sin t, -2 \cos t + 3 \rangle$, $\mathbf{r}(t) = \langle -\cos t + 2, -2 \sin t + 3t \rangle$

41. a. $\mathbf{v}(t) = \langle 30, -9.8t + 6 \rangle, \mathbf{r}(t) = \langle 30t, -4.9t^2 + 6t \rangle$

b. 

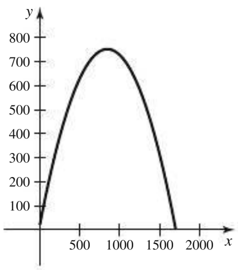
c. $T \approx 1.22$ s, range ≈ 36.7 m **d.** 1.84 m

43. a. $\mathbf{v}(t) = \langle 80, 10 - 32t \rangle, \mathbf{r}(t) = \langle 80t, -16t^2 + 10t + 6 \rangle$

b. 

c. 1 s, 80 ft **d.** Max height ≈ 7.56 ft

45. a. $\mathbf{v}(t) = \langle 125, -32t + 125\sqrt{3} \rangle$, $\mathbf{r}(t) = \langle 125t, -16t^2 + 125\sqrt{3}t + 20 \rangle$

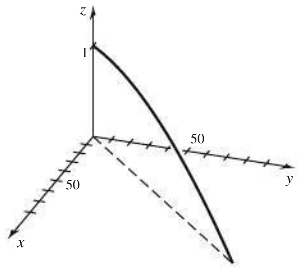
b. 

c. 13.6 s, 1702.5 ft **d.** 752.4 ft

47. $\mathbf{v}(t) = \langle 1, 5, 10t \rangle, \mathbf{r}(t) = \langle t, 5t + 5, 5t^2 \rangle$

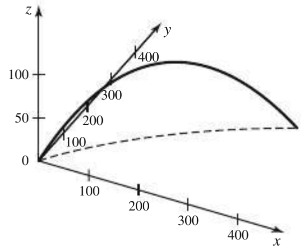
49. $\mathbf{v}(t) = \langle -\cos t + 1, \sin t + 2, t \rangle$, $\mathbf{r}(t) = \left\langle -\sin t + t, -\cos t + 2t + 1, \frac{t^2}{2} \right\rangle$

51. a. $\mathbf{v}(t) = \langle 200, 200, -9.8t \rangle$, $\mathbf{r}(t) = \langle 200t, 200t, -4.9t^2 + 1 \rangle$

b. 

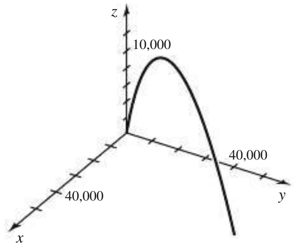
c. 0.452 s, 127.8 m **d.** 1 m

53. a. $\mathbf{v}(t) = \langle 60 + 10t, 80, 80 - 32t \rangle$, $\mathbf{r}(t) = \langle 60t + 5t^2, 80t, 80t - 16t^2 + 3 \rangle$

b. 

c. 5.04 s, 589 ft **d.** 103 ft

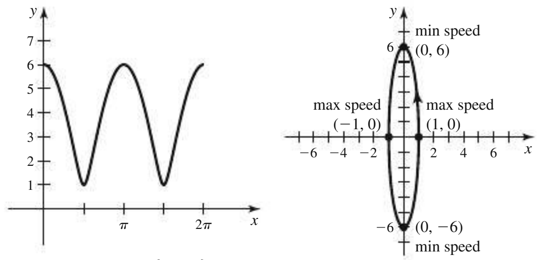
55. a. $\mathbf{v}(t) = \langle 300, 2.5t + 400, -9.8t + 500 \rangle$, $\mathbf{r}(t) = \langle 300t, 1.25t^2 + 400t, -4.9t^2 + 500t + 10 \rangle$

b. 

c. 102.1 s, 61,941.5 m **d.** 12,765.1 m

57. a. False **b.** True **c.** False **d.** True **e.** False **f.** True **g.** True **59.** 15.3 s, 1988.3 m, 287.0 m **61.** 21.7 s, 4330.1 ft, 1875 ft **63.** Approx. 27.4° and 62.6°

65. a. $\mathbf{v}(t) = \langle -a \sin t, b \cos t \rangle$; $|\mathbf{v}(t)| = \sqrt{a^2 \sin^2 t + b^2 \cos^2 t}$

b. 

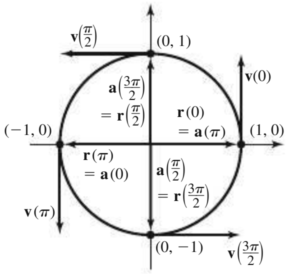
c. Yes **d.** Max $\left\{\frac{a}{b}, \frac{b}{a}\right\}$ **67.** Approx. 23.5° or 59.6°

69. 113.4 ft/s **71. a.** 1.2 ft, 0.46 s **b.** 0.88 ft/s **c.** 0.85 ft **d.** More curve in the second half **e.** $c = 28.17$ ft/s^2

73. $T = \dfrac{|\mathbf{v}_0| \sin \alpha + \sqrt{|\mathbf{v}_0|^2 \sin^2 \alpha + 2gy_0}}{g}$, range $= |\mathbf{v}_0| (\cos \alpha)\, T$, max height $= y_0 + \dfrac{|\mathbf{v}_0|^2 \sin^2 \alpha}{2g}$

75. a. $\left[0, \frac{2\pi}{\omega}\right]$ **b.** $\mathbf{v}(t) = \langle -A\omega \sin \omega t, A\omega \cos \omega t \rangle$ is not constant; $|\mathbf{v}(t)| = |A\omega|$ is constant. **c.** $\mathbf{a}(t) = \langle -A\omega^2 \cos \omega t, -A\omega^2 \sin \omega t \rangle$ **d.** $\mathbf{r}$ and $\mathbf{v}$ are orthogonal; $\mathbf{r}$ and $\mathbf{a}$ are in opposite directions.

e.



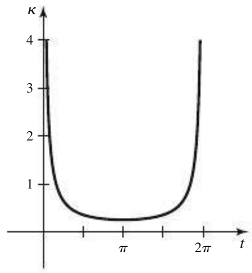
77. a. $\mathbf{r}(t) = \langle 5 \sin(\pi t/6), 5 \cos(\pi t/6) \rangle$
b. $\mathbf{r}(t) = \left\langle 5 \sin\left(\frac{1 - e^{-t}}{5}\right), 5 \cos\left(\frac{1 - e^{-t}}{5}\right) \right\rangle$
79. $\{(\cos t, \sin t, c \sin t): t \in \mathbb{R}\}$ satisfies the equations $x^2 + y^2 = 1$ and $z - cy = 0$ so that $\langle \cos t, \sin t, c \sin t \rangle$ lies on the intersection of a right circular cylinder and a plane, which is an ellipse.
83. a. The direction of **r** does not change. **b.** Constant in direction, not in magnitude

Section 14.4 Exercises, pp. 900–902

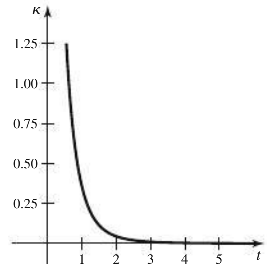
1. $\sqrt{5}(b - a)$ **3.** $\int_a^b |\mathbf{v}(t)|\, dt$ **5.** 20π **7.** If the parameter t used to describe a trajectory also measures the arc length s of the curve that is generated, we say the curve has been parameterized by its arc length. **9.** 5 **11.** 3π **13.** $\frac{\pi^2}{8}$ **15.** $5\sqrt{34}$ **17.** $4\pi\sqrt{65}$ **19.** 9 **21.** $\frac{3}{2}$
23. $3t^2\sqrt{30}$; $64\sqrt{30}$ **25.** 26; 26π
27. Approx. 66,626 mi/hr **29.** 19.38
31. 32.50 **33.** Yes **35.** No; $\mathbf{r}(s) = \left\langle \frac{s}{\sqrt{5}}, \frac{2s}{\sqrt{5}} \right\rangle, 0 \le s \le 3\sqrt{5}$
37. No; $\mathbf{r}(s) = \left\langle 2 \cos \frac{s}{2}, 2 \sin \frac{s}{2} \right\rangle, 0 \le s \le 4\pi$
39. No; $\mathbf{r}(s) = \langle \cos s, \sin s \rangle, 0 \le s \le \pi$
41. No; $\mathbf{r}(s) = \left\langle \frac{s}{\sqrt{3}} + 1, \frac{s}{\sqrt{3}} + 1, \frac{s}{\sqrt{3}} + 1 \right\rangle, s \ge 0$
43. a. True **b.** True **c.** True **d.** False **45. a.** If $a^2 = b^2 + c^2$, then $|\mathbf{r}(t)|^2 = (a \cos t)^2 + (b \sin t)^2 + (c \sin t)^2 = a^2$ so that $\mathbf{r}(t)$ is a circle centered at the origin of radius $|a|$. **b.** $2\pi a$
c. If $a^2 + c^2 + e^2 = b^2 + d^2 + f^2$ and $ab + cd + ef = 0$, then $\mathbf{r}(t)$ is a circle of radius $\sqrt{a^2 + c^2 + e^2}$ and its arc length is $2\pi\sqrt{a^2 + c^2 + e^2}$. **47. a.** $\int_a^b \sqrt{(Ah'(t))^2 + (Bh'(t))^2}\, dt$
$= \int_a^b \sqrt{(A^2 + B^2)(h'(t))^2}\, dt = \sqrt{A^2 + B^2} \int_a^b |h'(t)|\, dt$
b. $64\sqrt{29}$ **c.** $\frac{7\sqrt{29}}{4}$ **49. a.** 5.102 s
b. $\int_0^{5.102} \sqrt{400 + (25 - 9.8t)^2}\, dt$ **c.** 124.43 m **d.** 102.04 m
51. $|\mathbf{v}(t)| = \sqrt{a^2 + b^2 + c^2} = 1$, if $a^2 + b^2 + c^2 = 1$
53. $\int_a^b |\mathbf{r}'(t)|\, dt = \int_a^b \sqrt{(cf'(t))^2 + (cg'(t))^2}\, dt$
$= |c| \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2}\, dt = |c| L$

Section 14.5 Exercises, pp. 913–915

1. 0 **3.** $\kappa = \frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right|$ or $\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$ **5.** $\mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$
7. These three unit vectors are mutually orthogonal at all points of the curve. **9.** The torsion measures the rate at which the curve rises or twists out of the **TN**-plane at a point. **11.** $\mathbf{T} = \frac{\langle 1, 2, 3 \rangle}{\sqrt{14}}, \kappa = 0$
13. $\mathbf{T} = \frac{\langle 1, 2 \cos t, -2 \sin t \rangle}{\sqrt{5}}, \kappa = \frac{1}{5}$
15. $\mathbf{T} = \frac{\langle \sqrt{3} \cos t, \cos t, -2 \sin t \rangle}{2}, \kappa = \frac{1}{2}$
17. $\mathbf{T} = \frac{\langle 1, 4t \rangle}{\sqrt{1 + 16t^2}}, \kappa = \frac{4}{(1 + 16t^2)^{3/2}}$
19. $\mathbf{T} = \left\langle \cos\left(\frac{\pi t^2}{2}\right), \sin\left(\frac{\pi t^2}{2}\right) \right\rangle, \kappa = \pi t$
21. $\frac{1}{3}$ **23.** $\frac{2}{(4t^2 + 1)^{3/2}}$ **25.** $\frac{2\sqrt{5}}{(20 \sin^2 t + \cos^2 t)^{3/2}}$
27. $\mathbf{T} = \langle \cos t, -\sin t \rangle, \mathbf{N} = \langle -\sin t, -\cos t \rangle$
29. $\mathbf{T} = \frac{\langle t, -3, 0 \rangle}{\sqrt{t^2 + 9}}, \mathbf{N} = \frac{\langle 3, t, 0 \rangle}{\sqrt{t^2 + 9}}$
31. $\mathbf{T} = \langle -\sin t^2, \cos t^2 \rangle$, $\mathbf{N} = \langle -\cos t^2, -\sin t^2 \rangle$
33. $\mathbf{T} = \frac{\langle 2t, 1 \rangle}{\sqrt{4t^2 + 1}}, \mathbf{N} = \frac{\langle 1, -2t \rangle}{\sqrt{4t^2 + 1}}$ **35.** $a_N = a_T = 0$
37. $a_T = \sqrt{3}e^t; a_N = \sqrt{2}e^t$ **39.** $\mathbf{a} = \frac{6t}{\sqrt{9t^2 + 4}}\mathbf{N} + \frac{18t^2 + 4}{\sqrt{9t^2 + 4}}\mathbf{T}$
41. $\mathbf{B}(t) = \langle 0, 0, -1 \rangle, \tau = 0$ **43.** $\mathbf{B}(t) = \langle 0, 0, 1 \rangle, \tau = 0$
45. $\mathbf{B}(t) = \frac{\langle -\sin t, \cos t, 2 \rangle}{\sqrt{5}}, \tau = -\frac{1}{5}$
47. $\mathbf{B}(t) = \frac{\langle 5, 12 \sin t, -12 \cos t \rangle}{13}, \tau = \frac{12}{169}$ **49. a.** False
b. False **c.** False **d.** True **e.** False **f.** False **g.** False
51. $\kappa = \frac{2}{(1 + 4x^2)^{3/2}}$ **53.** $\kappa = \frac{x}{(x^2 + 1)^{3/2}}$
57. $\kappa = \frac{|ab|}{(a^2 \cos^2 t + b^2 \sin^2 t)^{3/2}}$ **59.** $\kappa = \frac{2|a|}{(1 + 4a^2 t^2)^{3/2}}$
61. b. $\mathbf{v}_A(t) = \langle 1, 2, 3 \rangle$, $\mathbf{a}_A(t) = \langle 0, 0, 0 \rangle$ and $\mathbf{v}_B(t) = \langle 2t, 4t, 6t \rangle$, $\mathbf{a}_B(t) = \langle 2, 4, 6 \rangle$; A has constant velocity and zero acceleration, while B has increasing speed and constant acceleration.
c. $\mathbf{a}_A(t) = 0\mathbf{N} + 0\mathbf{T}$, $\mathbf{a}_B(t) = 0\mathbf{N} + 2\sqrt{14}\,\mathbf{T}$; both normal components are zero since the path is a straight line ($\kappa = 0$).
63. b. $\mathbf{v}_A(t) = \langle -\sin t, \cos t \rangle$, $\mathbf{a}_A(t) = \langle -\cos t, -\sin t \rangle$
$\mathbf{v}_B(t) = \langle -2t \sin t^2, 2t \cos t^2 \rangle$
$\mathbf{a}_B(t) = \langle -4t^2 \cos t^2 - 2 \sin t^2, -4t^2 \sin t^2 + 2 \cos t^2 \rangle$
c. $\mathbf{a}_A(t) = \mathbf{N} + 0\mathbf{T}$, $\mathbf{a}_B(t) = 4t^2\,\mathbf{N} + 2\mathbf{T}$; for A, the acceleration is always normal to the curve, but this is not true for B.
65. b. $\kappa = \frac{1}{2\sqrt{2(1 - \cos t)}}$ **c.**



d. Minimum curvature at $t = \pi$
67. b. $\kappa = \frac{1}{t(1 + t^2)^{3/2}}$ **c.**



d. No maximum or minimum curvature

69. $\kappa = \dfrac{e^x}{(1 + e^{2x})^{3/2}}, \left(-\dfrac{\ln 2}{2}, \dfrac{1}{\sqrt{2}}\right), \dfrac{2\sqrt{3}}{9}$

71. $\dfrac{1}{\kappa} = \dfrac{1}{2}; x^2 + \left(y - \dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

73. $\dfrac{1}{\kappa} = 4; (x - \pi)^2 + (y + 2)^2 = 16$

75. $\kappa\left(\dfrac{\pi}{2n}\right) = n^2$; κ increases as n increases.

77. a. Speed $= \sqrt{V_0^2 - 2V_0\,gt \sin\alpha + g^2t^2}$

b. $\kappa(t) = \dfrac{gV_0 \cos\alpha}{(V_0^2 - 2V_0\,gt\sin\alpha + g^2t^2)^{3/2}}$

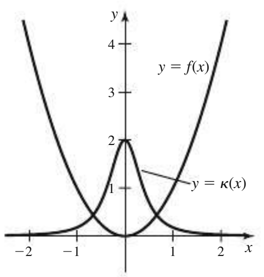
c. Speed has a minimum at $t = \dfrac{V_0 \sin\alpha}{g}$ and $\kappa(t)$ has a maximum at $t = \dfrac{V_0 \sin\alpha}{g}$. **79.** $\kappa = \dfrac{1}{|\mathbf{v}|}\cdot\left|\dfrac{d\mathbf{T}}{dt}\right|$, where $\mathbf{T} = \dfrac{\langle b, d, f\rangle}{\sqrt{b^2 + d^2 + f^2}}$ and b, d, and f are constant. Therefore, $\dfrac{d\mathbf{T}}{dt} = \mathbf{0}$ so $\kappa = 0$.

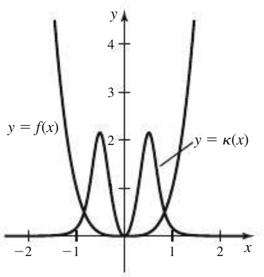
81. a. $\kappa_1(x) = \dfrac{2}{(1 + 4x^2)^{3/2}}$

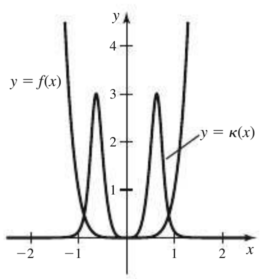
$\kappa_2(x) = \dfrac{12x^2}{(1 + 16x^6)^{3/2}}$

$\kappa_3(x) = \dfrac{30x^4}{(1 + 36x^{10})^{3/2}}$

b.





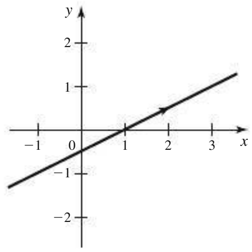


c. κ_1 has its maximum at $x = 0$, κ_2 has its maxima at $x = \pm\sqrt[6]{\frac{1}{56}}$, and κ_3 has its maxima at $x = \pm\sqrt[10]{\frac{1}{99}}$. **d.** $\lim\limits_{n\to\infty} z_n = 1$; the graphs of $y = f_n(x)$ show that as $n \to \infty$, the point corresponding to maximum curvature gets arbitrarily close to the point $(1, 0)$.

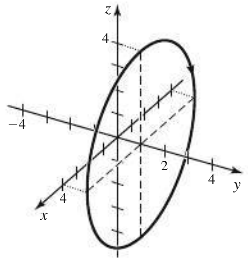
Chapter 14 Review Exercises, pp. 916–918

1. a. False **b.** True **c.** True **d.** True **e.** False **f.** False

3.

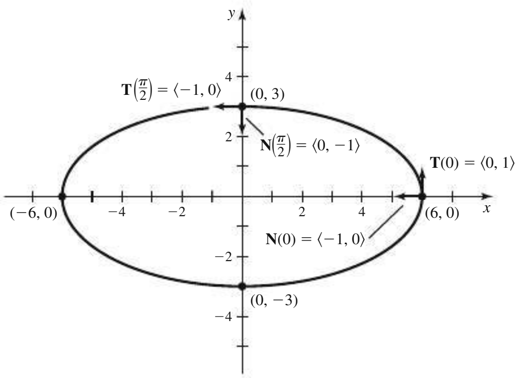


5.

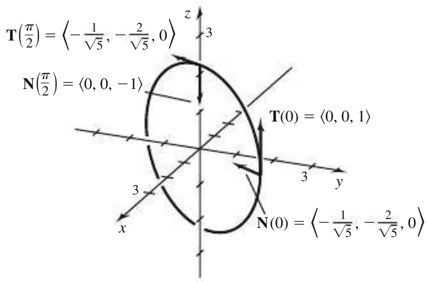


7. $x^2 + y^2 + z^2 = 2$; $y = z$; a tilted circle of radius $\sqrt{2}$ centered at $(0, 0, 0)$ **9.** $\mathbf{r}(t) = \langle 4 + 15t, -2 - t, 3 - 5t\rangle$
11. $\mathbf{r}(t) = \langle 2, 3\cos t, 4\sin t\rangle$, for $0 \le t \le 2\pi$
13. $\mathbf{r}(t) = \langle \cos t, \sin t, \sin t\rangle$, for $0 \le t \le 2\pi$
15. $\mathbf{r}(t) = \langle 3\cos t, \sin t, \sin t\rangle$, for $0 \le t \le 2\pi$
17. a. $\langle 1, 0\rangle$; $\langle 0, 1\rangle$ **b.** $\left\langle -\dfrac{2}{(2t+1)^2}, \dfrac{1}{(t+1)^2}\right\rangle$; $\langle -2, 1\rangle$
c. $\left\langle \dfrac{8}{(2t+1)^3}, -\dfrac{2}{(t+1)^3}\right\rangle$
d. $\left\langle \dfrac{1}{2}\ln|2t+1|, t - \ln|t+1|\right\rangle + \mathbf{C}$
19. a. $\langle 0, 3, 0\rangle$; does not exist
b. $\langle 2\cos 2t, -12\sin 4t, 1\rangle$; $\langle 2, 0, 1\rangle$ **c.** $\langle -4\sin 2t, -48\cos 4t, 0\rangle$
d. $\left\langle -\dfrac{1}{2}\cos 2t, \dfrac{3}{4}\sin 4t, \dfrac{1}{2}t^2\right\rangle + \mathbf{C}$ **21.** $2\mathbf{j} + \pi\mathbf{k}$
23. $23\mathbf{i} - 41\mathbf{k}$ **25.** $\mathbf{r}(t) = \left\langle t + 2, -\dfrac{1}{2}\cos 2t + \dfrac{5}{2}, \tan t + 2\right\rangle$
27. $\mathbf{r}(t) = \langle 4\tan^{-1} t - \pi, t^2 + t - 2, t^3 - 1\rangle$
29. $\mathbf{T}(t) = \left\langle \dfrac{2e^t}{2e^{2t}+1}, \dfrac{2e^{2t}}{2e^{2t}+1}, \dfrac{1}{2e^{2t}+1}\right\rangle$; $\left\langle \dfrac{2}{3}, \dfrac{2}{3}, \dfrac{1}{3}\right\rangle$
31. a. $\langle 4e^{4t}, 4e^{4t}, 2e^{4t}\rangle$; $6e^{4t}$ **b.** $\langle 16e^{4t}, 16e^{4t}, 8e^{4t}\rangle$
33. $\mathbf{v}(t) = \langle 2 + \sin t, 3 - 2\cos t\rangle$;
$\mathbf{r}(t) = \langle 2t + 2 - \cos t, 3t + 2 - 2\sin t\rangle$
35. a. $\mathbf{v}(t) = \langle 40, -32t + 40\sqrt{3}\rangle$;
$\mathbf{r}(t) = \langle 40t, -16t^2 + 40\sqrt{3}t + 3\rangle$
b. Approx. 4.37 s; approx. 174.9 ft **c.** 78 ft
37. a. $\mathbf{v}(t) = \langle 4t + 40, 20, 40 - 32t\rangle$;
$\mathbf{r}(t) = \langle 2t^2 + 40t, 20t, -16t^2 + 40t + 2\rangle$ **b.** 2.549 s
c. 126 ft **39. a.** (116, 30) **b.** 39.1 ft **c.** 2.315 s
d. $\displaystyle\int_0^{2.315}\sqrt{50^2 + (-32t + 50)^2}\,dt$ **e.** 129 ft **f.** 41.4° to 79.4°
41. (1.47, 3.15, 4.4) **43.** 12 **45.** Approx. 6.42
47. a. $\mathbf{v}(t) = \mathbf{i} + t\sqrt{2}\,\mathbf{j} + t^2\mathbf{k}$ **b.** 12
49. $\mathbf{r}(s) = \left\langle (\sqrt{1+s} - 1)^2, \dfrac{4\sqrt{2}}{3}(\sqrt{1+s} - 1)^{3/2}, 2(\sqrt{1+s} - 1)\right\rangle$, for $s \ge 0$ **51. a.** $\mathbf{v} = \langle -6\sin t, 3\cos t\rangle$,
$\mathbf{T} = \dfrac{\langle -2\sin t, \cos t\rangle}{\sqrt{1 + 3\sin^2 t}}$ **b.** $\kappa(t) = \dfrac{2}{3(1 + 3\sin^2 t)^{3/2}}$
c. $\mathbf{N} = \left\langle -\dfrac{\cos t}{\sqrt{1 + 3\sin^2 t}}, -\dfrac{2\sin t}{\sqrt{1 + 3\sin^2 t}}\right\rangle$
d. $|\mathbf{N}| = \sqrt{\dfrac{\cos^2 t + 4\sin^2 t}{1 + 3\sin^2 t}} = \sqrt{\dfrac{(\cos^2 t + \sin^2 t) + 3\sin^2 t}{1 + 3\sin^2 t}} = 1$;
$\mathbf{T}\cdot\mathbf{N} = \dfrac{2\sin t\cos t - 2\sin t\cos t}{1 + 3\sin^2 t} = 0$

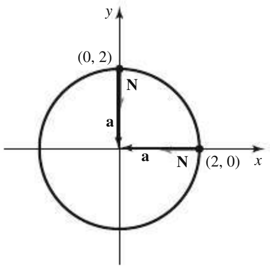
e.



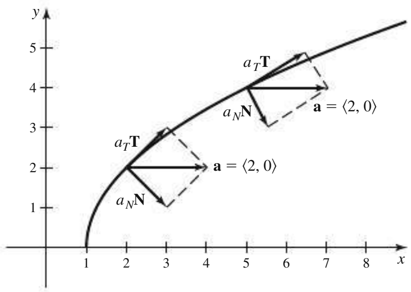
53. a. $\mathbf{v}(t) = \langle -\sin t, -2\sin t, \sqrt{5}\cos t\rangle$,
$\mathbf{T}(t) = \left\langle -\frac{1}{\sqrt{5}}\sin t, -\frac{2}{\sqrt{5}}\sin t, \cos t\right\rangle$ **b.** $\kappa(t) = \frac{1}{\sqrt{5}}$
c. $\mathbf{N}(t) = \left\langle -\frac{1}{\sqrt{5}}\cos t, -\frac{2}{\sqrt{5}}\cos t, -\sin t\right\rangle$
d. $|\mathbf{N}(t)| = \sqrt{\frac{1}{5}\cos^2 t + \frac{4}{5}\cos^2 t + \sin^2 t} = 1$;
$\mathbf{T}\cdot\mathbf{N} = \left(\frac{1}{5}\cos t\sin t + \frac{4}{5}\cos t\sin t\right) - \sin t\cos t = 0$
e.



55. a. $\mathbf{a}(t) = 2\mathbf{N} + 0\mathbf{T} = 2\langle -\cos t, -\sin t\rangle$
b.

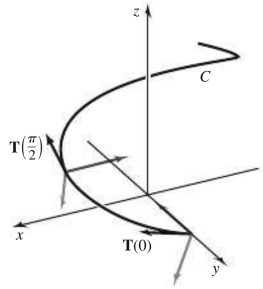


57. a. $a_T = \frac{2t}{\sqrt{t^2+1}}$ and $a_N = \frac{2}{\sqrt{t^2+1}}$
b.



59. $\mathbf{B}(1) = \frac{\langle 3, -3, 1\rangle}{\sqrt{19}}$; $\tau = \frac{3}{19}$
61. a. $\mathbf{T}(t) = \frac{1}{5}\langle 3\cos t, -3\sin t, 4\rangle$
b. $\mathbf{N}(t) = \langle -\sin t, -\cos t, 0\rangle$; $\kappa = \frac{3}{25}$

c.



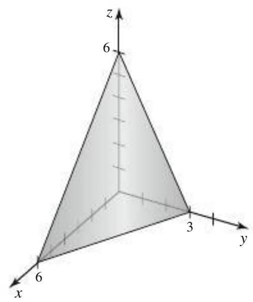
d. Yes
e. $\mathbf{B}(t) = \frac{1}{5}\langle 4\cos t, -4\sin t, -3\rangle$
f. See graph in part (c).
g. Check that **T**, **N**, and **B** have unit length and are mutually orthogonal.
h. $\tau = -\frac{4}{25}$

63. a. Consider first the case where $a_3 = b_3 = c_3 = 0$, and show that for all $s \neq t$ in I, $\mathbf{r}(t) \times \mathbf{r}(s)$ is a multiple of the constant vector $\langle b_1c_2 - b_2c_1, a_2c_1 - a_1c_2, a_1b_2 - a_2b_1\rangle$, which implies $\mathbf{r}(t) \times \mathbf{r}(s)$ is always orthogonal to the same vector, and therefore the vectors $\mathbf{r}(t)$ must all lie in the same plane. When a_3, b_3, and c_3 are not necessarily 0, the curve still lies in a plane because these constants represent a simple translation of the curve to a different location in $\mathbb{R}^3$.
b. Because the curve lies in a plane, **B** is always normal to the plane and has length 1. Therefore, $\frac{d\mathbf{B}}{ds} = \mathbf{0}$ and $\tau = 0$.

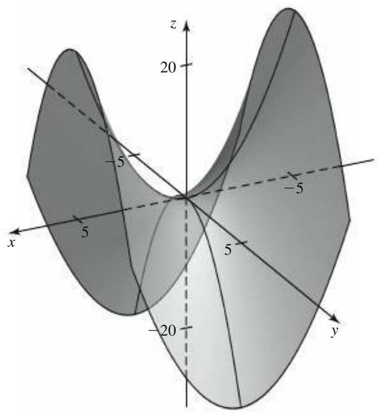
CHAPTER 15

Section 15.1 Exercises, pp. 927–930

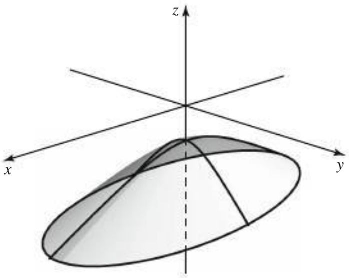
1. Independent: x and y; dependent: z
3. $D = \{(x, y): x \neq 0 \text{ and } y \neq 0\}$ **5.** Three **7.** 3; 4
9. a. 1300 ft **b.** Katie; Katie is 100 ft higher than Zeke. **11.** Circles
13. $n = 6$ **15.** $D = \mathbb{R}^2$ **17.** $D = \{(x, y): x^2 + y^2 \leq 25\}$
19. $D = \{(x, y): y \neq 0\}$ **21.** $D = \{(x, y): y < x^2\}$
23. $D = \{(x, y): xy \geq 0, (x, y) \neq (0, 0)\}$
25. Plane; $D = \mathbb{R}^2$, $R = \mathbb{R}$



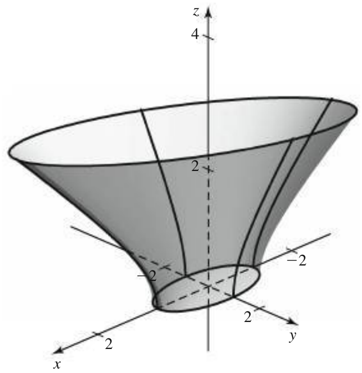
27. Hyperbolic paraboloid; $D = \mathbb{R}^2$, $R = \mathbb{R}$



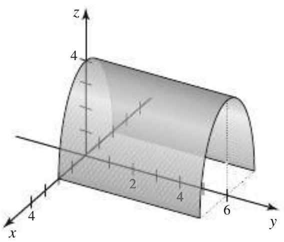
29. Lower part of a hyperboloid of two sheets; $D = \mathbb{R}^2, R = (-\infty, -1]$



31. Upper half of a hyperboloid of one sheet; $D = \{(x, y): x^2 + y^2 \geq 1\}, R = [0, \infty)$

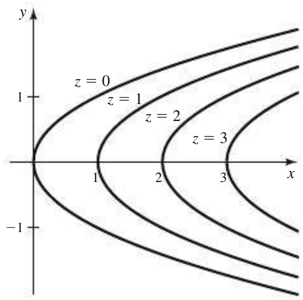


33. Upper half of an elliptical cylinder; $D = \{(x, y): -2 \leq x \leq 2, -\infty < y < \infty\}, R = [0, 4]$



35. a. A **b.** D **c.** B **d.** C

37.



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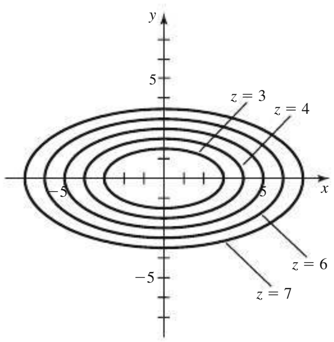

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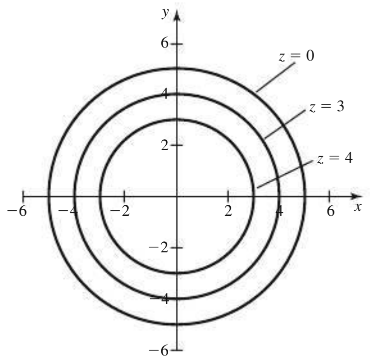
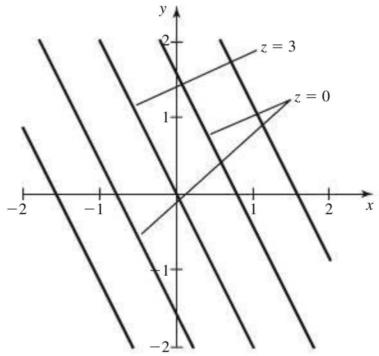
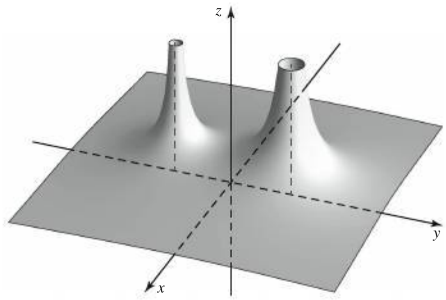


43.

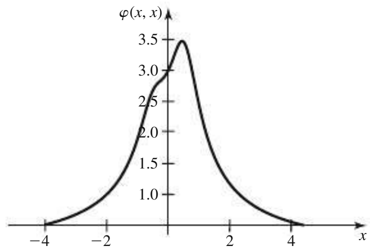


45. a.

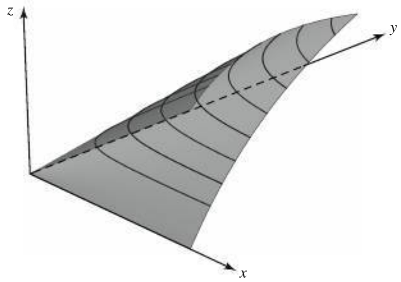


b. $\mathbb{R}^2$ excluding the points $(0, 1)$ and $(0, -1)$

c. $\varphi(2, 3)$ is greater. **d.**

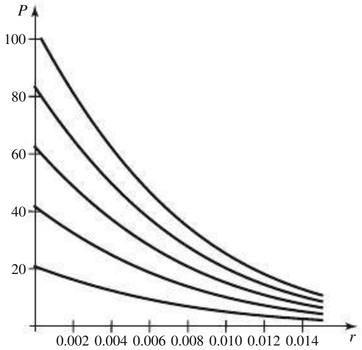


47. a.

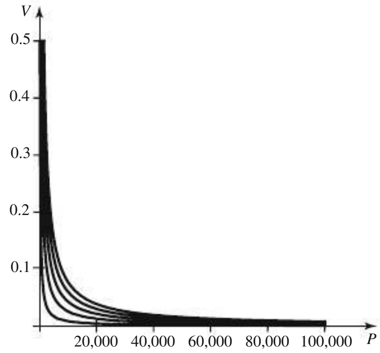


b. $R(10, 10) = 5$
c. $R(x, y) = R(y, x)$

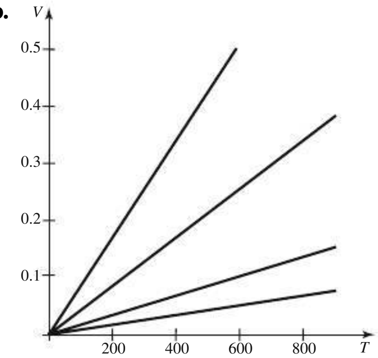
49. a. $P = \dfrac{20{,}000r}{(1+r)^{240}-1}$ **b.** $P = \dfrac{Br}{(1+r)^{240}-1}$, with $B = 5000, 10{,}000, 15{,}000, 25{,}000$



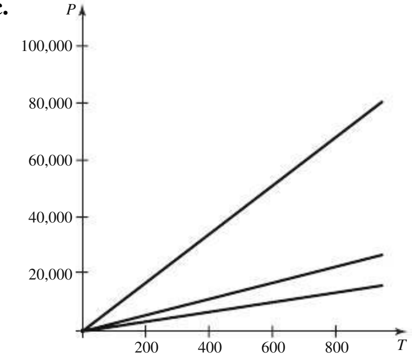
51. $D = \{(x, y, z): x \neq z\}$; all points not on the plane $x = z$
53. $D = \{(x, y, z): y \geq z\}$; all points on or below the plane $y = z$
55. $D = \{(x, y, z): x^2 \leq y\}$; all points on the side of the vertical cylinder $y = x^2$ that contains the positive y-axis **57. a.** False **b.** False **c.** True
59. a.



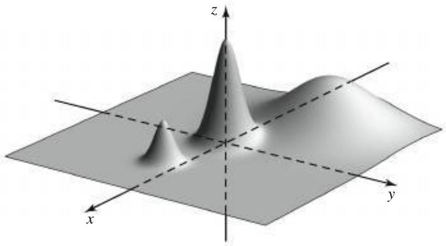
b.



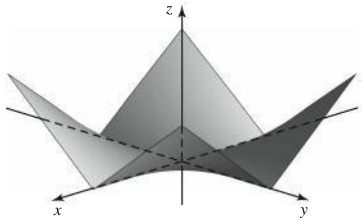
c.



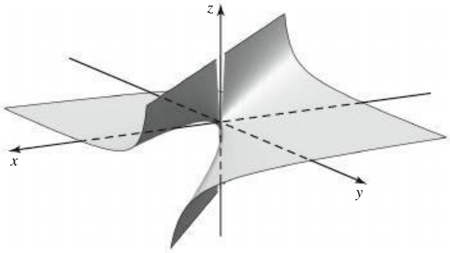
61. a.



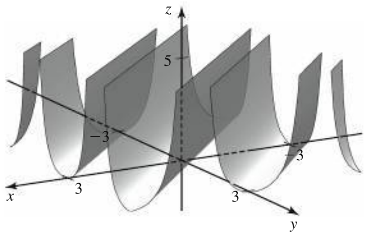
b. $(0, 0), (-5, 3), (4, -1)$
c. $f(0,0) = 10.17, f(-5,3) = 5.00, f(4,-1) = 4.00$
63. a. $D = \mathbb{R}^2, R = [0, \infty)$
b.



65. a. $D = \{(x, y): x \neq y\}, R = \mathbb{R}$
b.



67. a. $D = \{(x, y): y \neq x + \pi/2 + n\pi$ for any integer $n\}$, $R = [0, \infty)$ **b.**



69. Peak at the origin **71.** Depression at $(1, 0)$ **73.** The level curves are $ax + by = d - cz_0$, where z_0 is a constant, which are lines with slope $-a/b$ if $b \neq 0$ or vertical lines if $b = 0$.
75. $z = x^2 + y^2 - C$; paraboloids with vertices at $(0, 0, -C)$
77. $x^2 + 2z^2 = C$; elliptic cylinders parallel to the y-axis
79. $D = \{(x, y): x - 1 \leq y \leq x + 1\}$
81. $D = \{(x, y, z): (x \leq z$ and $y \geq -z)$ or $(x \geq z$ and $y \leq -z)\}$

Section 15.2 Exercises, pp. 937–939

1. The values of $f(x, y)$ are arbitrarily close to L for all (x, y) sufficiently close to (a, b). **3.** Because polynomials of n variables are continuous on all of $\mathbb{R}^n$, limits of polynomials can be evaluated with direct substitution. **5.** If the function approaches different values along different paths, the limit does not exist. **7.** f must be defined, the limit must exist, and the limit must equal the function value.
9. At any point where the denominator is nonzero **11.** 10 **13.** 101
15. 27 **17.** $1/(2\pi)$ **19.** 2 **21.** 6 **23.** -1 **25.** 2
27. $1/(2\sqrt{2}) = \sqrt{2}/4$ **29.** $L = 1$ along $y = 0$, and $L = -1$ along $x = 0$ **31.** $L = 1$ along $x = 0$, and $L = -2$ along $y = 0$
33. $L = 2$ along $y = x$, and $L = 0$ along $y = -x$ **35.** $\mathbb{R}^2$
37. All points except $(0, 0)$ **39.** $\{(x, y): x \neq 0\}$ **41.** All points except $(0, 0)$ **43.** $\mathbb{R}^2$ **45.** $\mathbb{R}^2$ **47.** $\mathbb{R}^2$ **49.** All points except $(0, 0)$ **51.** $\mathbb{R}^2$ **53.** $\mathbb{R}^2$ **55.** 6 **57.** -1 **59.** 2 **61. a.** False

b. False **c.** True **d.** False **63.** $\frac{1}{2}$ **65.** 0 **67.** Does not exist **69.** $\frac{1}{4}$ **71.** 1 **73.** 1 **75.** 5 **77.** $b = 1$ **79.** 0 **81.** 1 **85.** 0

Section 15.3 Exercises, pp. 948–951

1. $f_x(a, b)$ is the slope of the surface in the direction parallel to the positive x-axis, $f_y(a, b)$ is the slope of the surface in the direction parallel to the positive y-axis, both taken at (a, b). **3. a.** Negative **b.** Negative **c.** Negative **d.** Positive **5.** $f_x(x, y) = 6xy$; $f_y(x, y) = 3x^2$ **7.** $f_{xy} = 0 = f_{yx}$ **9.** $f_x(x, y, z) = y + z$; $f_y(x, y, z) = x + z$; $f_z(x, y, z) = x + y$

11. $f_x(x, y) = 5y$; $f_y(x, y) = 5x$ **13.** $f_x(x, y) = \dfrac{1}{y}$; $f_y(x, y) = -\dfrac{x}{y^2}$

15. $f_x(x, y) = e^y$; $f_y(x, y) = xe^y$

17. $f_x(x, y) = 2xye^{x^2y}$; $f_y(x, y) = x^2e^{x^2y}$

19. $f_w(w, z) = \dfrac{z^2 - w^2}{(w^2 + z^2)^2}$; $f_z(w, z) = -\dfrac{2wz}{(w^2 + z^2)^2}$

21. $f_x(x, y) = \cos xy - xy \sin xy$; $f_y(x, y) = -x^2 \sin xy$

23. $s_y(y, z) = z^3 \sec^2 yz$; $s_z(y, z) = 2z \tan yz + yz^2 \sec^2 yz$

25. $G_s(s, t) = \dfrac{\sqrt{st}(t - s)}{2s(s + t)^2}$; $G_t(s, t) = \dfrac{\sqrt{st}(s - t)}{2t(s + t)^2}$

27. $f_x(x, y) = 2yx^{2y-1}$; $f_y(x, y) = 2x^{2y} \ln x$

29. $h_x(x, y) = \dfrac{\sqrt{x^2 - 4y} - x}{\sqrt{x^2 - 4y}}$; $h_y(x, y) = \dfrac{2}{\sqrt{x^2 - 4y}}$

31. $f_x(x, y) = -e^{x^2}$; $f_y(x, y) = 3y^2e^{y^6}$

33. $f_x(x, y) = -\dfrac{2x}{1 + (x^2 + y^2)^2}$; $f_y(x, y) = -\dfrac{2y}{1 + (x^2 + y^2)^2}$

35. $h_x(x, y) = (1 + 2y)^x \ln(1 + 2y)$; $h_y(x, y) = 2x(1 + 2y)^{x-1}$

37. $f_x(x, y) = -h(x)$; $f_y(x, y) = h(y)$

39. $h_{xx}(x, y) = 6x$; $h_{xy}(x, y) = 2y = h_{yx}(x, y)$; $h_{yy}(x, y) = 2x$

41. $f_{xx}(x, y) = -16y^3 \sin 4x$; $f_{xy}(x, y) = 12y^2 \cos 4x = f_{yx}(x, y)$; $f_{yy}(x, y) = 6y \sin 4x$

43. $p_{uu}(u, v) = \dfrac{-2u^2 + 2v^2 + 8}{(u^2 + v^2 + 4)^2}$;

$p_{uv}(u, v) = -\dfrac{4uv}{(u^2 + v^2 + 4)^2} = p_{vu}(u, v)$;

$p_{vv}(u, v) = \dfrac{2u^2 - 2v^2 + 8}{(u^2 + v^2 + 4)^2}$

45. $F_{rr}(r, s) = 0$; $F_{rs}(r, s) = e^s = F_{sr}(r, s)$; $F_{ss}(r, s) = re^s$

47. $f_{xx}(x, y) = \dfrac{6xy^2(1 - 2x^6y^4)}{(1 + x^6y^4)^2}$;

$f_{xy}(x, y) = \dfrac{6x^2y(1 - x^6y^4)}{(1 + x^6y^4)^2} = f_{yx}(x, y)$;

$f_{yy}(x, y) = \dfrac{2x^3(1 - 3x^6y^4)}{(1 + x^6y^4)^2}$

49. $f_{xy} = e^{x+y} = f_{yx}$ **51.** $f_{xy} = -(xy \cos xy + \sin xy) = f_{yx}$

53. $f_{xy}(x, y) = -72y^2(2x - y^3)^2 = f_{yx}(x, y)$

55. $h_x(x, y, z) = h_y(x, y, z) = h_z(x, y, z) = -\sin(x + y + z)$

57. $F_u(u, v, w) = \dfrac{1}{v + w}$; $F_v(u, v, w) = F_w(u, v, w) = -\dfrac{u}{(v + w)^2}$

59. $G_r(r, s, t) = \dfrac{s^3t^5}{2\sqrt{rs^3t^5}}$;

$G_s(r, s, t) = \dfrac{3rs^2t^5}{2\sqrt{rs^3t^5}}$;

$G_t(r, s, t) = \dfrac{5rs^3t^4}{2\sqrt{rs^3t^5}}$

61. $h_w(w, x, y, z) = \dfrac{z}{xy}$; $h_x(w, x, y, z) = -\dfrac{wz}{x^2y}$;

$h_y(w, x, y, z) = -\dfrac{wz}{xy^2}$; $h_z(w, x, y, z) = \dfrac{w}{xy}$

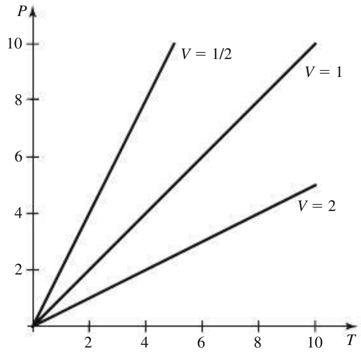
63. b. $g_x(x, y, z) = -\dfrac{8z}{3(2x - y + z)^2}$;

$g_y(x, y, z) = \dfrac{4z}{3(2x - y + z)^2}$;

$g_z(x, y, z) = \dfrac{4(2x - y)}{3(2x - y + z)^2}$

65. 1.41 **67.** 1.55 (answer will vary) **69. a.** $\dfrac{\partial V}{\partial P} = -\dfrac{kT}{P^2}$; volume decreases with pressure at fixed temperature. **b.** $\dfrac{\partial V}{\partial T} = \dfrac{k}{P}$; volume increases with temperature at fixed pressure.

c.



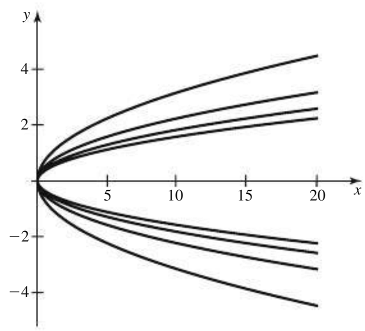
71. a. $\dfrac{\partial R}{\partial R_1} = \dfrac{R_2^2}{(R_1 + R_2)^2}$; $\dfrac{\partial R}{\partial R_2} = \dfrac{R_1^2}{(R_1 + R_2)^2}$

b. $\dfrac{\partial R}{\partial R_1} = \dfrac{R^2}{R_1^2}$; $\dfrac{\partial R}{\partial R_2} = \dfrac{R^2}{R_2^2}$ **c.** Increase **d.** Decrease

73. $u_t = -16e^{-4t} \cos 2x = u_{xx}$ **75.** $u_t = -a^2Ae^{-a^2t} \cos ax = u_{xx}$ **77. a.** No **b.** No **c.** $f_x(0, 0) = f_y(0, 0) = 0$ **d.** f_x and f_y are not continuous at $(0, 0)$. **79. a.** False **b.** False **c.** True

81. a. $z_x(x, y) = \dfrac{1}{y^2}$; $z_y(x, y) = -\dfrac{2x}{y^3}$

b.



c. z increases as x increases. **d.** z increases as y increases when $y < 0$, z is undefined for $y = 0$, and z decreases as y increases for $y > 0$.

83. a. $\varphi_x(x, y) = -\dfrac{2x}{(x^2 + (y - 1)^2)^{3/2}} - \dfrac{x}{(x^2 + (y + 1)^2)^{3/2}}$;

$\varphi_y(x, y) = -\dfrac{2(y - 1)}{(x^2 + (y - 1)^2)^{3/2}} - \dfrac{y + 1}{(x^2 + (y + 1)^2)^{3/2}}$

b. They both approach zero. **c.** $\varphi_x(0, y) = 0$

d. $\varphi_y(x, 0) = \dfrac{1}{(x^2 + 1)^{3/2}}$

87. $\dfrac{\partial^2 u}{\partial t^2} = -4c^2 \cos(2(x + ct)) = c^2 \dfrac{\partial^2 u}{\partial x^2}$

89. $\dfrac{\partial^2 u}{\partial t^2} = c^2 A f''(x + ct) + c^2 B g''(x - ct) = c^2 \dfrac{\partial^2 u}{\partial x^2}$

91. $u_{xx} = 6x$; $u_{yy} = -6x$

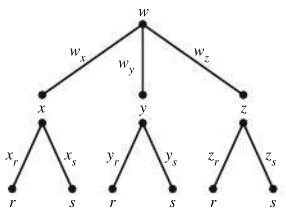
93. $u_{xx} = \dfrac{2(x-1)y}{((x-1)^2 + y^2)^2} - \dfrac{2(x+1)y}{((x+1)^2 + y^2)^2}$;
$u_{yy} = -\dfrac{2(x-1)y}{((x-1)^2 + y^2)^2} + \dfrac{2(x+1)y}{((x+1)^2 + y^2)^2}$

95. $\varepsilon_1 = \Delta y, \varepsilon_2 = 0$ or $\varepsilon_1 = 0, \varepsilon_2 = \Delta x$ **97. a.** f is continuous at $(0, 0)$. **b.** f is not differentiable at $(0, 0)$. **c.** $f_x(0, 0) = f_y(0, 0) = 0$ **d.** f_x and f_y are not continuous at $(0, 0)$. **e.** Theorem 15.5 does not apply because f_x and f_y are not continuous at $(0, 0)$; Theorem 15.6 does not apply because f is not differentiable at $(0, 0)$. **99.** $f_x(x, y) = yh(xy)$; $f_y(x, y) = xh(xy)$

Section 15.4 Exercises, pp. 957–961

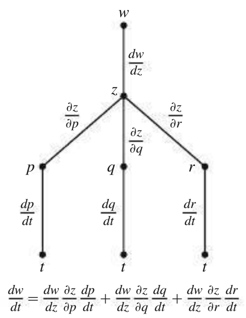
1. One dependent, two intermediate, and one independent variable
3. Multiply each of the partial derivatives of w by the t-derivative of the corresponding function and add all these expressions.

5.

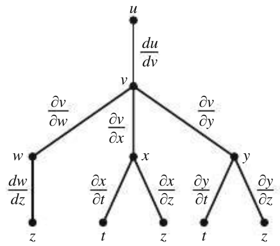


7. $4t^3 + 3t^2$
9. $z'(t) = 2t \sin 4t^3 + 12t^4 \cos 4t^3$
11. $w'(t) = -\sin t \sin 3t^4 + 12t^3 \cos t \cos 3t^4$
13. $z'(t) = 20(\sin^2 t + 2(3t + 4)^5)^9 (\sin t \cos t + 15(3t + 4)^4)$
15. $w'(t) = 20t^4 \sin(t + 1) + 4t^5 \cos(t + 1)$
17. $V'(t) = e^t((2t + 5)\sin t + (2t + 3)\cos t)$
19. $z_s = 2(s - t)\sin t^2$; $z_t = 2(s - t)(t(s - t)\cos t^2 - \sin t^2)$
21. $z_s = 2s - 3s^2 - 2st + t^2$; $z_t = -s^2 - 2t + 2st + 3t^2$
23. $z_s = (t + 1)e^{st+s+t}$; $z_t = (s + 1)e^{st+s+t}$
25. $w_s = -\dfrac{2t(t+1)}{(st + s - t)^2}$; $w_t = \dfrac{2s}{(st + s - t)^2}$
27. a. $V'(t) = 2\pi r(t)h(t)r'(t) + \pi r(t)^2 h'(t)$ **b.** $V'(t) = 0$ **c.** The volume remains constant.
29. $z'(t) = -\dfrac{2t + 2}{(t + 2t)} - \dfrac{3t^2}{(t^3 - 2)}$

31.



33.



$\dfrac{\partial u}{\partial z} = \dfrac{du}{dv}\left(\dfrac{\partial v}{\partial w}\dfrac{dw}{dz} + \dfrac{\partial v}{\partial x}\dfrac{\partial x}{\partial z} + \dfrac{\partial v}{\partial y}\dfrac{\partial y}{\partial z}\right)$

35. $\dfrac{dy}{dx} = \dfrac{x}{2y}$ **37.** $\dfrac{dy}{dx} = -\dfrac{y}{x}$ **39.** $\dfrac{dy}{dx} = -\dfrac{x + y}{2y^3 + x}$

41. $\dfrac{\partial s}{\partial x} = \dfrac{2x}{\sqrt{x^2 + y^2}}$; $\dfrac{\partial s}{\partial y} = \dfrac{2y}{\sqrt{x^2 + y^2}}$

43. $f_{ss} = 2(3s + t)$; $f_{st} = 2(s - t)$; $f_{tt} = -2(s + 3t)$

45. $f_{ss} = \dfrac{4t^2(-3s^2 + t^2)}{(s^2 + t^2)^3}$; $f_{st} = \dfrac{8st(s^2 - t^2)}{(s^2 + t^2)^3}$; $f_{tt} = -\dfrac{4(s^4 - 3s^2t^2)}{(s^2 + t^2)^3}$

47. $f''(s) = 4\left(\dfrac{6}{s^4} - \dfrac{2}{s^3} - 1 - 9s + 9s^2\right)$ **49. a.** False **b.** False

51. $w'(t) = 0$ **53.** $\dfrac{\partial z}{\partial x} = -\dfrac{z^2}{x^2}$ **55. a.** $w'(t) = af_x + bf_y + cf_z$
b. $w'(t) = ayz + bxz + cxy = 3abct^2$
c. $w'(t) = \sqrt{a^2 + b^2 + c^2}\,\dfrac{t}{|t|}$
d. $w''(t) = a^2 f_{xx} + b^2 f_{yy} + c^2 f_{zz} + 2ab f_{xy} + 2ac f_{xz} + 2bc f_{yz}$

57. $\dfrac{\partial z}{\partial x} = -\dfrac{y + z}{x + y}$; $\dfrac{\partial z}{\partial y} = -\dfrac{x + z}{x + y}$ **59.** $\dfrac{\partial z}{\partial x} = -\dfrac{yz + 1}{xy - 1}$;
$\dfrac{\partial z}{\partial y} = -\dfrac{xz + 1}{xy - 1}$ **61. a.** $z'(t) = -2x \sin t + 8y \cos t = 3 \sin 2t$
b. $0 < t < \pi/2$ and $\pi < t < 3\pi/2$

63. a. $z'(t) = \dfrac{(x + y)e^{-t}}{\sqrt{1 - x^2 - y^2}} = \dfrac{2e^{-2t}}{\sqrt{1 - 2e^{-2t}}}$ **b.** All $t \geq \frac{1}{2}\ln 2$

65. $E'(t) = mx'x'' + my'y'' + mgy' = 0$
67. a. The volume increases. **b.** The volume decreases.
69. a. $\dfrac{\partial P}{\partial V} = -\dfrac{P}{V}$; $\dfrac{\partial T}{\partial P} = \dfrac{V}{k}$; $\dfrac{\partial V}{\partial T} = \dfrac{k}{P}$ **b.** Follows directly from part (a)

71. a. $w'(t) = \dfrac{2t(t^2 + 1)\cos 2t - (t^2 - 1)\sin 2t}{2(t^2 + 1)^2}$
b. Max value of $t \approx 0.838$, $(x, y, z) \approx (0.669, 0.743, 0.838)$

73. a. $z_x = \dfrac{x}{r} z_r - \dfrac{y}{r^2} z_\theta$; $z_y = \dfrac{y}{r} z_r + \dfrac{x}{r^2} z_\theta$

b. $z_{xx} = \dfrac{x^2}{r^2} z_{rr} + \dfrac{y^2}{r^4} z_{\theta\theta} - \dfrac{2xy}{r^3} z_{r\theta} + \dfrac{y^2}{r^3} z_r + \dfrac{2xy}{r^4} z_\theta$

c. $z_{yy} = \dfrac{y^2}{r^2} z_{rr} + \dfrac{x^2}{r^4} z_{\theta\theta} + \dfrac{2xy}{r^3} z_{r\theta} + \dfrac{x^2}{r^3} z_r - \dfrac{2xy}{r^4} z_\theta$

75. a. $\left(\dfrac{\partial z}{\partial x}\right)_y = -\dfrac{F_x}{F_z}$ **b.** $\left(\dfrac{\partial y}{\partial z}\right)_x = -\dfrac{F_z}{F_y}$; $\left(\dfrac{\partial x}{\partial y}\right)_z = -\dfrac{F_y}{F_x}$

d. $\left(\dfrac{\partial w}{\partial x}\right)_{y,z}\left(\dfrac{\partial z}{\partial w}\right)_{x,y}\left(\dfrac{\partial y}{\partial z}\right)_{x,w}\left(\dfrac{\partial x}{\partial y}\right)_{z,w} = 1$

77. a. $\left(\dfrac{\partial w}{\partial x}\right)_y = f_x + f_z\dfrac{dz}{dx} = 18$ **b.** $\left(\dfrac{\partial w}{\partial x}\right)_z = f_x + f_y\dfrac{dy}{dx} = 8$

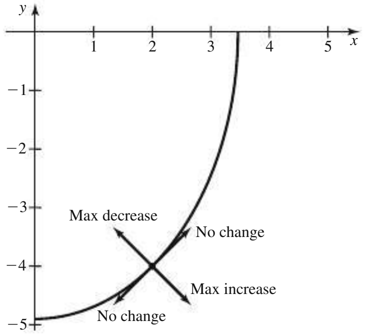
d. $\left(\dfrac{\partial w}{\partial y}\right)_x = -5$; $\left(\dfrac{\partial w}{\partial y}\right)_z = 4$; $\left(\dfrac{\partial w}{\partial z}\right)_x = \dfrac{5}{2}$; $\left(\dfrac{\partial w}{\partial z}\right)_y = \dfrac{9}{2}$

Section 15.5 Exercises, pp. 970–973

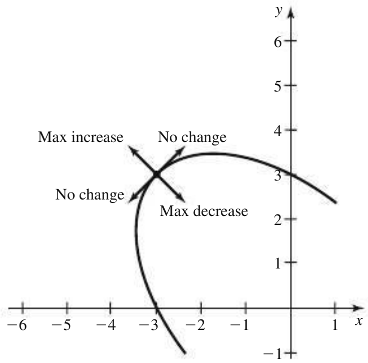
1. Form the dot product between the unit direction vector **u** and the gradient of the function. **3.** Direction of steepest ascent
5. The gradient is orthogonal to the level curves of f.
7. -2 **9.** $-7; 0$
11. a.

	$(a,b) = (2,0)$	$(a,b) = (0,2)$	$(a,b) = (1,1)$
$\mathbf{u} = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$	$-\sqrt{2}$	$-2\sqrt{2}$	$-3\sqrt{2}/2$
$\mathbf{v} = \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$	$\sqrt{2}$	$-2\sqrt{2}$	$-\sqrt{2}/2$
$\mathbf{w} = \langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$	$\sqrt{2}$	$2\sqrt{2}$	$3\sqrt{2}/2$

b. The function is decreasing at $(2, 0)$ in the direction of **u** and increasing at $(2, 0)$ in the directions of **v** and **w**.
13. $\nabla f(x, y) = \langle 6x, -10y \rangle, \nabla f(2, -1) = \langle 12, 10 \rangle$
15. $\nabla g(x, y) = \langle 2(x - 4xy - 4y^2), -4x(x + 4y) \rangle$, $\nabla g(-1, 2) = \langle -18, 28 \rangle$ **17.** $\nabla f(x, y) = e^{2xy} \langle 1 + 2xy, 2x^2 \rangle$, $\nabla f(1, 0) = \langle 1, 2 \rangle$ **19.** $\nabla F(x, y) = -2e^{-x^2-2y^2} \langle x, 2y \rangle$, $\nabla F(-1, 2) = 2e^{-9} \langle 1, -4 \rangle$ **21.** -6 **23.** $\frac{27}{2} - 6\sqrt{3}$
25. $-\dfrac{2}{\sqrt{5}}$ **27.** -2 **29.** 0 **31. a.** Direction of steepest ascent: $\dfrac{1}{\sqrt{65}}\langle 1, 8 \rangle$; direction of steepest descent: $-\dfrac{1}{\sqrt{65}}\langle 1, 8 \rangle$
b. $\langle -8, 1 \rangle$ **33. a.** Direction of steepest ascent: $\dfrac{1}{\sqrt{5}}\langle -2, 1 \rangle$; direction of steepest descent: $\dfrac{1}{\sqrt{5}}\langle 2, -1 \rangle$ **b.** $\langle 1, 2 \rangle$
35. a. Direction of steepest ascent: $\dfrac{1}{\sqrt{2}}\langle 1, -1 \rangle$; direction of steepest descent: $\dfrac{1}{\sqrt{2}}\langle -1, 1 \rangle$ **b.** $\langle 1, 1 \rangle$
37. a. $\nabla f(3, 2) = -12\mathbf{i} - 12\mathbf{j}$
b. Direction of max increase: $\theta = \dfrac{5\pi}{4}$; direction of max decrease: $\theta = \dfrac{\pi}{4}$; directions of no change: $\theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$
c. $g(\theta) = -12 \cos \theta - 12 \sin \theta$ **d.** $\theta = \frac{5\pi}{4}, g\left(\frac{5\pi}{4}\right) = 12\sqrt{2}$
e. $\nabla f(3, 2) = 12\sqrt{2} \left\langle \cos \frac{5\pi}{4}, \sin \frac{5\pi}{4} \right\rangle, |\nabla f(3, 2)| = 12\sqrt{2}$
39. a. $\nabla f(\sqrt{3}, 1) = \frac{\sqrt{6}}{6}\langle \sqrt{3}, 1 \rangle$ **b.** Direction of max increase: $\theta = \frac{\pi}{6}$; direction of max decrease: $\theta = \frac{7\pi}{6}$; directions of no change: $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$ **c.** $g(\theta) = \frac{\sqrt{2}}{2} \cos \theta + \frac{\sqrt{6}}{6} \sin \theta$ **d.** $\theta = \frac{\pi}{6}, g\left(\frac{\pi}{6}\right) = \frac{\sqrt{6}}{3}$
e. $\nabla f(\sqrt{3}, 1) = \frac{\sqrt{6}}{3}\langle \cos \frac{\pi}{6}, \sin \frac{\pi}{6} \rangle, |\nabla f(\sqrt{3}, 1)| = \frac{\sqrt{6}}{3}$
41. a. $\nabla F(-1, 0) = \frac{2}{e}\,\mathbf{i}$ **b.** Direction of max increase: $\theta = 0$; direction of max decrease: $\theta = \pi$; directions of no change: $\theta = \pm\frac{\pi}{2}$
c. $g(\theta) = \frac{2}{e} \cos \theta$ **d.** $\theta = 0, g(0) = \frac{2}{e}$
e. $\nabla F(-1, 0) = \frac{2}{e}\langle \cos 0, \sin 0 \rangle, |\nabla F(-1, 0)| = \frac{2}{e}$
43.

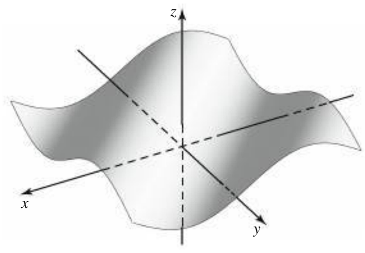


45.



47. $y' = 0$
49. Vertical tangent
51. $y' = -2/\sqrt{3}$
53. Vertical tangent

55. a. $\nabla f = \langle 1, 0 \rangle$ **b.** $x = 4 - t, y = 4, t \geq 0$
c. $\mathbf{r}(t) = \langle 4 - t, 4, 8 - t \rangle$, for $t \geq 0$
57. a. $\nabla f = \langle -2x, -4y \rangle$ **b.** $y = x^2, x \geq 1$
c. $\mathbf{r}(t) = \langle t, t^2, 4 - t^2 - 2t^4 \rangle$, for $t \geq 1$
59. a. $\nabla f(x, y, z) = 2x\,\mathbf{i} + 4y\,\mathbf{j} + 8z\,\mathbf{k}, \nabla f(1, 0, 4) = 2\mathbf{i} + 32\mathbf{k}$
b. $\dfrac{1}{\sqrt{257}}(\mathbf{i} + 16\mathbf{k})$ **c.** $2\sqrt{257}$ **d.** $17\sqrt{2}$
61. a. $\nabla f(x, y, z) = 4yz\,\mathbf{i} + 4xz\,\mathbf{j} + 4xy\,\mathbf{k}$, $\nabla f(1,-1,-1) = 4\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$ **b.** $\dfrac{1}{\sqrt{3}}(\mathbf{i} - \mathbf{j} - \mathbf{k})$ **c.** $4\sqrt{3}$
d. $\dfrac{4}{\sqrt{3}}$ **63. a.** $\nabla f(x, y, z) = \cos(x + 2y - z)(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, $\nabla f\left(\dfrac{\pi}{6}, \dfrac{\pi}{6}, -\dfrac{\pi}{6}\right) = -\dfrac{1}{2}\mathbf{i} - \mathbf{j} + \dfrac{1}{2}\mathbf{k}$ **b.** $\dfrac{1}{\sqrt{6}}(-\mathbf{i} - 2\mathbf{j} + \mathbf{k})$
c. $\sqrt{6}/2$ **d.** $-\dfrac{1}{2}$
65. a. $\nabla f(x, y, z) = \dfrac{2}{1 + x^2 + y^2 + z^2}(x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k})$, $\nabla f(1, 1, -1) = \dfrac{1}{2}\mathbf{i} + \dfrac{1}{2}\mathbf{j} - \dfrac{1}{2}\mathbf{k}$ **b.** $\dfrac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} - \mathbf{k})$
c. $\dfrac{\sqrt{3}}{2}$ **d.** $\dfrac{5}{6}$ **67. a.** False **b.** False **c.** False **d.** True
69. $\pm\dfrac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$ **71.** $\pm\dfrac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$
73. $x = x_0 + at, y = y_0 + bt$ **75. a.** $\nabla f(x, y, z) = \langle 2x, 2y, 2z \rangle$, $\nabla f(1, 1, 1) = \langle 2, 2, 2 \rangle$ **b.** $x + y + z = 3$
77. a. $\nabla f(x, y, z) = e^{x+y-z}\langle 1, 1, -1 \rangle, \nabla f(1, 1, 2) = \langle 1, 1, -1 \rangle$
b. $x + y - z = 0$
79. a.



b. $\mathbf{v} = \pm\langle 1, 1 \rangle$
c. $\mathbf{v} = \pm\langle 1, -1 \rangle$

83. $\langle u, v \rangle = \langle \pi \cos \pi x \sin 2\pi y, 2\pi \sin \pi x \cos 2\pi y \rangle$
87. $\nabla f(x, y) = \dfrac{1}{(x^2 + y^2)^2} \langle y^2 - x^2 - 2xy, x^2 - y^2 - 2xy \rangle$
89. $\nabla f(x, y, z) = -\dfrac{1}{\sqrt{25 - x^2 - y^2 - z^2}}\langle x, y, z \rangle$
91. $\nabla f(x, y, z) = \dfrac{(y + xz)\langle 1, z, y \rangle - (x + yz)\langle z, 1, x \rangle}{(y + xz)^2}$

$$= \frac{1}{(y + xz)^2} \langle y(1 - z^2), x(z^2 - 1), y^2 - x^2 \rangle$$

Section 15.6 Exercises, pp. 980–983

1. The gradient of f is a multiple of $\mathbf{n}$.
3. $F_x(a, b, c)(x - a) + F_y(a, b, c)(y - b) + F_z(a, b, c)(z - c) = 0$
5. Multiply the change in x by $f_x(a, b)$ and the change in y by $f_y(a, b)$, and add both terms to f. **7.** $dz = f_x(x, y)\,dx + f_y(x, y)\,dy$
9. $z = 5x - 3y + 5$ **11.** $3x - y + 6z = 4$ **13.** $2x + y + z = 4$; $4x + y + z = 7$ **15.** $x + y + z = 6$; $3x + 4y + z = 12$
17. $z = -8x - 4y + 16$ and $z = 4x + 2y + 7$ **19.** $z = y + 1$ and $z = x + 1$ **21.** $x + \frac{1}{2}y + \sqrt{3}z = 2 + \frac{\sqrt{3}\pi}{6}$ and $\frac{1}{2}x + y + \sqrt{3}z = \frac{5\sqrt{3}\pi}{6} - 2$ **23.** $\frac{1}{2}x + \frac{2}{3}y + 2\sqrt{3}z = -2$ and $x - 2y + 2\sqrt{14}z = 2$ **25.** $z = 8x - 4y - 4$ and $z = -x - y - 1$
27. $z = \frac{7}{25}x - \frac{1}{25}y - \frac{2}{5}$ and $z = -\frac{7}{25}x + \frac{1}{25}y + \frac{6}{5}$
29. $z = \frac{1}{2}x + \frac{1}{2}y + \frac{\pi}{4} - 1$
31. $\frac{1}{6}(x - \pi) + \frac{\pi}{6}(y - 1) + \pi\left(z - \frac{1}{6}\right) = 0$
33. a. $L(x, y) = 4x + y - 6$ **b.** $L(2.1, 2.99) = 5.39$
35. a. $L(x, y) = -6x - 4y + 7$ **b.** $L(3.1, -1.04) = -7.44$
37. a. $L(x, y, z) = x + y + 2z$ **b.** $L(0.1, -0.2, 0.2) = 0.3$
39. $dz = -6dx - 5dy = -0.1$ **41.** $dz = dx + dy = 0.05$
43. a. The surface area decreases. **b.** Impossible to say **c.** $\Delta S \approx 53.3$ **d.** $\Delta S \approx 33.95$ **e.** $R\,dR = r\,dr$ **45.** $\frac{\Delta A}{A} \approx 3.5\%$
47. $dw = (y^2 + 2xz)\,dx + (2xy + z^2)\,dy + (x^2 + 2yz)\,dz$
49. $dw = \frac{dx}{y + z} - \frac{u + x}{(y + z)^2}dy - \frac{u + x}{(y + z)^2}dz + \frac{du}{y + z}$
51. a. $\Delta c \approx 0.035$ **b.** When $\theta = \frac{\pi}{20}$ **53. a.** True **b.** True **c.** False **55.** $(1, -1, 1)$ and $(1, -1, -1)$
57. Points with $x = 0, \pm\frac{\pi}{2}, \pm\pi$ and $y = \pm\frac{\pi}{2}$, or points with $x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$ and $y = 0, \pm\pi$
59. a. $\Delta S \approx 0.749$ **b.** More sensitive to changes in r
61. a. $\Delta A \approx \frac{2}{1225} = 0.00163$ **b.** No. The batting average increases more if the batter gets a hit than it decreases if he fails to get a hit. **c.** Yes. The answer depends on whether A is less than 0.500 or greater than 0.500. **63. a.** $\Delta V \approx \frac{21}{5000} = 0.0042$ **b.** $\frac{\Delta V}{V} \approx -4\%$ **c.** $2p$
65. a. $f_r = n(1 - r)^{n-1}$, $f_n = -(1 - r)^n \ln(1 - r)$
b. $\Delta P \approx 0.027$ **c.** $\Delta P \approx 2 \times 10^{-20}$ **67.** $\Delta R \approx 7/540 \approx 0.013\Omega$
69. a. Apply the Chain Rule. **b.** Follows directly from (a)
c. $d(\ln(xy)) = \frac{dx}{x} + \frac{dy}{y}$ **d.** $d(\ln(x/y)) = \frac{dx}{x} - \frac{dy}{y}$
e. $\frac{df}{f} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2} + \cdots + \frac{dx_n}{x_n}$

Section 15.7 Exercises, pp. 993–996

1. The local maximum occurs at the highest point on the surface; you cannot get to a higher point in any direction. **3.** The partial derivatives are both zero or do not exist. **5.** The discriminant is a determinant; it is defined as $D(a, b) = f_{xx}(a, b)\,f_{yy}(a, b) - f_{xy}^2(a, b)$.
7. f has an absolute minimum value on R at (a, b) if $f(a, b) \le f(x, y)$ for all (x, y) in R. **9.** Saddle point **11.** Local min **13.** $(0, 0)$
15. $(0, 1), (0, -1)$ **17.** $(0, 0), (2, 2)$, and $(-2, -2)$
19. $(0, 2), (\pm 1, 2)$ **21.** $(3, 0), (-15, 6)$ **23.** Saddle point at $(0, 0)$
25. Local min at $(0, 0)$ **27.** Saddle point at $(0, 0)$; local min at $(1, 1)$ and $(-1, -1)$ **29.** $(0, 0)$; Second Derivative Test is inconclusive; absolute min of 4 at $(0, 0)$ **31.** Local min at $(2, 0)$
33. Saddle point at $(0, 0)$; local max at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$; local min at $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
35. Local min at $(-1, 0)$; local max at $(1, 0)$ **37.** Saddle point at $(0, 1)$; local min at $(\pm 2, 0)$ **39.** Saddle point at $(0, 0)$ **41.** Saddle point **43.** Height $= 32$ in, base is 16 in $\times$ 16 in; volume is 8192 in^3
45. 2 m $\times$ 2 m $\times$ 1 m **47.** Absolute min: $0 = f(0, 1)$; absolute max: $9 = f(0, -2)$ **49.** Absolute min: $4 = f(0, 0)$; absolute max: $7 = f(\pm 1, \pm 1)$ **51.** Absolute min: $0 = f(1, 0)$; absolute max: $3 = f(1, 1) = f(1, -1)$ **53.** Absolute min: $1 = f(1, -2) = f(1, 0)$; absolute max: $4 = f(1, -1)$
55. Absolute min: $0 = f(0, 0)$; absolute max: $\frac{7}{8} = f\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$
57. a. 1.83; 82.6° **b.** 14% **59.** Absolute min: $-4 = f(0, 0)$; no absolute max on R **61.** Absolute max: $2 = f(0, 0)$; no absolute min on R **63.** $P\left(\frac{4}{3}, \frac{2}{3}, \frac{4}{3}\right)$ **65.** $(3, 4, 5), (3, 4, -5)$
67. a. True **b.** False **c.** True **d.** True
69. Local min at $(0.3, -0.3)$; saddle point at $(0, 0)$
71. a.–d. $x = y = z = \frac{200}{3}$
73. a. $P\left(1, \frac{1}{3}\right)$ **b.** $P\left(\frac{1}{3}(x_1 + x_2 + x_3), \frac{1}{3}(y_1 + y_2 + y_3)\right)$
c. $P(\bar{x}, \bar{y})$, where $\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k$ and $\bar{y} = \frac{1}{n}\sum_{k=1}^{n} y_k$
d. $d(x, y) = \sqrt{x^2 + y^2} + \sqrt{(x - 2)^2 + y^2} + \sqrt{(x - 1)^2 + (y - 1)^2}$. Absolute max: $1 + \sqrt{3} = f\left(1, \frac{1}{\sqrt{3}}\right)$
77. $y = \frac{22}{13}x + \frac{46}{13}$ **79.** $a = b = c = 3$
81. a. $\nabla d_1(x, y) = \frac{x - x_1}{d_1(x, y)}\mathbf{i} + \frac{y - y_1}{d_1(x, y)}\mathbf{j}$
b. $\nabla d_2(x, y) = \frac{x - x_2}{d_2(x, y)}\mathbf{i} + \frac{y - y_2}{d_2(x, y)}\mathbf{j}$;
$\nabla d_3(x, y) = \frac{x - x_3}{d_3(x, y)}\mathbf{i} + \frac{y - y_3}{d_3(x, y)}\mathbf{j}$
c. Follows from $\nabla f = \nabla d_1 + \nabla d_2 + \nabla d_3$ **d.** Three unit vectors add to zero. **e.** P is the vertex at the large angle.
f. $P(0.255457, 0.304504)$ **83. a.** Local max at $(1, 0), (-1, 0)$ **b.** Local max at $(1, 0)$ and $(-1, 0)$ **85.** $\frac{abc\sqrt{3}}{2}$

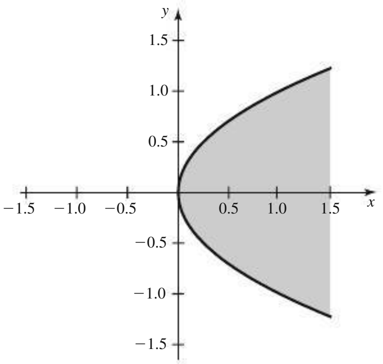
Section 15.8 Exercises, pp. 1002–1004

1. The level curve of f is tangent to the curve $g = 0$ at the optimal point; therefore, the gradients are parallel.
3. $1 = 2\lambda x$, $4 = 2\lambda y$, $x^2 + y^2 - 1 = 0$
5. Abs. min: 1; abs. max: 8 **7.** Abs. min: $-2\sqrt{5}$ at $\left(-\frac{2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$; abs. max: $2\sqrt{5}$ at $\left(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$
9. Abs. min: -2 at $(-1, -1)$; abs. max: 2 at $(1, 1)$
11. Abs. min: -3 at $(-\sqrt{3}, \sqrt{3})$ and $(\sqrt{3}, -\sqrt{3})$; abs. max: 9 at $(3, 3)$ and $(-3, -3)$
13. Abs. min: e^{-9} at $(-3, 3)$ and $(3, -3)$; abs. max: e^3 at $(\sqrt{3}, \sqrt{3})$ and $(-\sqrt{3}, -\sqrt{3})$ **15.** Abs. min: 9 at $(0, 3)$; abs. max: 34 at $(-\sqrt{15}, -2)$ and $(\sqrt{15}, -2)$ **17.** Abs. min: $-2\sqrt{11}$ at $\left(-\frac{2}{\sqrt{11}}, -\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}\right)$; abs. max: $2\sqrt{11}$ at $\left(\frac{2}{\sqrt{11}}, \frac{6}{\sqrt{11}}, -\frac{2}{\sqrt{11}}\right)$

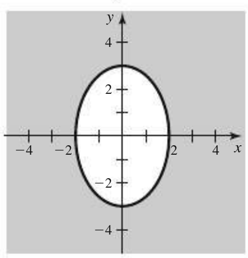
19. Abs. min: $-\frac{\sqrt{5}}{2}$ at $\left(-\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right)$; abs. max: $\frac{\sqrt{5}}{2}$ at $\left(\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right)$
21. Abs. min: -5 at $(-2, -2, -1)$; abs. max: 5 at $(2, 2, 1)$
23. Abs. min: -10 at $(-5, 0, 0)$; abs. max: $\frac{29}{2}$ at $\left(2, 0, \pm\sqrt{\frac{21}{2}}\right)$
25. Abs. min: $-\sqrt{3}$ at $\left(0, -\frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right)$; abs. max: $\frac{7}{4}$ at $\left(\frac{1}{2}, \frac{1}{2}, 1\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}, 1\right)$ **27.** 18 in $\times$ 18 in $\times$ 36 in **29.** Abs. min: 0.6731; abs. max: 1.1230 **31.** 2×1 **33.** $\left(-\frac{3}{17}, \frac{29}{17}, -3\right)$
35. Abs. min: $\sqrt{38 - 6\sqrt{29}}$ (or $\sqrt{29} - 3$); abs. max: $\sqrt{38 + 6\sqrt{29}}$ (or $\sqrt{29} + 3$) **37.** $\ell = 3$ and $g = \frac{3}{2}$; $U = 15\sqrt{2}$
39. $\ell = \frac{16}{5}$ and $g = 1$; $U = 20.287$ **41. a.** True **b.** False
43. $\frac{\sqrt{6}}{3}$ m $\times$ $\frac{\sqrt{6}}{3}$ m $\times$ $\frac{\sqrt{6}}{6}$ m **45.** $2 \times 1 \times \frac{2}{3}$ **47.** $P\left(\frac{4}{3}, \frac{2}{3}, \frac{4}{3}\right)$
49. Abs. min: 1; abs. max: 9 **51.** Abs. min: 0; abs. max: 3
53. $K = 7.5$ and $L = 5$ **57.** Abs. max: 8 **59.** Abs. max: $\sqrt{a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2}$ **61. a.** Gradients are perpendicular to level surfaces. **b.** If the gradient were not in the plane spanned by ∇g and ∇h, f could be increased (decreased) by moving the point slightly. **c.** ∇f is a linear combination of ∇g and ∇h, since it belongs to the plane spanned by these two vectors. **d.** The gradient condition from part (c), as well as the constraints, must be satisfied.
63. Abs. min: $2 - 4\sqrt{2}$; abs. max: $2 + 4\sqrt{2}$
65. a. $y + 1 = \lambda y, x + 1 = \lambda x, xy = 4$ **c.** Abs. min of 108 over the curve C_1 **d.** Abs. max of 100 over the curve C_2
e. The constraint curve is unbounded, so there is no guarantee that an abs. min or max occurs over the curve $xy = 4$.

Chapter 15 Review Exercises, pp. 1005–1007

1. a. True **b.** False **c.** False **d.** False
3. $D = \{(x, y): x \geq y^2\}$

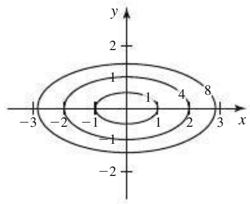


5. $D = \left\{(x, y): \frac{x^2}{4} + \frac{y^2}{9} \geq 1\right\}$



7. $D = \{(x, y): x^2 + y^2 \geq 1\}$; $R = (-\infty, 0]$; lower half of the hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$

9.



11. 2 **13.** Does not exist
15. $\frac{2}{3}$ **17.** 4
19. $\{(x, y): y > x^2 + 1\}$
21. $f_x = 6xy^5$; $f_y = 15x^2y^4$
23. $f_x = \frac{2xy^2}{(x^2 + y^2)^2}$; $f_y = -\frac{2x^2y}{(x^2 + y^2)^2}$
25. $f_x = y(1 + xy)e^{xy}$; $f_y = x(1 + xy)e^{xy}$ **27.** $f_{xx} = 4y^2e^{2xy}$; $f_{xy} = 2e^{2xy}(2xy + 1) = f_{yx}$; $f_{yy} = 4x^2e^{2xy}$
29. $\frac{\partial^2 u}{\partial x^2} = 6y = -\frac{\partial^2 u}{\partial y^2}$ **31. a.** V increases with R if r is fixed, $V_R > 0$; V decreases if r increases and R is fixed, $V_r < 0$.
b. $V_r = -4\pi r^2$; $V_R = 4\pi R^2$ **c.** The volume increases more if R is increased. **33.** $4t + 2 \ln 5$
35. $w_r = \frac{3r + s}{r(r + s)}$; $w_s = \frac{r + 3s}{s(r + s)}$; $w_t = \frac{1}{t}$
37. $\frac{dy}{dx} = -\frac{2xy}{2y^2 + (x^2 + y^2)\ln(x^2 + y^2)}$
39. a. $z'(t) = -24 \sin t \cos t = -12 \sin 2t$
b. $z'(t) > 0$ for $\frac{\pi}{2} < t < \pi$ and $\frac{3\pi}{2} < t < 2\pi$
41. a.

	$(a, b) = (0, 0)$	$(a, b) = (2, 0)$	$(a, b) = (1, 1)$
$\mathbf{u} = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$	0	$4\sqrt{2}$	$-2\sqrt{2}$
$\mathbf{v} = \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$	0	$-4\sqrt{2}$	$-6\sqrt{2}$
$\mathbf{w} = \langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$	0	$-4\sqrt{2}$	$2\sqrt{2}$

b. The function is increasing at $(2, 0)$ in the direction of **u** and decreasing at $(2, 0)$ in the directions of **v** and **w**.
43. $\nabla g = \langle 2xy^3, 3x^2y^2 \rangle$; $\nabla g(-1, 1) = \langle -2, 3 \rangle$; $D_{\mathbf{u}}\, g(-1, 1) = 2$
45. $\nabla h = \left\langle \frac{x}{\sqrt{2 + x^2 + 2y^2}}, \frac{2y}{\sqrt{2 + x^2 + 2y^2}} \right\rangle$; $\nabla h(2, 1) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$; $D_{\mathbf{u}}\, h(2, 1) = \frac{7\sqrt{2}}{10}$
47. $\nabla f = \langle y \cos xy, x \cos xy, -\sin z \rangle$; $\nabla f(1, \pi, 0) = \langle -\pi, -1, 0 \rangle$; $D_{\mathbf{u}}\, f(1, \pi, 0) = -\frac{1}{7}(3 + 2\pi)$
49. a. Direction of steepest ascent: $\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$; direction of steepest descent: $\mathbf{u} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$
b. No change: $\mathbf{u} = \pm\left(\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}\right)$
51. Tangent line is vertical; $\nabla f(2, 0) = -8\mathbf{i}$
53. $E = \frac{kx}{x^2 + y^2}\mathbf{i} + \frac{ky}{x^2 + y^2}\mathbf{j}$
55. $y = 2$ and $12x + 3y - 2z = 12$
57. $x + 2y + 3z = 6$ and $x - 2y + 3z = 6$
59. $x + y - z = 0$ and $x + y - z = 0$
61. a. $L(x, y) = x + 5y$ **b.** $L(1.95, 0.05) = 2.2$ **63.** Approx. -4%
65. a. $\Delta V \approx -0.1\pi$ m^3 **b.** $\Delta S \approx -0.05\pi$ m^2
67. Saddle point at $(0, 0)$; local min at $(2, -2)$

69. Saddle points at $(0, 0)$ and $(-2, 2)$; local max at $(0, 2)$; local min at $(-2, 0)$ **71.** Abs. min: $-1 = f(1, 1) = f(-1, -1)$; abs. max: $49 = f(2, -2) = f(-2, 2)$

73. Abs. min: $-\frac{1}{2} = f\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; abs. max: $\frac{1}{2} = f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

75. Abs. min: $\frac{23}{2} = f\left(\frac{1}{3}, \frac{5}{6}\right)$ abs. max: $\frac{29}{2} = f\left(\frac{5}{3}, \frac{7}{6}\right)$;

77. Abs. min: $-\sqrt{6} = f\left(-\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}\right)$;
abs. max: $\sqrt{6} = f\left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6}\right)$

79. $\frac{2a^2}{\sqrt{a^2 + b^2}}$ by $\frac{2b^2}{\sqrt{a^2 + b^2}}$

81. $x = \frac{1}{2} + \frac{\sqrt{10}}{20}, y = \frac{3}{2} + \frac{3\sqrt{10}}{20} = 3x, z = \frac{1}{2} + \frac{\sqrt{10}}{2} = \sqrt{10}x$

83. $(1, 2, 5)$

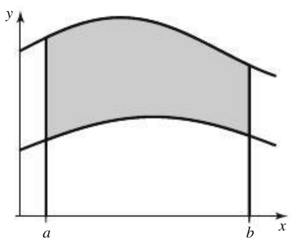
CHAPTER 16

Section 16.1 Exercises, pp. 1015–1017

1. $\int_0^2 \int_1^3 xy\,dy\,dx$ or $\int_1^3 \int_0^2 xy\,dx\,dy$ **3.** $\int_{-2}^4 \int_1^5 f(x, y)\,dy\,dx$ or $\int_1^5 \int_{-2}^4 f(x, y)\,dx\,dy$ **5.** 48 **7.** 4 **9.** $\frac{32}{3}$ **11.** 4 **13.** $\frac{224}{9}$ **15.** $10 - 2e$ **17.** $\frac{1}{2}$ **19.** $e^2 + 3$ **21.** $\frac{1}{2}$ **23.** $10\sqrt{5} - 4\sqrt{2} - 14$ **25.** $\frac{117}{2}$ **27.** $\frac{\pi^2}{4} + 1$ **29.** $\frac{4}{3}$ **31.** $\frac{9 - e^2}{2}$ **33.** $\frac{4}{11}$ **35.** $\frac{1}{4}$ **37.** 136 **39.** 3 **41.** $e^2 - 3$ **43.** $e^{16} - 17$ **45.** $\ln \frac{5}{3}$ **47.** $\frac{1}{2 \ln 2}$ **49.** $\frac{8}{3}$ **51. a.** True **b.** False **c.** True **53. a.** 1475 **b.** The sum of products of population densities and areas is a Riemann sum. **55.** $\int_c^d \int_a^b f(x)\,dy\,dx = (c - d) \int_a^b f(x)\,dx$. The integral is the area of the cross section of S. **57.** $a = \pi/6, 5\pi/6$ **59.** $a = \sqrt{6}$ **61. a.** $\frac{1}{2}\pi^2 + \pi$ **b.** $\frac{1}{2}\pi^2 + \pi$ **c.** $\frac{1}{2}\pi^2 + 2$ **63.** $f(a, b) - f(a, 0) - f(0, b) + f(0, 0)$

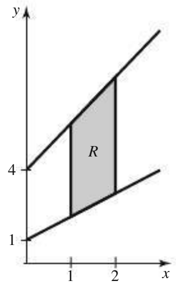
Section 16.2 Exercises, pp. 1024–1027

1.



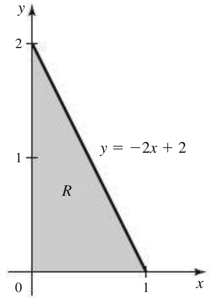
3. $dx\,dy$ **5.** $\int_0^1 \int_{x^2}^{\sqrt{x}} f(x, y)\,dy\,dx$ **7.** 4 **9.** $\int_0^2 \int_{x^3}^{4x} f(x, y)\,dy\,dx$ **11.** 2 **13.** $\frac{8}{3}$ **15.** 0 **17.** $e - 1$ **19.** $\frac{\ln^3 2}{6}$ **21.** 2 **23.** $\frac{\pi}{2} - 1$ **25.** 0 **27.** $\pi - 1$

29.



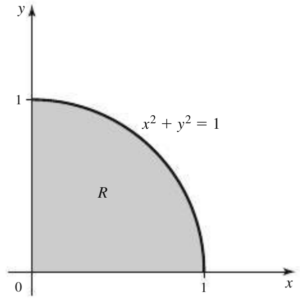
$\int_1^2 \int_{x+1}^{2x+4} f(x, y)\,dy\,dx$

31.



$\int_0^1 \int_0^{-2x+2} f(x, y)\,dy\,dx$

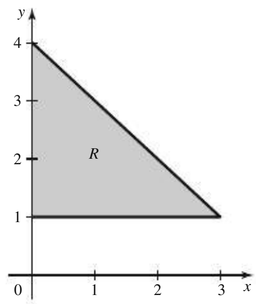
33.



$\int_0^1 \int_0^{\sqrt{1-x^2}} f(x, y)\,dy\,dx$

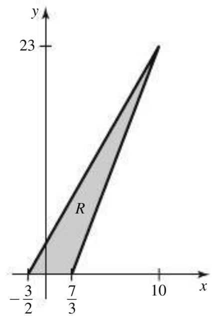
35. $\int_0^{18} \int_{y/2}^{(y+9)/3} f(x, y)\,dx\,dy$

37.



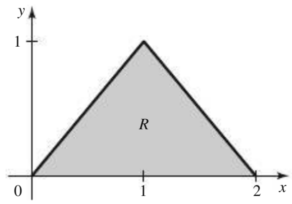
$\int_1^4 \int_0^{4-y} f(x, y)\,dx\,dy$

39.



$\int_0^{23} \int_{(y-3)/2}^{(y+7)/3} f(x, y)\,dx\,dy$

41.



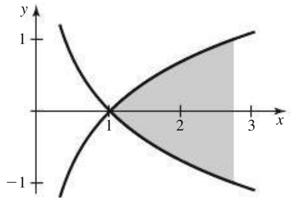
$\int_0^1 \int_y^{2-y} f(x, y)\,dx\,dy$

43. 2 **45.** 12 **47.** 5 **49.** 14 **51.** 32 **53.** $\frac{9}{8}$ **55.** $\frac{1}{4}\ln 2$

57. $\int_0^4\int_{y/2}^{\sqrt{y}} f(x, y)\,dx\,dy$ **59.** $\int_0^{\ln 2}\int_{1/2}^{e^{-x}} f(x, y)\,dy\,dx$

61. $\int_0^{\pi/2}\int_0^{\cos x} f(x, y)\,dy\,dx$ **63.** $\frac{1}{2}(e - 1)$ **65.** 0 **67.** $\frac{2}{3}$

69.

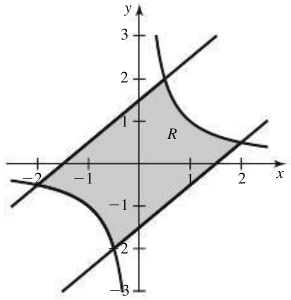


$\int_1^e\int_{-\ln x}^{\ln x} f(x, y)\,dy\,dx$

71. $\frac{11}{12}$ **73.** $\frac{32}{3}$ **75.** 12π **77.** $\frac{43}{6}$ **79.** $\frac{2}{3}$ **81.** 16 **83.** $4a\pi$

85. $\frac{32}{3}$ **87.** 1 **89.** $\frac{140}{3}$ **91. a.** False **b.** False **c.** False

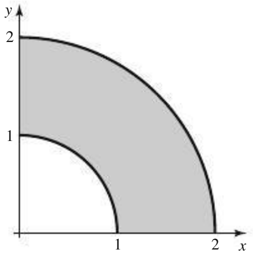
93. 30 **95.** $\frac{a}{3}$ **97. a.**



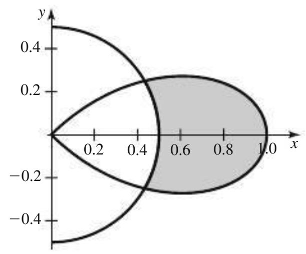
b. $\frac{15}{4} + 4\ln 2$ **c.** $2\ln 2 - \frac{5}{64}$ **99.** $\frac{3}{8e^2}$ **101.** 1

Section 16.3 Exercises, pp. 1033–1036

1. It is called a polar rectangle because r and θ vary between two constants.

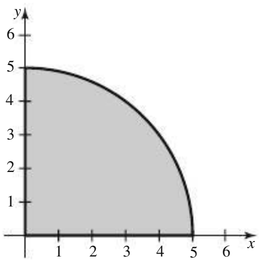


3.

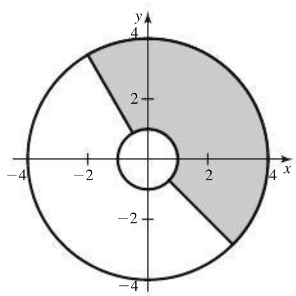


5. Evaluate the integral $\int_\alpha^\beta\int_{g(\theta)}^{h(\theta)} r\,dr\,d\theta$.

7.



9.

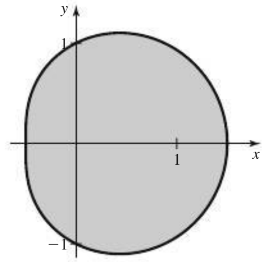


11. $\frac{64\pi}{3}$ **13.** $(8 - 24e^{-2})\pi$ **15.** $\frac{7\pi}{2}$ **17.** $\frac{9\pi}{2}$ **19.** $\frac{37\pi}{3}$

21. 128π **23.** 0 **25.** $(2 - \sqrt{3})\pi$ **27.** $2\pi/5$ **29.** $\frac{14\pi}{3}$

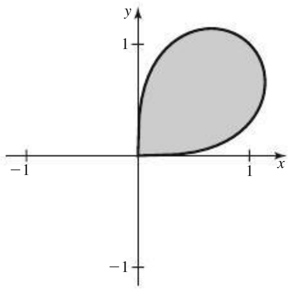
31. $\frac{81\pi}{2}$ **33.** π **35.** 8π **37.** 81π **39.** $\frac{2\pi}{3}(7\sqrt{7} - 15)$

41.



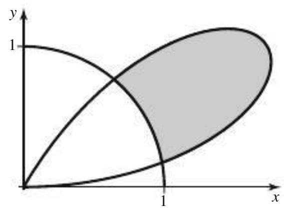
$\int_0^{2\pi}\int_0^{1+\frac{1}{2}\cos\theta} g(r, \theta)r\,dr\,d\theta$

43.



$\int_0^{\pi/2}\int_0^{\sqrt{2\sin 2\theta}} g(r, \theta)r\,dr\,d\theta$

45.



$\int_{\pi/18}^{5\pi/18}\int_1^{2\sin 3\theta} g(r, \theta)r\,dr\,d\theta$

47. $3\pi/2$ **49.** π **51.** $\frac{3\pi}{2} - 2\sqrt{2}$ **53.** $2a/3$

55. a. False **b.** True **c.** True **57.** The hyperboloid $\left(V = \frac{112\pi}{3}\right)$

59. a. $R = \{(r, \theta): -\pi/4 \le \theta \le \pi/4 \text{ or } 3\pi/4 \le \theta \le 5\pi/4\}$

b. $\frac{a^4}{4}$ **61.** $\frac{32}{9}$ **63.** $2\pi\left(1 - 2\ln\frac{3}{2}\right)$ **65.** 1

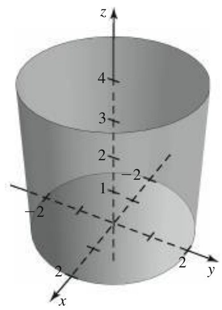
67. $\pi/4$ **69. a.** $\frac{16\pi}{3}$ **b.** 2.78 **71.** $30\pi + 42$

73. c. $\sqrt{\pi}/2$, $1/2$, and $\sqrt{\pi}/4$ **75. a.** $I = \frac{\sqrt{2}}{2}\tan^{-1}\frac{\sqrt{2}}{2}$

b. $I = \frac{\sqrt{2}}{4}\tan^{-1}\frac{\sqrt{2}a}{2} + \frac{a}{2\sqrt{a^2 + 1}}\tan^{-1}\frac{1}{\sqrt{a^2 + 1}}$ **c.** $\frac{\sqrt{2}\pi}{8}$

Section 16.4 Exercises, pp. 1043–1047

1.

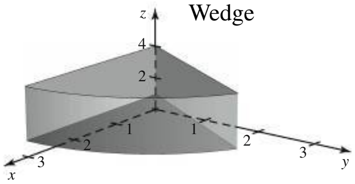


3. $\int_{-9}^{9}\int_{-\sqrt{81-x^2}}^{\sqrt{81-x^2}}\int_{-\sqrt{81-x^2-y^2}}^{\sqrt{81-x^2-y^2}} f(x, y, z)\,dz\,dy\,dx$

5. $\int_0^1\int_0^{\sqrt{1-z^2}}\int_0^{\sqrt{1-z^2-x^2}} f(x,y,z)\,dy\,dx\,dz$ **7.** 24 **9.** 8 **11.** $\frac{2}{\pi}$
13. 0 **15.** 8 **17.** $\frac{16}{3}$ **19.** $1-\ln 2$
21. $\frac{2\pi(1+19\sqrt{19}-20\sqrt{10})}{3}$ **23.** $\frac{27\pi}{2}$ **25.** 12π
27. $\frac{5}{12}$ **29.** 8 **31.** $\int_0^1\int_y^1\int_0^{2\sqrt{1-x^2}} f(x,y,z)\,dz\,dx\,dy$
33. $\int_0^1\int_0^{2\sqrt{1-x^2}}\int_0^{x} f(x,y,z)\,dy\,dz\,dx$
35. $\int_0^1\int_0^{2\sqrt{1-y^2}}\int_{y^2}^{\frac{1}{2}\sqrt{4-z^2}} f(x,y,z)\,dx\,dz\,dy$
37. $\int_0^1\int_0^2\int_0^{1-y} dz\,dx\,dy$, $\int_0^2\int_0^1\int_0^{1-z} dy\,dz\,dx$, $\int_0^1\int_0^2\int_0^{1-z} dy\,dx\,dz$, $\int_0^1\int_0^{1-y}\int_0^2 dx\,dz\,dy$, $\int_0^1\int_0^{1-z}\int_0^2 dx\,dy\,dz$ **39.** $\frac{256}{9}$ **41.** $\frac{2}{3}$
43. $(10\sqrt{10}-1)\frac{\pi}{6}$ **45.** $\frac{3\ln 2}{2}+\frac{e}{16}-1$
47. $\int_0^4\int_{y/4-1}^0\int_0^5 dz\,dx\,dy = 10$ **49.** $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2}} dz\,dy\,dx = \frac{2}{3}$
51. $\frac{8}{\pi}$ **53.** $\frac{10}{3}$ **55. a.** False **b.** False **c.** False **57.** 2
59. 1 **61.** $\frac{16}{3}$ **63.** $\frac{16}{3}$ **65.** $a=2\sqrt{2}$ **67.** $V=\frac{\pi r^2 h}{3}$
69. $V=\frac{\pi h^2}{3}(3R-h)$ **71.** $V=\frac{4\pi abc}{3}$ **73.** $\frac{1}{24}$

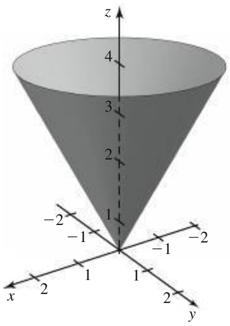
Section 16.5 Exercises, pp. 1059–1063

1. r measures the distance from the point to the z axis, θ is the angle that the segment from the point to the z-axis makes with the positive xz-plane, and z is the directed distance from the point to the xy-plane. **3.** A cone **5.** It approximates the volume of the cylindrical wedge formed by the changes Δr, $\Delta\theta$, and Δz.
7. $\int_\alpha^\beta\int_{g(\theta)}^{h(\theta)}\int_{G(r,\theta)}^{H(r,\theta)} w(r,\theta,z)\,r\,dz\,dr\,d\theta$ **9.** Cylindrical coordinates
11.



Wedge

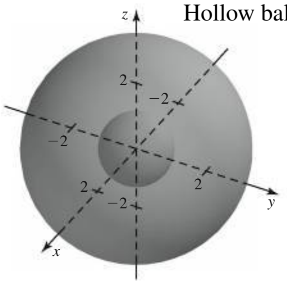
13.



Solid bounded by cone and plane

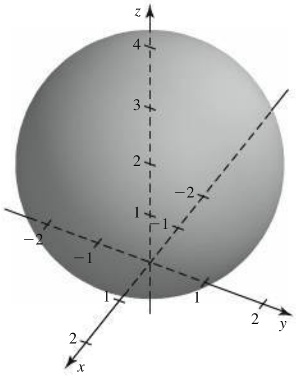
15. 2π **17.** $4\pi/5$ **19.** $\pi(1-e^{-1})/2$ **21.** $9\pi/4$
23. 560π **25.** 396π **27.** The paraboloid ($V=44\pi/3$)
29. $\frac{20\pi}{3}$ **31.** $\frac{(16+17\sqrt{29})\pi}{3}$ **33.** $\frac{1}{3}$

35.



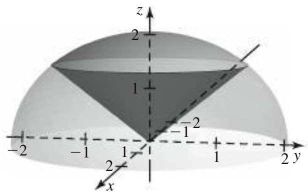
Hollow ball

37.



Sphere of radius $r=2$, centered at $(0, 0, 2)$

39. a. $(3960, 0.74, -2.13)$, $(-1426.85, -2257.05, 2924.28)$
b. $(3960, 0.84, 0.22)$, $(2877.61, 637.95, 2644.62)$ **c.** 5666 mi
41. $\frac{\pi}{2}$ **43.** $4\pi\ln 2$ **45.** $\pi\left(\frac{188}{9}-\frac{32\sqrt{3}}{3}\right)$ **47.** $\frac{32\pi\sqrt{3}}{9}$
49. $\frac{5\pi}{12}$ **51.** $\frac{8\pi}{3}$ **53.** $\frac{8\pi}{3}(9\sqrt{3}-11)$ **55. a.** True **b.** True
57. $z=\sqrt{x^2+y^2-1}$; upper half of a hyperboloid of one sheet
59. $\frac{8\pi}{3}(1-e^{-512})\approx\frac{8\pi}{3}$ **61.** 32π
63.



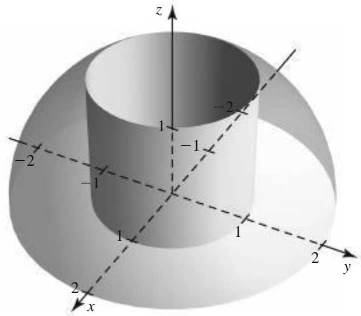
$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} g(r,\theta,z)\,r\,dz\,dr\,d\theta$,

$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_0^{z} g(r,\theta,z)\,r\,dr\,dz\,d\theta$

$+\int_0^{2\pi}\int_{\sqrt{2}}^{2}\int_0^{\sqrt{4-z^2}} g(r,\theta,z)\,r\,dr\,dz\,d\theta$,

$\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}}\int_0^{2\pi} g(r,\theta,z)\,r\,d\theta\,dz\,dr$

65.



$\int_{\pi/6}^{\pi/2}\int_0^{2\pi}\int_{\csc\varphi}^{2} g(\rho, \varphi, \theta)\rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi$,

$\int_{\pi/6}^{\pi/2}\int_{\csc\varphi}^{2}\int_0^{2\pi} g(\rho, \varphi, \theta)\rho^2 \sin\varphi \, d\theta \, d\rho \, d\varphi$

67. $32\sqrt{3}\pi/9$ **69.** $2\sqrt{2}/3$ **71.** $7\pi/2$ **73.** 95.6036

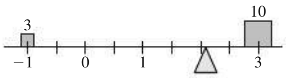
77. $V = \dfrac{\pi r^2 h}{3}$ **79.** $V = \dfrac{\pi}{3}(R^2 + rR + r^2)h$

81. $V = \dfrac{\pi R^3(8r - 3R)}{12r}$

Section 16.6 Exercises, pp. 1070–1072

1. The pivot should be located at the center of mass of the system.
3. Use a double integral. Integrate the density function over the region occupied by the plate. **5.** Use a triple integral to find the mass of the object and the three moments.

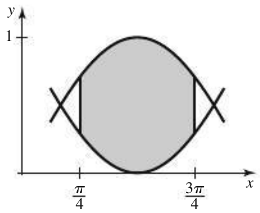
7.



$\frac{27}{13}$ **9.** Mass is $2 + \pi$; $\bar{x} = \frac{\pi}{2}$

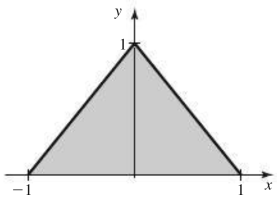
11. Mass is $\frac{20}{3}$; $\bar{x} = \frac{9}{5}$ **13.** Mass is 10; $\bar{x} = \frac{8}{3}$

15.



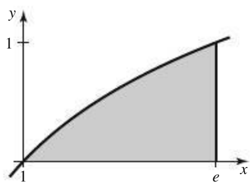
$\left(\dfrac{\pi}{2}, \dfrac{1}{2}\right)$

17.



$\left(0, \dfrac{1}{3}\right)$

19.

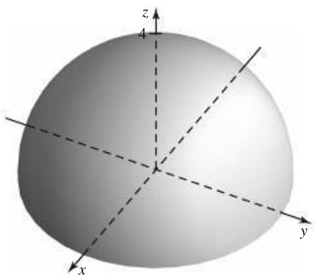


$\left(\dfrac{1}{4}(e^2 + 1), \dfrac{e}{2} - 1\right) \approx (2.10, 0.36)$

21. $\left(\frac{7}{3}, 1\right)$; density increases to the right. **23.** $\left(\frac{16}{11}, \frac{16}{11}\right)$; density increases toward the hypotenuse of the triangle.

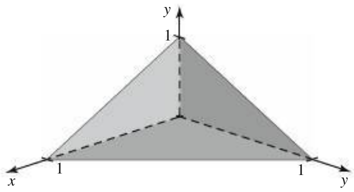
25. $\left(0, \frac{16 + 3\pi}{16 + 12\pi}\right) \approx (0, 0.4735)$; density increases away from the x-axis.

27.



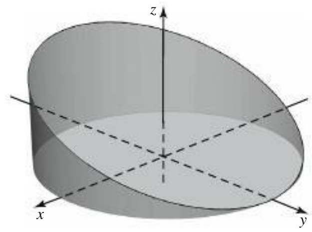
$\left(0, 0, \frac{3}{2}\right)$

29.



$\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

31.



$\left(0, -\frac{1}{4}, \frac{5}{8}\right)$

33. $\left(\frac{7}{3}, \frac{1}{2}, \frac{1}{2}\right)$ **35.** $\left(0, 0, \frac{198}{85}\right)$ **37.** $\left(\frac{2}{3}, \frac{7}{3}, \frac{1}{3}\right)$ **39. a.** False **b.** True **c.** False **d.** False **41.** $\bar{x} = \dfrac{\ln(1 + L^2)}{2\tan^{-1} L}$, $\lim_{L\to\infty} \bar{x} = \infty$

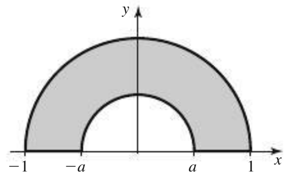
43. $\left(0, \dfrac{8}{9}\right)$ **45.** $\left(0, \dfrac{8}{3\pi}\right)$ **47.** $\left(\dfrac{5}{6}, 0\right)$ **49.** $\left(\dfrac{128}{105\pi}, \dfrac{128}{105\pi}\right)$

51. On the line of symmetry, $2a/\pi$ units above the diameter

53. $\left(\dfrac{2a}{3(4 - \pi)}, \dfrac{2a}{3(4 - \pi)}\right)$ **55.** $h/4$ units

57. $h/3$ units, where h is the height of the triangle **59.** $3a/8$ units

61. a. $\left(0, \dfrac{4(1 + a + a^2)}{3(1 + a)\pi}\right)$



b. $a = \dfrac{1}{2}\left(-1 + \sqrt{1 + \dfrac{16}{3\pi - 4}}\right) \approx 0.4937$

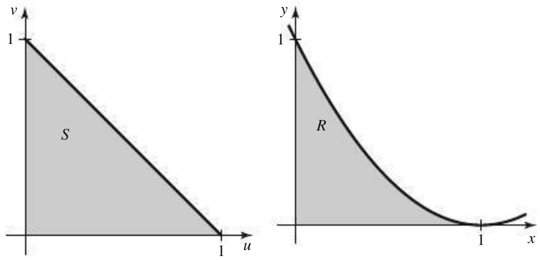
63. Depth $= \dfrac{40\sqrt{10} - 4}{333}$ cm $\approx$ 0.3678 cm

65. a. $(\bar{x}, \bar{y}) = \left(\dfrac{-r^2}{R + r}, 0\right)$ (origin at center of large circle);
$(\bar{x}, \bar{y}) = \left(\dfrac{R^2 + Rr + r^2}{R + r}, 0\right)$ (origin at common point of the circles)
b. *Hint:* Solve $\bar{x} = R - 2r$.

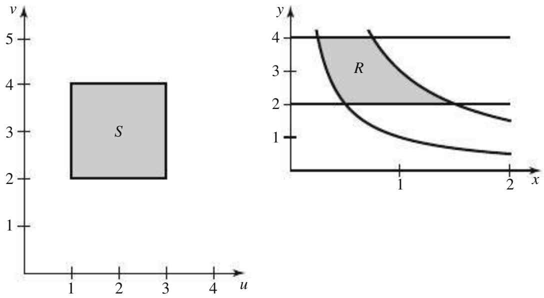
Section 16.7 Exercises, pp. 1082–1084

1. The image of S is the 2×2 square with vertices at (0, 0), (2, 0), (2, 2), and (0, 2). **3.** $\int_0^1\int_0^1 f(u + v, u - v)\, 2 \, du \, dv$
5. The rectangle with vertices at (0, 0), (2, 0), $\left(2, \frac{1}{2}\right)$, and $\left(0, \frac{1}{2}\right)$
7. The square with vertices at (0, 0), $\left(\frac{1}{2}, \frac{1}{2}\right)$, (1, 0), and $\left(\frac{1}{2}, -\frac{1}{2}\right)$
9. The region above the x-axis and bounded by the curves $y^2 = 4 \pm 4x$ **11.** The upper half of the unit circle

13.



15.

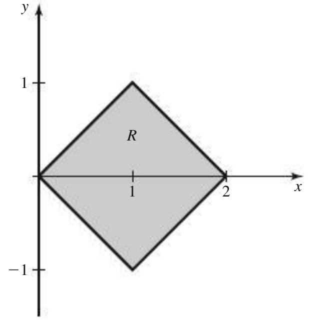


17. -9 **19.** $-4(u^2 + v^2)$ **21.** -1

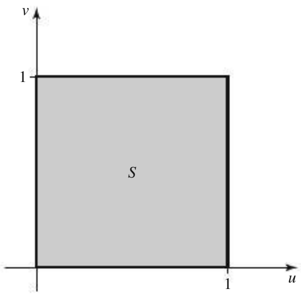
23. $x = (u + v)/3, y = (2u - v)/3; -1/3$

25. $x = -(u + 3v), y = -(u + 2v); -1$

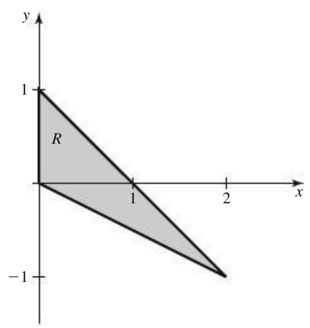
27. a.



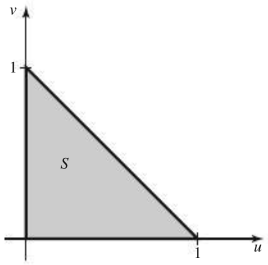
b. $0 \le u \le 1, 0 \le v \le 1$ **c.** $J(u, v) = -2$ **d.** 0



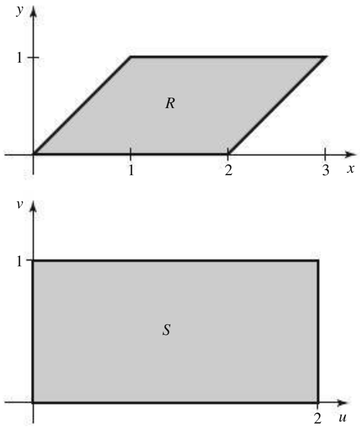
29. a.



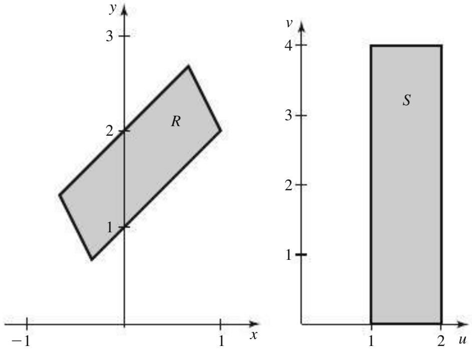
b. $0 \le u \le 1, 0 \le v \le 1 - u$ **c.** $J(u, v) = 2$ **d.** $256\sqrt{2}/945$



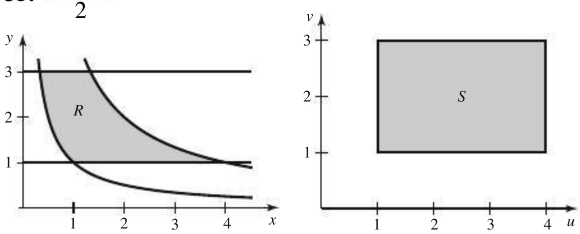
31. $4\sqrt{2}/3$



33. $3844/5625$



35. $\dfrac{15 \ln 3}{2}$



37. 2 **39.** $2w(u^2 - v^2)$ **41.** 5 **43.** $1024\pi/3$

45. a. True **b.** True **c.** True

47. *Hint:* $J(\rho, \varphi, \theta) = \begin{vmatrix} \sin\varphi\cos\theta & \rho\cos\varphi\cos\theta & -\rho\sin\varphi\sin\theta \\ \sin\varphi\sin\theta & \rho\cos\varphi\sin\theta & \rho\sin\varphi\cos\theta \\ \cos\varphi & -\rho\sin\varphi & 0 \end{vmatrix}$

49. $a^2b^2/2$ **51.** $(a^2 + b^2)/4$ **53.** $4\pi abc/3$

55. $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \dfrac{3c}{8}\right)$ **57. a.** $x = a^2 - \dfrac{y^2}{4a^2}$

b. $x = \dfrac{y^2}{4b^2} - b^2$ **c.** $J(u, v) = 4(u^2 + v^2)$ **d.** $\dfrac{80}{3}$ **e.** 160

f. Vertical lines become parabolas opening downward with vertices on the positive y-axis, and horizontal lines become parabolas opening upward with vertices on the negative y-axis. **59. a.** S is stretched in the positive u- and v-directions but not in the w-direction. The amount of stretching increases with u and v. **b.** $J(u, v, w) = ad$

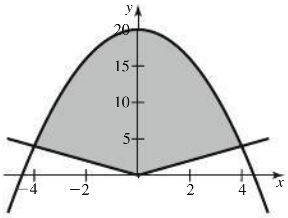
c. Volume $= ad$ **d.** $\left(\frac{a+b+c}{2}, \frac{d+e}{2}, \frac{1}{2}\right)$

Chapter 16 Review Exercises, pp. 1084–1088

1. a. False **b.** True **c.** False **d.** False **3.** $\frac{26}{3}$

5. $\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y)\, dx\, dy$ **7.** $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x, y)\, dy\, dx$

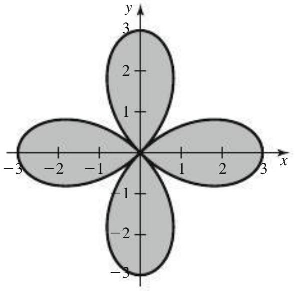
9. $\frac{304}{3}$



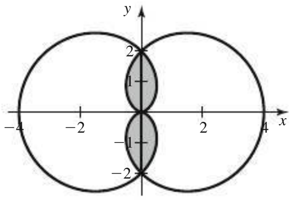
11. $\frac{\sqrt{17} - \sqrt{2}}{2}$

13. 8π **15.** $\frac{2}{7\pi^2}$ **17.** $\frac{1}{5}$

19. $\frac{9\pi}{2}$



21. $6\pi - 16$



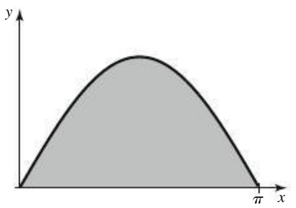
23. 2 **25.** $\int_0^1 \int_{2y}^2 \int_0^{\sqrt{z^2-4y^2}/2} f(x, y, z)\, dx\, dz\, dy$ **27.** $\pi - \frac{4}{3}$

29. $8 \sin^2 2 = 4(1 - \cos 4)$ **31.** $\frac{848}{9}$ **33.** $\frac{8}{15}$ **35.** $\frac{16}{3}$

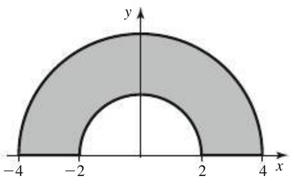
37. $\frac{128}{3}$ **39.** $\frac{\pi}{6} - \frac{\sqrt{3}}{2} + \frac{1}{2}$ **41.** $\frac{1}{3}$ **43.** $\frac{1}{3}$ **45.** π

47. 4π **49.** $\frac{28\pi}{3}$ **51.** $\frac{2048\pi}{105}$

53. $(\bar{x}, \bar{y}) = \left(\frac{\pi}{2}, \frac{\pi}{8}\right)$



55. $(\bar{x}, \bar{y}) = \left(0, \frac{56}{9\pi}\right)$



57. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 24)$ **59.** $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{63}{10}\right)$

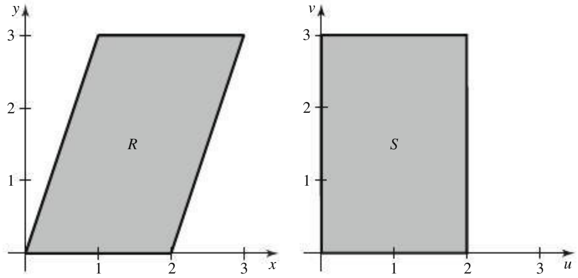
61. $\frac{h}{3}$ **63.** $\left(\frac{4\sqrt{2}a}{3\pi}, \frac{4(2 - \sqrt{2})a}{3\pi}\right)$ **65. a.** $\frac{4\pi}{3}$ **b.** $\frac{16Q}{3}$

67. $R = \{(x, y): 0 \le x \le 1, 0 \le y \le 1\}$

69. The parallelogram with vertices $(0, 0)$, $(3, 1)$, $(4, 4)$, and $(1, 3)$

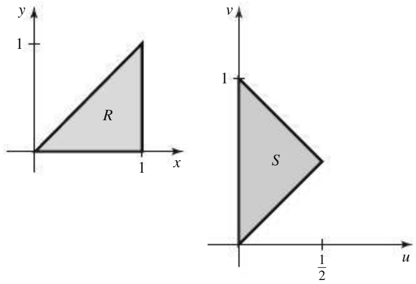
71. 10 **73.** 6

75. a.



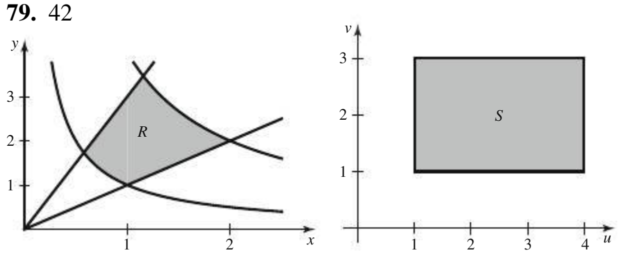
b. $0 \le u \le 2, 0 \le v \le 3$ **c.** $J(u, v) = 1$ **d.** $\frac{63}{2}$

77. a.



b. $u \le v \le 1 - u, 0 \le u \le \frac{1}{2}$ **c.** $J(u, v) = 2$ **d.** $\frac{1}{60}$

79. 42



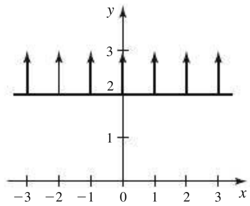
81. $-\frac{7}{16}$

CHAPTER 17

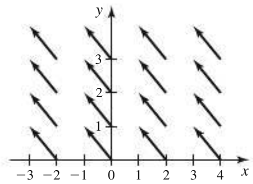
Section 17.1 Exercises, pp. 1096–1098

1. $\mathbf{F} = \langle f, g, h \rangle$ evaluated at (x, y, z) is the velocity vector of an air particle at (x, y, z) at a fixed point in time. **3.** At selected points (a, b), plot the vector $\langle f(a, b), g(a, b) \rangle$. **5.** It shows the direction in which the temperature increases the fastest and the amount of increase.

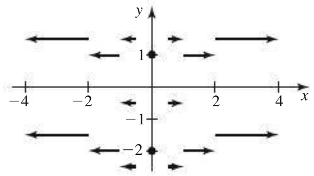
7.



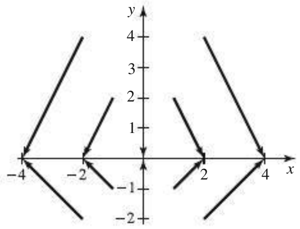
9.



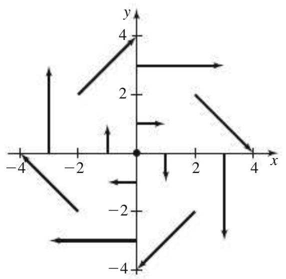
11.



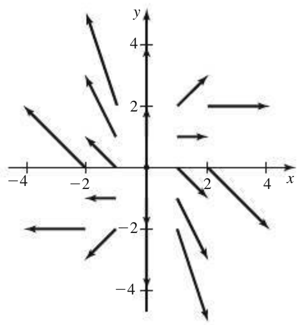
13.



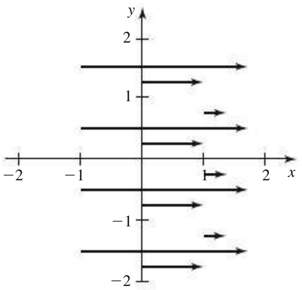
15.



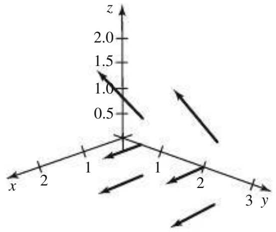
17.



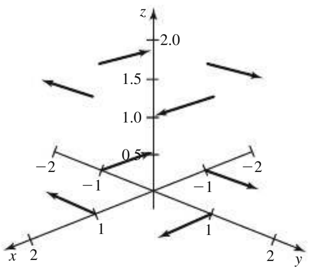
19.



21.

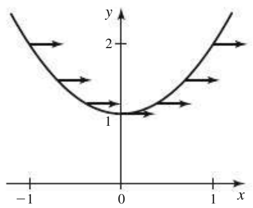


23.



25. a. $(0, 1)$ **b.** None

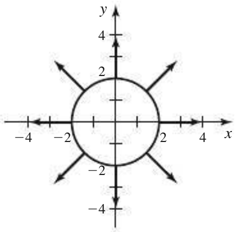
c.



27. a. None

b. At all points on C

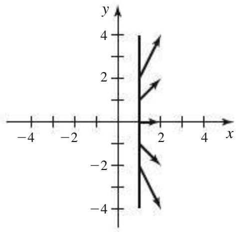
c.



29. a. None

b. $(1, 0)$

c.



31. $\mathbf{F} = \langle -y, x \rangle$ or $\mathbf{F} = \langle -1, 1 \rangle$

33. $\mathbf{F}(x, y) = \dfrac{\langle x, y \rangle}{\sqrt{x^2 + y^2}} = \dfrac{\mathbf{r}}{|\mathbf{r}|}$, $\mathbf{F}(0, 0) = \mathbf{0}$

35. $\nabla\varphi(x, y) = \langle 2xy - y^2, x^2 - 2xy \rangle$

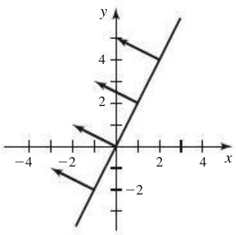
37. $\nabla\varphi(x, y) = \langle 1/y, -x/y^2 \rangle$ **39.** $\nabla\varphi(x, y, z) = \langle x, y, z \rangle = \mathbf{r}$

41. $\nabla\varphi(x, y, z) = -(x^2 + y^2 + z^2)^{-3/2} \langle x, y, z \rangle = -\dfrac{\mathbf{r}}{|\mathbf{r}|^3}$

43. a. $\mathbf{F} = \langle -2, 1 \rangle$

b. $\mathbf{F}(-1, -2) = \mathbf{F}(0, 0) = \mathbf{F}(1, 2) = \mathbf{F}(2, 4) = \langle -2, 1 \rangle$

c.

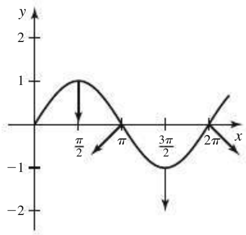


45. a. $\mathbf{F} = \langle \cos x, -1 \rangle$

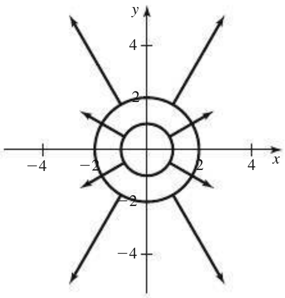
b. $\mathbf{F}\left(\dfrac{\pi}{2}, 1\right) = \langle 0, -1 \rangle$; $\mathbf{F}(\pi, 0) = \langle -1, -1 \rangle$;

$\mathbf{F}\left(\dfrac{3\pi}{2}, -1\right) = \langle 0, -1 \rangle$; $\mathbf{F}(2\pi, 0) = \langle 1, -1 \rangle$

c.



47. $\nabla\varphi(x, y) = 2\langle x, y \rangle$

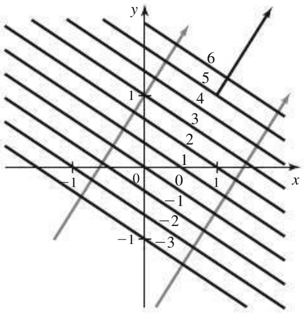


49. a. $\nabla\varphi(x, y) = \langle 2, 3 \rangle$

b. $y' = -2/3$, $\left\langle 1, -\frac{2}{3} \right\rangle \cdot \nabla\varphi(1, 1) = 0$

c. $y' = -2/3$, $\left\langle 1, -\frac{2}{3} \right\rangle \cdot \nabla\varphi(x, y) = 0$

d.

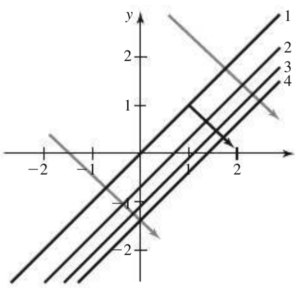


51. a. $\nabla\varphi(x, y) = \langle e^{x-y}, -e^{x-y} \rangle = e^{x-y} \langle 1, -1 \rangle$

b. $y' = 1$, $\langle 1, 1 \rangle \cdot \nabla\varphi(1, 1) = 0$

c. $y' = 1$, $\langle 1, 1 \rangle \cdot \nabla\varphi(x, y) = 0$

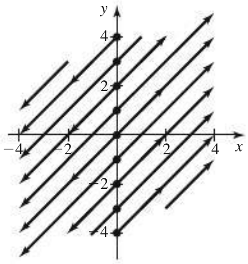
d.



53. a. True **b.** False **c.** True **55. a.** $\mathbf{E} = \dfrac{c}{x^2 + y^2} \langle x, y \rangle$

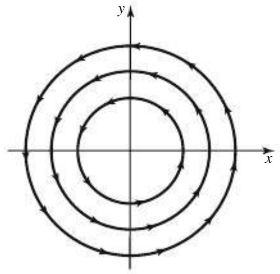
b. $|\mathbf{E}| = \left| \dfrac{c}{|\mathbf{r}|^2} \mathbf{r} \right| = \dfrac{c}{r}$ **c.** *Hint:* The equipotential curves are circles centered at the origin. **57.** The slope of the streamline at (x, y) is $y'(x)$, which equals the slope of the vector $\mathbf{F}(x, y)$, which is g/f. Therefore, $y'(x) = g/f$.

59.



$y = x + C$

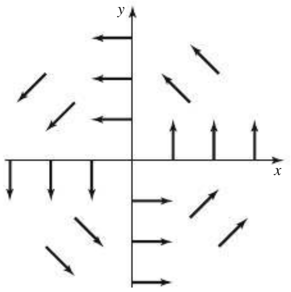
61.



$x^2 + y^2 = C$

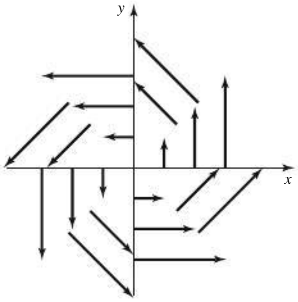
63. For $\theta = 0$: $\mathbf{u}_r = \mathbf{i}$ and $\mathbf{u}_\theta = \mathbf{j}$
for $\theta = \frac{\pi}{2}$: $\mathbf{u}_r = \mathbf{j}$ and $\mathbf{u}_\theta = -\mathbf{i}$
for $\theta = \pi$: $\mathbf{u}_r = -\mathbf{i}$ and $\mathbf{u}_\theta = -\mathbf{j}$
for $\theta = \frac{3\pi}{2}$: $\mathbf{u}_r = -\mathbf{j}$ and $\mathbf{u}_\theta = \mathbf{i}$

65.



$\mathbf{F} = \dfrac{1}{\sqrt{x^2 + y^2}} \langle -y, x \rangle$

67.



$\mathbf{F} = r\,\mathbf{u}_\theta$

Section 17.2 Exercises, pp. 1110–1114

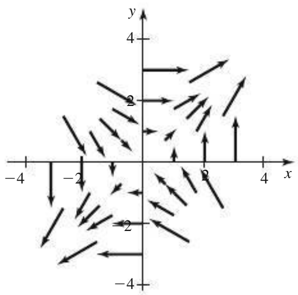
1. A line integral is taken along a curve; an ordinary single-variable integral is taken along an interval. **3.** $\displaystyle\int_{\pi/2}^{\pi} \frac{1}{t} \cos t \sqrt{\sin^2 t + 1}\, dt$
5. $\mathbf{r}(t) = \langle 1 + 4t, 2 + 2t, 3 - 3t \rangle$, for $0 \le t \le 1$
7. $\mathbf{r}(t) = \langle t^2 + 1, t \rangle$, for $2 \le t \le 4$
9. a. $\displaystyle\int_0^2 (t + 6t^5)\,dt$ **b.** 66 **11.** $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ and $\displaystyle\int_C f\,dx + g\,dy + h\,dz$
13. 7 **15.** Take the line integral of $\mathbf{F} \cdot \mathbf{T}$ along the curve with arc length as the parameter. **17.** 0 **19.** 100 **21.** 8 **23.** $-40\pi^2$
25. 128π **27.** $\dfrac{\sqrt{2}}{2} \ln 10$ **29.** $\dfrac{112}{9}$ **31.** 8 **33.** 414 **35.** 409.5
37. $\dfrac{15}{2}$ **39.** $\sqrt{101}$ **41.** $\dfrac{17}{2}$ **43.** 49 **45.** $\dfrac{3}{4\sqrt{10}}$ **47. a.** Negative
b. Positive **49.** 0 **51.** 16 **53.** 0 **55.** $\dfrac{3\sqrt{3}}{10}$ **57. b.** 0
59. a. Negative **b.** -4π **61. a.** True **b.** True **c.** True **d.** True
63. a. Both paths require the same work: $W = 28{,}200$.
b. Both paths require the same work: $W = 28{,}200$.
65. a. $\dfrac{5\sqrt{5} - 1}{12}$ **b.** $\dfrac{5\sqrt{5} - 1}{12}$ **c.** The results are identical.
67. The work equals zero for all three paths.
69. $8\pi(48 + 7\pi - 128\pi^2) \approx -29{,}991.4$ **71.** 2π **73. a.** 4 **b.** -4
e. 0 **75.** *Hint:* Show that $\displaystyle\int_C \mathbf{F} \cdot \mathbf{T}\, ds = \pi r^2(c - b)$.
77. *Hint:* Show that $\displaystyle\int_C \mathbf{F} \cdot \mathbf{n}\, ds = \pi r^2(a + d)$. **79. a.** $\ln a$ **b.** No
c. $\dfrac{1}{6}\left(1 - \dfrac{1}{a^2}\right)$ **d.** Yes **e.** $W = \dfrac{3^{1-p/2}}{2 - p}(a^{2-p} - 1)$, for $p \ne 2$; otherwise, $W = \ln a$. **f.** $p > 2$ **81.** ab

Section 17.3 Exercises, pp. 1121–1123

1. A simple curve has no self-intersections; the initial and terminal points of a closed curve are identical. **3.** Test for equality of partial derivatives as given in Theorem 17.3. **5.** Integrate f with respect to x and make the constant of integration a function of y to obtain $\varphi = \int f\,dx + h(y)$; finally, set $\frac{\partial\varphi}{\partial y} = g$ to determine h. **7.** 0
9. Conservative **11.** Not conservative **13.** Conservative
15. Conservative **17.** $\varphi(x, y) = \frac{1}{2}(x^2 + y^2)$ **19.** Not conservative
21. $\varphi(x, y) = \sqrt{x^2 + y^2}$ **23.** $\varphi(x, y, z) = xz + y$
25. Not conservative **27.** $\varphi(x, y, z) = xy + yz + zx$
29. $\varphi(x, y) = \sqrt{x^2 + y^2 + z^2}$ **31. a, b.** 0 **33. a, b.** 2 **35.** 3
37. -10 **39.** 24 **41.** $-\frac{1}{2}$ **43.** $-\pi^2$ **45.** 0 **47.** 0 **49.** 0
51. -5 **53.** 0 **55.** 1 **57. a.** False **b.** True **c.** True **d.** True
e. True **59.** 10 **61.** 25 **63. a.** Negative **b.** Positive **c.** No
67. a. Compare partial derivatives.
b. $\varphi(x, y, z) = \dfrac{GMm}{\sqrt{x^2 + y^2 + z^2}} = \dfrac{GMm}{|\mathbf{r}|}$
c. $\varphi(B) - \varphi(A) = GMm\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$ **d.** No
69. a. $\dfrac{\partial}{\partial y}\left(\dfrac{-y}{(x^2 + y^2)^{p/2}}\right) = \dfrac{-x^2 + (p - 1)y^2}{(x^2 + y^2)^{1+p/2}}$ and
$\dfrac{\partial}{\partial x}\left(\dfrac{x}{(x^2 + y^2)^{p/2}}\right) = \dfrac{(1 - p)x^2 + y^2}{(x^2 + y^2)^{1+p/2}}$
b. The two partial derivatives in (a) are equal if $p = 2$.
c. $\varphi(x, y) = \tan^{-1}(y/x)$ **73.** $\varphi(x, y) = \frac{1}{2}(x^2 + y^2)$
75. $\varphi(x, y) = \frac{1}{2}(x^4 + x^2y^2 + y^4)$

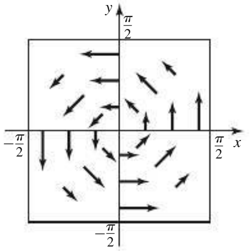
Section 17.4 Exercises, pp. 1133–1136

1. In both forms, the integral of a *derivative* is computed from boundary data. **3.** Area $= \frac{1}{2}\oint_C x\,dy - y\,dx$, where C encloses the region **5.** The integral in the flux form of Green's Theorem vanishes.
7. $\mathbf{F} = \langle y, x \rangle$

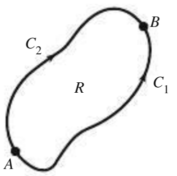


9. a. 0 **b.** 2 **c.** Yes **d.** No **11. a.** -4 **b.** 0 **c.** No **d.** Yes
13. a. y^2 **b.** $12x^2y + 2xy$ **c.** No **d.** No **15. a.** 1; no
b. $\mathbf{r}_1(t) = \langle t, t^2 \rangle$, for $0 \le t \le 1$, and $\mathbf{r}_2(t) = \langle 1 - t, 1 - t \rangle$, for $0 \le t \le 1$ (answers may vary) **c.** Both integrals equal $\dfrac{1}{6}$. **d.** 0
17. a. -4 **b.** Both integrals equal -8. **19. a.** $4x$ **b.** $\dfrac{16}{3}$
21. 25π **23.** 16π **25.** 32 **27. a.** 2 **b.** Both integrals equal 8π.
29. a. $2y$ **b.** $\dfrac{16}{15}$ **31.** 104 **33.** $\dfrac{31 - 3e^4}{6}$ **35.** 6 **37.** $\dfrac{8}{3}$
39. $8 - \dfrac{\pi}{2}$ **41. a.** 0 **b.** 3π **43. a.** 0 **b.** $-\dfrac{15\pi}{2}$ **45. a.** 0
b. 2π **47. a.** $\dfrac{16}{3}$ **b.** 0 **49. a.** True **b.** False **c.** True

51. Note: $\dfrac{\partial f}{\partial y} = 0 = \dfrac{\partial g}{\partial x}$ **53.** The integral becomes $\iint_R 2\,dA$.
55. a. $f_x = g_y = 0$ **b.** $\psi(x, y) = -2x + 4y$
57. a. $f_x = e^{-x}\sin y = -g_y$ **b.** $\psi(x, y) = e^{-x}\cos y$
59. a. *Hint:* $f_x = e^x\cos y$, $f_y = -e^x\sin y$,
$g_x = -e^x\sin y$, $g_y = -e^x\cos y$
b. $\varphi(x, y) = e^x\cos y$, $\psi(x, y) = e^x\sin y$
61. a. *Hint:* $f_x = -\dfrac{y}{x^2 + y^2}$, $f_y = \dfrac{x}{x^2 + y^2}$,
$g_x = \dfrac{x}{x^2 + y^2}$, $g_y = \dfrac{y}{x^2 + y^2}$
b. $\varphi(x, y) = x\tan^{-1}\dfrac{y}{x} + \dfrac{y}{2}\ln(x^2 + y^2) - y$,
$\psi(x, y) = y\tan^{-1}\dfrac{y}{x} - \dfrac{x}{2}\ln(x^2 + y^2) + x$
63. a.



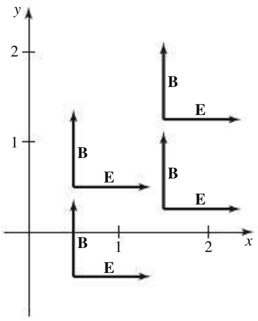
$\mathbf{F} = \langle -4\cos x\sin y, 4\sin x\cos y\rangle$ **b.** Yes, the divergence equals zero. **c.** No, the two-dimensional curl equals $8\cos x\cos y$. **d.** 0 **e.** 32
67. c. The vector field is undefined at the origin.
69.



Basic ideas: Let C_1 and C_2 be two smooth simple curves from A to B.
$$\int_{C_1}\mathbf{F}\cdot\mathbf{n}\,ds - \int_{C_2}\mathbf{F}\cdot\mathbf{n}\,ds = \oint_C \mathbf{F}\cdot\mathbf{n}\,ds = \iint_R (f_x + g_y)\,dA = 0$$
and $\displaystyle\int_{C_1}\mathbf{F}\cdot\mathbf{n}\,ds = \int_{C_1}\psi_x\,dx + \psi_y\,dy = \int_{C_1} d\psi = \psi(B) - \psi(A)$
71. Use $\nabla\varphi\cdot\nabla\psi = \langle f, g\rangle\cdot\langle -g, f\rangle = 0$.

Section 17.5 Exercises, pp. 1143–1146

1. Compute $f_x + g_y + h_z$. **3.** There is no source or sink. **5.** It indicates the axis and the angular speed of the circulation at a point. **7.** 0 **9.** 3 **11.** 0 **13.** $2(x + y + z)$
15. $\dfrac{x^2 + y^2 + 3}{(1 + x^2 + y^2)^2}$ **17.** $\dfrac{1}{|\mathbf{r}|^2}$ **19.** $-\dfrac{1}{|\mathbf{r}|^4}$ **21. a.** Positive for both points **b.** div $\mathbf{F} = 2$ **c.** Outward everywhere **d.** Positive
23. a. curl $\mathbf{F} = 2\mathbf{i}$ **b.** $|\text{curl } \mathbf{F}| = 2$
25. a. curl $\mathbf{F} = 2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ **b.** $|\text{curl } \mathbf{F}| = 2\sqrt{3}$ **27.** $3y\,\mathbf{k}$
29. $-4z\,\mathbf{j}$ **31. 0** **33. 0** **35.** Follows from partial differentiation of $\dfrac{1}{(x^2 + y^2 + z^2)^{3/2}}$ **37.** Combine Exercise 36 with Theorem 17.10.
39. a. False **b.** False **c.** False **d.** False **e.** False **41. a.** No **b.** No **c.** Yes, scalar function **d.** No **e.** No **f.** No **g.** Yes, vector field **h.** No **i.** Yes, vector field **43. a.** At $(0, 1, 1)$, $\mathbf{F}$ points in the positive x-direction; at $(1, 1, 0)$, $\mathbf{F}$ points in the negative z-direction; at $(0, 1, -1)$, $\mathbf{F}$ points in the negative x-direction; and at $(-1, 1, 0)$, $\mathbf{F}$ points in the positive z-direction. These vectors circle the y-axis in the counterclockwise direction looking along $\mathbf{a}$ from head to tail. **b.** The argument in part (a) can be repeated in any plane perpendicular to the y-axis to show that the vectors of $\mathbf{F}$ circle the y-axis in the counterclockwise direction looking along $\mathbf{a}$ from head to tail. Alternatively, computing the cross product, we find that $\mathbf{F} = \mathbf{a}\times\mathbf{r} = \langle z, 0, -x\rangle$, which is a rotation field in any plane perpendicular to $\mathbf{a}$. **45.** Compute an explicit expression for $\mathbf{a}\times\mathbf{r}$ and then take the required partial derivatives. **47.** div $\mathbf{F}$ has a maximum value of 6 at $(1, 1, 1)$, $(1, -1, -1)$, $(-1, 1, 1)$, and $(-1, -1, -1)$. **49.** $\mathbf{n} = \langle a, b, 2a + b\rangle$, where a and b are real numbers **51.** $\mathbf{F} = \frac{1}{2}(y^2 + z^2)\,\mathbf{i}$; no **53. a.** The wheel does not spin. **b.** Clockwise, looking in the positive y-direction **c.** The wheel does not spin. **55.** $\omega = \dfrac{10}{\sqrt{3}}$, or $\dfrac{5}{\sqrt{3}\pi} \approx 0.9189$ revolution per unit time
57. $\mathbf{F} = -200ke^{-x^2+y^2+z^2}(-x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k})$
$\nabla\cdot\mathbf{F} = -200k(1 + 2(x^2 + y^2 + z^2))e^{-x^2+y^2+z^2}$
59. a. $\mathbf{F} = -\dfrac{GMm\mathbf{r}}{|\mathbf{r}|^3}$ **b.** See Theorem 17.11.
61.
$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$
$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$
$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$
63. a. Use $\nabla\times\mathbf{B} = -Ak\cos(kz - \omega t)\,\mathbf{i}$ and $\dfrac{\partial\mathbf{E}}{\partial t} = -A\omega\cos(kz - \omega t)\,\mathbf{i}$. **b.**



Section 17.6 Exercises, pp. 1159–1161

1. $\mathbf{r}(u, v) = \langle a\cos u, a\sin u, v\rangle$, $0 \le u \le 2\pi$, $0 \le v \le h$
3. $\mathbf{r}(u, v) = \langle a\sin u\cos v, a\sin u\sin v, a\cos u\rangle$, $0 \le u \le \pi$, $0 \le v \le 2\pi$ **5.** Use the parameterization from Exercise 3 and compute $\displaystyle\int_0^{\pi}\int_0^{2\pi} f(a\sin u\cos v, a\sin u\sin v, a\cos u)\,a^2\sin u\,dv\,du$.
7. The normal vectors point outward. **9.** $\langle u, v, \frac{1}{3}(16 - 2u + 4v)\rangle$, $|u| < \infty$, $|v| < \infty$ **11.** $\langle v\cos u, v\sin u, v\rangle$, $0 \le u \le 2\pi$, $2 \le v \le 8$ **13.** $\langle 3\cos u, 3\sin u, v\rangle$, $0 \le u \le \dfrac{\pi}{2}$, $0 \le v \le 3$
15. The plane $z = 2x + 3y - 1$ **17.** Part of the upper half of the cone $z^2 = 16x^2 + 16y^2$ of height 12 and radius 3 (with $y \ge 0$)
19. 28π **21.** $16\sqrt{3}$ **23.** $\pi r\sqrt{r^2 + h^2}$ **25.** 1728π **27.** 0
29. 12 **31.** $4\pi\sqrt{5}$ **33.** $\dfrac{(65\sqrt{65} - 1)\pi}{24}$ **35.** $\dfrac{2\sqrt{3}}{3}$ **37.** $\dfrac{1250\pi}{3}$
39. $e - 1$ **41.** $\dfrac{1}{4\pi}$ **43.** -8 **45.** 0 **47.** 4π **49. a.** True **b.** False **c.** True **d.** True **51.** $8\pi(4\sqrt{17} + \ln(\sqrt{17} + 4))$
53. $8\pi a$ **55.** 8 **57. a.** 0 **b.** 0; the flow is tangent to the surface (radial flow). **59.** $2\pi ah$ **61.** $-400\left(e - \dfrac{1}{e}\right)^2$ **63.** $8\pi a$

65. a. $4\pi(b^3 - a^3)$ **b.** The net flux is zero. **67.** $\left(0, 0, \frac{2}{3}h\right)$

69. $\left(0, 0, \frac{7}{6}\right)$ **73.** Flux $= \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_R dA$

Section 17.7 Exercises, pp. 1169–1171

1. The integral measures the circulation along the closed curve C. **3.** Under certain conditions, the accumulated rotation of the vector field over the surface S equals the net circulation on the boundary of S. **5.** Both integrals equal -2π. **7.** Both integrals equal zero. **9.** Both integrals equal -18π. **11.** -24π **13.** $-\frac{128}{3}$ **15.** 15π **17.** 0 **19.** 0 **21.** -2π **23.** -4π **25.** $\nabla \times \mathbf{v} = \langle 1, 0, 0 \rangle$; a paddle wheel with its axis aligned with the x-axis will spin with maximum angular speed counterclockwise (looking in the negative x-direction) at all points. **27.** $\nabla \times \mathbf{v} = \langle 0, -2, 0 \rangle$; a paddle wheel with its axis aligned with the y-axis will spin with maximum angular speed clockwise (looking in the negative y-direction) at all points. **29. a.** False **b.** False **c.** True **d.** True **31.** 0 **33.** 0 **35.** 2π **37.** $\pi(\cos\varphi - \sin\varphi)$; maximum for $\varphi = 0$ **39.** The circulation is 48π; it depends on the radius of the circle but not on the center. **41. a.** The normal vectors point toward the z-axis on the curved surface of S and in the direction of $\langle 0, 1, 0 \rangle$ on the flat surface of S. **b.** 2π **c.** 2π **43.** The integral is π for all a. **45. a.** 0 **b.** 0 **47. b.** 2π for any circle of radius r centered at the origin **c.** $\mathbf{F}$ is not differentiable along the z-axis.

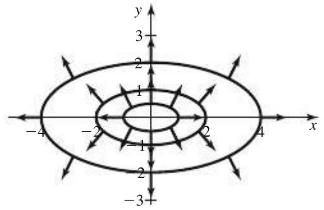
49. Apply the Chain Rule. **51.** $\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}\right) dA$

Section 17.8 Exercises, pp. 1179–1182

1. The surface integral measures the flow across the boundary. **3.** The flux across the boundary equals the cumulative expansion or contraction of the vector field inside the region. **5.** 32π **7.** The outward fluxes are equal. **9.** Both integrals equal 96π. **11.** Both integrals equal zero. **13.** 0 **15.** 0 **17.** $16\sqrt{6}\pi$ **19.** $\frac{2}{3}$ **21.** $-\frac{128}{3}\pi$ **23.** 24π **25.** -224π **27.** 12π **29.** 20 **31. a.** False **b.** False **c.** True **33.** 0 **35.** $\frac{3}{2}$ **37. b.** The net flux between the two spheres is $4\pi(a^2 - \varepsilon^2)$. **39. b.** Use $\nabla \cdot \mathbf{E} = 0$. **c.** The flux across S is the sum of the contributions from the individual charges. **d.** For an arbitrary volume, we find $\frac{1}{\varepsilon_0}\iiint_D q(x, y, z)\, dV = \iint_S \mathbf{E} \cdot \mathbf{n}\, dS = \iiint_D \nabla \cdot \mathbf{E}\, dV.$ **e.** Use $\nabla^2\varphi = \nabla \cdot \nabla\varphi$. **41.** 0 **43.** $e^{-1} - 1$ **45.** $800\pi a^3 e^{-a^2}$

Chapter 17 Review Exercises, pp. 1182–1184

1. a. False **b.** True **c.** False **d.** False **e.** True
3. $\nabla\varphi = \langle 2x, 8y \rangle$



5. $-\frac{\mathbf{r}}{|\mathbf{r}|^3}$ **7.** $\mathbf{n} = \frac{1}{2}\langle x, y \rangle$ **9.** $\frac{7}{8}(e^{48} - 1)$ **11.** Both integrals equal zero. **13.** 0 **15.** The circulation is -4π; the outward flux is zero. **17.** The circulation is zero; the outward flux is 2π.

19. $\frac{4v_0 L^3}{3}$ **21.** $\varphi(x, y, z) = xy + yz^2$ **23.** $\varphi(x, y, z) = xye^z$ **25.** 0 for both methods **27. a.** $-\pi$ **b.** $\mathbf{F}$ is not conservative. **29.** 0 **31.** $\frac{20}{3}$ **33.** 8π **35.** The circulation is zero; the outward flux equals 2π. **37. a.** $b = c$ **b.** $a = -d$ **c.** $a = -d$ and $b = c$ **39.** $\nabla \cdot \mathbf{F} = 4\sqrt{x^2 + y^2 + z^2} = 4|\mathbf{r}|$, $\nabla \times \mathbf{F} = \mathbf{0}$, $\nabla \cdot \mathbf{F} \neq 0$; irrotational but not source free **41.** $\nabla \cdot \mathbf{F} = 2y + 12xz^2$, $\nabla \times \mathbf{F} = \mathbf{0}$, $\nabla \cdot \mathbf{F} \neq 0$; irrotational but not source free

43. a. -1 **b.** 0 **c.** $\mathbf{n} = \frac{1}{\sqrt{3}}\langle -1, 1, 1 \rangle$ **45.** 18π **47.** $4\sqrt{3}$

49. $\frac{8\sqrt{3}}{3}$ **51.** 8π **53.** $4\pi a^2$ **55. a.** Use $x = y = 0$ to confirm the highest point; use $z = 0$ to confirm the base. **b.** The hemisphere S has the greater surface area—$2\pi a^2$ for S versus $\frac{5\sqrt{5} - 1}{6}\pi a^2$ for T.

57. 0 **59.** 99π **61.** 0 **63.** $\frac{972}{5}\pi$ **65.** $\frac{124}{5}\pi$ **67.** $\frac{32}{3}$

Answers

CHAPTER D2

Section D2.1, pp. D2-10–D2-13

1. The order of a differential equation is the order of the highest order derivative in the equation. **3.** A differential equation of the form $y''(t) + p(t)y'(t) + q(t)y(t) = f(t)$ is homogeneous if $f(t) = 0$ on the interval of interest. **5.** If one function is a nonzero constant multiple of the other on the interval, then the two functions are linearly dependent. **7.** Find two linearly independent solutions, y_1 and y_2, of the corresponding homogeneous differential equation. Find any particular solution, y_p, of the original differential equation. The general solution is then $y = c_1y_1 + c_2y_2 + y_p$, where c_1 and c_2 are arbitrary constants.
9. order 2; linear; nonhomogeneous **11.** order 2; nonlinear; nonhomogeneous
13. $y'' - 4y = (3e^{2t} - 5e^{-2t})'' - 4(3e^{2t} - 5e^{-2t}) = 12e^{2t} - 20e^{-2t} - 12e^{2t} + 20e^{-2t} = 0$
15. $y'' - 9y = (4e^{3t} + 3e^{-3t} - 2t)'' - 9(4e^{3t} + 3e^{-3t} - 2t) = 36e^{3t} + 27e^{-3t} - 36e^{3t} - 27e^{-3t} + 18t = 18t$
17. $y'' - y' - 2y = (c_1e^{-t} + c_2e^{2t})'' - (c_1e^{-t} + c_2e^{2t})' - 2(c_1e^{-t} + c_2e^{2t}) = c_1e^{-t} + 4c_2e^{2t} - (-c_1e^{-t} + 2c_2e^{2t}) - 2c_1e^{-t} - 2c_2e^{2t} = 0$
19. $y'' + 6y' + 25y$
$= (c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t)'' + 6(c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t)' + 25(c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t)$
$= (-7c_1e^{-3t}\sin 4t - 24c_1e^{-3t}\cos 4t + 24c_2e^{-3t}\sin 4t - 7c_2e^{-3t}\cos 4t) + 6(-3c_1e^{-3t}\sin 4t + 4c_1e^{-3t}\cos 4t - 4c_2e^{-3t}\sin 4t - 3c_2e^{-3t}\cos 4t) + 25(c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t)$
$= (-7 - 18 + 25)c_1e^{-3t}\sin 4t + (-24 + 24)c_1e^{-3t}\cos 4t + (24 - 24)c_2e^{-3t}\sin 4t + (-7 - 18 + 25)c_2e^{-3t}\cos 4t$
$= 0$
21. $ty'' - (t + 1)y' + y = t[c_1e^t + c_2(t + 1)]'' - (t + 1)\cdot[c_1e^t + c_2(t + 1)]' + [c_1e^t + c_2(t + 1)]$
$= t(c_1e^t) - (t + 1)(c_1e^t + c_2) + [c_1e^t + c_2(t + 1)]$
$= tc_1e^t - tc_1e^t - c_1e^t - (t + 1)c_2 + c_1e^t + c_2(t + 1)$
$= 0$
23. $y = c_1e^{-6t} + c_2e^{6t}$
25. $y = c_1e^{-t} + c_2te^{-t}$
27. $y'' - y = (e^{-3t})'' - e^{-3t} = 9e^{-3t} - e^{-3t} = 8e^{-3t}$
29. $y'' - 4y' + 4y = (t^2e^{2t})'' - 4(t^2e^{2t})' + 4t^2e^{2t} = (4t^2e^{2t} + 8te^{2t} + 2e^{2t}) - 4(2t^2e^{2t} + 2te^{2t}) + 4t^2e^{2t} = (4 - 8 + 4)t^2e^{2t} + (8 - 8)te^{2t} + 2e^{2t} = 2e^{2t}$
31. Let $y_1 = \dfrac{e^{-t}}{2}$, $y_2 = \dfrac{e^{-t}}{2} + 3e^{7t}$, and $y_3 = y_2 - y_1 = 3e^{7t}$. Then

$$y_1'' - 49y_1 = \left(\frac{e^{-t}}{2}\right)'' - 49\left(\frac{e^{-t}}{2}\right) = \frac{e^{-t}}{2} - \frac{49e^{-t}}{2} = -24e^{-t}$$

$$y_2'' - 49y_2 = \left(\frac{e^{-t}}{2} + 3e^{7t}\right)'' - 49\left(\frac{e^{-t}}{2} + 3e^{7t}\right) = \frac{e^{-t}}{2} + 147e^{7t} - \frac{49e^{-t}}{2} - 147e^{7t} = -24e^{-t}$$

$y_3'' - 49y_3 = (3e^{7t})'' - 49(3e^{7t}) = 147e^{7t} - 147e^{7t} = 0$
33. Let $y_1 = -e^t$, $y_2 = 6e^{4t} - e^t$, and $y_3 = y_2 - y_1 = 6e^{4t} - e^t - (-e^t) = 6e^{4t}$. Then
$y_1'' - y_1' - 12y_1 = (-e^t)'' - (-e^t)' - 12(-e^t) = -e^t + e^t + 12e^t = 12e^t$
$y_2'' - y_2' - 12y_2 = (6e^{4t} - e^t)'' - (6e^{4t} - e^t)' - 12(6e^{4t} - e^t) = (96e^{4t} - e^t) - (24e^{4t} - e^t) - 72e^{4t} + 12e^t = 12e^t$
$y_3'' - y_3' - 12y_3 = (6e^{4t})'' - (6e^{4t})' - 12(6e^{4t}) = 96e^{4t} - 24e^{4t} - 72e^{4t} = 0$
35. homogeneous solutions: $\sin\sqrt{2}t$, $\cos\sqrt{2}t$
general solution: $y = c_1\sin\sqrt{2}t + c_2\cos\sqrt{2}t + e^t$
37. homogeneous solutions: $e^{\frac{3}{2}t}\sin 4t$, $e^{\frac{3}{2}t}\cos 4t$
general solution: $y = c_1e^{\frac{3}{2}t}\sin 4t + c_2e^{\frac{3}{2}t}\cos 4t + 100t + 48$
39. $y = 4\cos 3t$ **41.** $y = -e^{5t} - 2e^{-4t}$
43. $y = \dfrac{1}{16}e^{4t} + \dfrac{1}{16}e^{-4t} - t^2 - \dfrac{1}{8}$ **45.** $y = \dfrac{3}{4}t^{-2} + \dfrac{1}{4}t^2$
47. a. False **b.** True **c.** False **d.** False **e.** False
49. $y'' - 12y' + 36y = (c_1e^{6t} + c_2te^{6t} + t^2e^{6t})'' - 12(c_1e^{6t} + c_2te^{6t} + t^2e^{6t})' + 36(c_1e^{6t} + c_2te^{6t} + t^2e^{6t})$
$= (36c_1e^{6t} + 12c_2e^{6t} + 36c_2te^{6t} + 2e^{6t} + 24te^{6t} + 36t^2e^{6t}) - 12(6c_1e^{6t} + c_2e^{6t} + 6c_2te^{6t} + 2te^{6t} + 6t^2e^{6t}) + 36(c_1e^{6t} + c_2te^{6t} + t^2e^{6t})$
$= (36 - 72 + 36)c_1e^{6t} + (12 - 12)c_2e^{6t} + (36 - 72 + 36)c_2te^{6t} + 2e^{6t} + (24 - 24)te^{6t} + (36 - 72 + 36)t^2e^{6t}$
$= 2e^{6t}$
51. $t^2y'' - 3ty' + 4y$
$= t^2(c_1t^2 + c_2t^2\ln t)'' - 3t(c_1t^2 + c_2t^2\ln t)' + 4(c_1t^2 + c_2t^2\ln t)$
$= t^2(2c_1 + 3c_2 + 2c_2\ln t) - 3t(2c_1t + c_2t + 2c_2t\ln t) + 4(c_1t^2 + c_2t^2\ln t) = (2c_1 + 3c_2 - 6c_1 - 3c_2 + 4c_1)t^2 + (2c_2 - 6c_2 + 4c_2)t^2\ln t$
$= 0$
53. $t^2y'' + ty' + \left(t^2 - \dfrac{1}{4}\right)y$
$= t^2(c_1t^{-1/2}\cos t + c_2t^{-1/2}\sin t)'' + t(c_1t^{-1/2}\cos t + c_2t^{-1/2}\sin t)' + \left(t^2 - \dfrac{1}{4}\right)(c_1t^{-1/2}\cos t + c_2t^{-1/2}\sin t)$
$= t^2\left(\dfrac{3}{4}c_1t^{-5/2}\cos t + \dfrac{3}{4}c_2t^{-5/2}\sin t - c_2t^{-3/2}\cos t + c_1t^{-3/2}\sin t - c_1t^{-1/2}\cos t - c_2t^{-1/2}\sin t\right) + t\left(-c_1t^{-1/2}\sin t + c_2t^{-1/2}\cos t - \dfrac{1}{2}c_1t^{-3/2}\cos t - \dfrac{1}{2}c_2t^{-3/2}\sin t\right) + \left(t^2 - \dfrac{1}{4}\right)(c_1t^{-1/2}\cos t + c_2t^{-1/2}\sin t)$

$= \left(\frac{3}{4} - \frac{1}{2} - \frac{1}{4}\right)c_1 t^{-1/2}\cos t + \left(\frac{3}{4} - \frac{1}{2} - \frac{1}{4}\right)c_2 t^{-1/2}\sin t$
$\quad + (-1 + 1)c_2 t^{1/2}\cos t + (1 - 1)c_1 t^{1/2}\sin t$
$\quad + (-1 + 1)c_1 t^{3/2}\cos t + (-1 + 1)c_2 t^{3/2}\sin t$
$= 0$

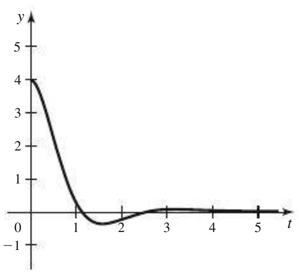
55. **a.** $y'' - y = (e^t)'' - e^t = e^t - e^t = 0$
$y'' - y = (e^{-t})'' - e^{-t} = e^{-t} - e^{-t} = 0$
The functions are not multiples of each other and so are a linearly independent set of solutions. **b.** $y = \sinh t$ and $y = \cosh t$ are linear combinations of the solutions e^{-t} and e^t and so must be solutions themselves by the Superposition Principle. They are independent since one is not a multiple of the other. **c.** $(\sinh t)' = \dfrac{e^t + e^{-t}}{2} = \cosh t$ and $(\cosh t)' = \dfrac{e^t - e^{-t}}{2} = \sinh t$ so $(\sinh t)'' - \sinh t =$
$(\cosh t)' - \sinh t = \sinh t - \sinh t = 0$
$(\cosh t)'' - \cosh t = (\sinh t)' - \cosh t = \cosh t - \cosh t = 0$
d. $y = c_1 e^{-t} + c_2 e^t$, $y = c_3 \sinh t + c_4 \cosh t$ **e.** For $y = e^{kt}$, $y'' - k^2 y = (e^{kt})'' - k^2 e^{kt} = (ke^{kt})' - k^2 e^{kt} = k^2 e^{kt} - k^2 e^{kt} = 0$. For $y = e^{kt}$, $y'' - k^2 y = (e^{-kt})'' - k^2 e^{-kt} = (-ke^{-kt})' - k^2 e^{-kt} = k^2 e^{-kt} - k^2 e^{-kt} = 0$. **f.** $y = c_1 e^{-kt} + c_2 e^{kt}$, $y = c_3 \sinh kt + c_4 \cosh kt$

57. $y^{(4)} - 16y$
$= (c_1 e^{-2t} + c_2 e^{2t} + c_3 \sin 2t + c_4 \cos 2t)^{(4)}$
$\quad - 16(c_1 e^{-2t} + c_2 e^{2t} + c_3 \sin 2t + c_4 \cos 2t)$
$= (16c_1 e^{-2t} + 16c_2 e^{2t} + 16c_3 \sin 2t + 16c_4 \cos 2t)$
$\quad - 16(c_1 e^{-2t} + c_2 e^{2t} + c_3 \sin 2t + c_4 \cos 2t)$
$= 0$

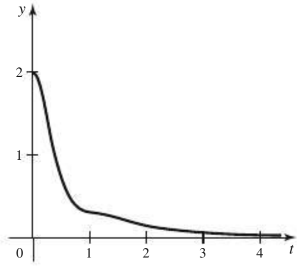
59. **a.** By the chain rule $\dfrac{d}{dt}(y'(t)^2) = 2y'(t)\dfrac{d}{dt}y'(t) = 2y'(t)y''(t)$
b. $y''y' = 1 \Rightarrow 2y''y' = 2 \Rightarrow (y'(t)^2)' = 2$ using part (a).
c. $(y'(t)^2)' = 2 \Rightarrow y'(t)^2 = \int 2\,dt = 2t + c_1$ so $y'(t) = \pm\sqrt{2t + c_1}$ **d.** From part (c), $y(t) = \int \pm\sqrt{2t + c_1}\,dt = \pm\left(\frac{1}{2}\right)\frac{2}{3}(2t + c_1)^{3/2} + c_2 = c_2 \pm \frac{1}{3}(2t + c_1)^{3/2}$

61. $y = c_1 e^{3t} + c_2 - \frac{4}{3}t$ **63.** $y = -\dfrac{1}{\sqrt{c_1}}\tan^{-1}\dfrac{t}{\sqrt{c_1}} + c_2$ if $c_1 > 0$; $y = -\dfrac{1}{2\sqrt{|c_1|}}\ln\left|\dfrac{t - \sqrt{|c_1|}}{t + \sqrt{|c_1|}}\right| + c_2$ if $c_1 < 0$

65. **a.** $y'' + 3y' + \frac{25}{4}y$
$= [e^{-3t/2}(c_1 \sin 2t + c_2 \cos 2t)]'' + 3[e^{-3t/2}(c_1 \sin 2t + c_2 \cos 2t)]'$
$\quad + \frac{25}{4}[e^{-3t/2}(c_1 \sin 2t + c_2 \cos 2t)]$
$= \left(6c_2 - \frac{7}{4}c_1\right)e^{-3t/2}\sin 2t + \left(-6c_1 - \frac{7}{4}c_2\right)e^{-3t/2}\cos 2t$
$\quad + \left(-6c_2 - \frac{9}{2}c_1\right)e^{-3t/2}\sin 2t + \left(6c_1 - \frac{9}{2}c_2\right)e^{-3t/2}\cos 2t$
$\quad + \frac{25}{4}c_1 e^{-3t/2}\sin 2t + \frac{25}{4}c_2 e^{-3t/2}\cos 2t$
$= 0$
b. $y = e^{-3t/2}(3 \sin 2t + 4 \cos 2t)$ **c.**



67. **a.** $y'' + 6y' + 25y$
$= [e^{-3t}(c_1 \sin 4t + c_2 \cos 4t)]'' + 6[e^{-3t}(c_1 \sin 4t + c_2 \cos 4t)]'$
$\quad + 25[e^{-3t}(c_1 \sin 4t + c_2 \cos 4t)]$
$= (24c_2 - 7c_1)e^{-3t}\sin 4t + (-24c_1 - 7c_2)e^{-3t}\cos 4t + e^{-t}$
$\quad + (-24c_2 - 18c_1)e^{-3t}\sin 4t + (24c_1 - 18c_2)e^{-3t}\cos 4t - 6e^{-t}$
$\quad + 25c_1 e^{-3t}\sin 4t + 25c_2 e^{-3t}\cos 4t + 25e^{-t}$
$= 20e^{-t}$
b. $y = e^{-3t}(\sin 4t + \cos 4t) + e^{-t}$
c.



69. **a.** By the Chain Rule $\dfrac{d}{dt}\varphi(x) = \dfrac{d\varphi}{dx}\dfrac{dx}{dt} = -F(x)x'(t)$, then
$\dfrac{d}{dt}\left[\frac{1}{2}m(x'(t))^2 + \varphi(x)\right] = 2\left(\frac{1}{2}m\right)x'(t)x''(t) - F(x)x'(t)$
$= mx'(t)x''(t) - F(x)x'(t)$
$= x'(t)(mx''(t) - F(x))$
$= x'(t)(0) = 0$
Since $v = x'(t)$, this relation can be written $\dfrac{d}{dt}\left[\frac{1}{2}mv^2 + \varphi(x)\right] = 0$.
b. By part a., the derivative of $E(t)$ is identically 0, so $E(t)$ is constant.

Section D2.2, pp. D2-23–D2-25

1. $y = e^{rt}$
3. Case 1: real distinct roots
Case 2: real repeated roots
Case 3: complex roots
5. $y = c_1 e^{rt} + c_2 t e^{rt}$ **7.** $y = c_1 e^{-2t}\sin 3t + c_2 e^{-2t}\cos 3t$
9. $y = c_1 e^{-5t} + c_2 e^{5t}$ **11.** $y = c_1 e^{3t} + c_2$
13. $y = c_1 e^{-5t} + c_2 e^{2t}$ **15.** $y = c_1 e^{-6t} + c_2 e^{6t}$; $y = \frac{3}{2}e^{-6t} + \frac{3}{2}e^{6t}$
17. $y = c_1 e^{-3t} + c_2 e^{6t}$; $y = -\frac{4}{9}e^{-3t} + \frac{4}{9}e^{6t}$
19. $y = c_1 e^{-t/2} + c_2 e^{5t/2}$; $y = \frac{5}{2}e^{-t/2} + \frac{1}{2}e^{5t/2}$
21. $y = c_1 e^t + c_2 t e^t$; $y = 4e^t - 4te^t$
23. $y = c_1 e^{t/2} + c_2 t e^{t/2}$; $y = e^{t/2} + \frac{3}{2}te^{t/2}$
25. $y = c_1 e^{-2t} + c_2 t e^{-2t}$; $y = e^{-2t} + 2te^{-2t}$
27. $y = c_1 \sin 3t + c_2 \cos 3t$; $y = -\frac{8}{3}\sin 3t + 8 \cos 3t$
29. $y = c_1 e^t \sin 2t + c_2 e^t \cos 2t$; $y = -e^t \sin 2t + e^t \cos 2t$
31. $y = c_1 e^{-3t}\sin t + c_2 e^{-3t}\cos t$; $y = 6e^{-3t}\sin t$
33. $y = t^{-1} + t$ **35.** $y = \frac{31}{8}t^{-3} + \frac{17}{8}t^5$ **37.** $y = 4t^{-3} - 4t^{-2}$
39. **a.** False **b.** False **c.** False **d.** True **e.** False

41. $A\cos(\omega t+\varphi) = A\cos\omega t\cos\varphi - A\sin\omega t\sin\varphi$
$= c_1\sin\omega t + c_2\cos\omega t$
$\Rightarrow c_1 = -A\sin\varphi,\ c_2 = A\cos\varphi$
$\Rightarrow \sqrt{c_1^2+c_2^2} = \sqrt{A^2\sin^2\varphi + A^2\cos^2\varphi} = \sqrt{A^2(1)} = A$
$\Rightarrow -\dfrac{c_1}{c_2} = \dfrac{A\sin\varphi}{A\cos\varphi} = \tan\varphi$
43. $y = 3\sqrt{2}\sin\left(4t+\dfrac{3\pi}{4}\right)$ **45.** $y = 2\sin\left(2t+\dfrac{2\pi}{3}\right)$
47. $y = c_1e^{-2t}+c_2e^{3t}+c_3$ **49.** $y = c_1e^{2t}+c_2e^{4t}+c_3$
51. $y = c_1\sin t + c_2\cos t + c_3\sin 2t + c_4\cos 2t$
53. $y = c_1t^{-1}+c_2t^{-1}\ln t$ **55.** $y = c_1t^{-3}+c_2t^{-3}\ln t$
57. $y = c_1t^{-3}\sin(4\ln t) + c_2t^{-3}\cos(4\ln t)$
59. $y = \sqrt{t}\left(c_1\sin\dfrac{\ln t}{2} + c_2\cos\dfrac{\ln t}{2}\right)$
61. $y = c_1\sin kt + c_2\cos kt + c_3t + c_4$
63. a. As seen in the text, the associated quadratic equation is $p^2+(a-1)p+b=0$. If there is a repeated root, the discriminant in the quadratic formula is 0, and the root is $p = \dfrac{-(a-1)\pm\sqrt{0}}{2} = \dfrac{1-a}{2}$. **b.** If $y = t^pv(t)$ then
$t^2y''+aty'+by$
$= t^2(t^pv)''+at(t^pv)'+b(t^pv)$
$= t^2(t^pv''+2pt^{p-1}v'+p(p-1)t^{p-2}v)+at(t^pv'+pt^{p-1}v)+bt^pv$
$= t^p[t^2v''+(2p+a)tv'+(p^2+(a-1)p+b)v]$
$= t^p[t^2v''+(1)tv'+(0)v]$ (using part (a) and the fact that p satisfies $p^2+(a-1)p+b=0$)
$= t^p(t^2v''+tv') = 0$
so $t^2v''+tv' = 0$.
c. If $w = v'$ then $t^2v''+tv' = 0 \Rightarrow t^2w'+tw = 0$
$\Rightarrow t^2\dfrac{dw}{dt}+tw = 0 \Rightarrow \dfrac{dw}{w} = -\dfrac{dt}{t}$. Integrating both sides gives
$\ln w = -\ln t + c \Rightarrow e^{\ln w} = w = e^{-\ln t + c} = e^ce^{-\ln t} = c_1\left(\dfrac{1}{t}\right)$
so $w = \dfrac{c_1}{t}$. **d.** Integrating both sides gives: $w = v' = \dfrac{c_1}{t} \Rightarrow$
$v = c_1\ln t + c_2$. Then $y = t^pv(t) = t^p(c_1\ln t + c_2) = c_1t^p\ln t + c_2t^p$ is the general solution. **65. a.** u has the form of the general solution with $c_1 = \dfrac{1}{r_1-r_2}$ and $c_2 = -\dfrac{1}{r_1-r_2}$. **b.** Apply L'Hôpital's rule, treating r_1 as a variable: $\lim\limits_{r_1\to r_2}\dfrac{e^{r_1t}-e^{r_2t}}{r_1-r_2} =$

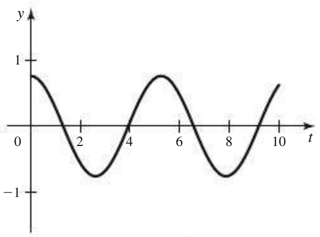
$\lim\limits_{r_1\to r_2}\dfrac{te^{r_1t}}{1} = te^{r_2t}$. If $r_1 = r_2$ the linearly independent solutions are e^{r_1t} and te^{r_1t}.

Section D2.3, pp. D2-33–D2-34

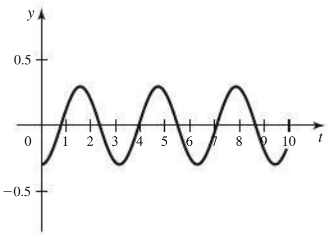
1. Find two linearly independent solutions, y_1 and y_2, of the corresponding homogeneous equation. Find a particular solution, y_p, of the equation. Form the general solution, $y = c_1y_1 + c_2y_2 + y_p$.
3. $y_p = Ae^{-4t}$ **5.** $y_p = Ae^{-t}\sin 4t + Be^{-t}\cos 4t$
7. $y_p = (At^2+Bt+C)e^{-t}$ **9.** $y_p = -\dfrac{2}{9}t - \dfrac{1}{9}$
11. $y_p = -\dfrac{1}{9}t^4 - \dfrac{2}{27}t^3 - \dfrac{1}{18}t^2 - \dfrac{7}{162}t - \dfrac{13}{972}$ **13.** $y_p = \dfrac{1}{3}e^{-2t}$
15. $y_p = -\dfrac{6}{11}e^{-3t}$ **17.** $y_p = -\dfrac{3}{5}\sin 2t$ **19.** $y_p = -\dfrac{1}{5}\sin t - \dfrac{2}{5}\cos t$
21. $y_p = -\dfrac{2}{3}e^t + \dfrac{1}{4}$ **23.** $y_p = \left(-\dfrac{1}{6}t + \dfrac{1}{72}\right)e^{-t}$
25. $y_p = \dfrac{1}{36}\sin 2t + \dfrac{1}{12}t\cos 2t$
27. $y_p = \left(\dfrac{1}{5}t + \dfrac{13}{50}\right)\sin t - \left(\dfrac{2}{5}t - \dfrac{9}{50}\right)\cos t$ **29.** $y_p = \dfrac{3}{2}te^t$
31. $y_p = -\dfrac{4}{5}te^{-3t}$ **33.** $y_p = 2te^{-2t}$ **35. a.** False **b.** False **c.** True
37. $y = c_1\sin t + c_2\cos t - \dfrac{4}{3}\sin 2t$; $y = \dfrac{8}{3}\sin t + \cos t - \dfrac{4}{3}\sin 2t$
39. $y = c_1e^{-2t}\sin t + c_2e^{-2t}\cos t + \dfrac{12}{5}$;
$y = -\dfrac{19}{5}e^{-2t}\sin t - \dfrac{7}{5}e^{-2t}\cos t + \dfrac{12}{5}$
41. $y = c_1\sin 3t + c_2\cos 3t + t\sin 3t$; $y = t\sin 3t$
43. $y_p = -3t^4 - 9t^3 - 63t^2 - 181t - 307$
45. $y_p = \left(-\dfrac{25}{13}t - \dfrac{1550}{1521}\right)e^{-t}\sin 3t + \left(\dfrac{50}{39}t - \dfrac{150}{169}\right)e^{-t}\cos 3t$
47. $y_p = -e^t$ **49.** $y_p = \left(t - \dfrac{10}{3}\right)e^{2t}$
51. Assume $y_{p1}''+py_{p1}'+qy_{p1} = f(t)$ and $y_{p2}''+py_{p2}'+qy_{p2} = g(t)$. Then
$(y_{p1}+y_{p2})''+p(y_{p1}+y_{p2})'+q(y_{p1}+y_{p2})$
$= (y_{p1}''+y_{p2}'')+p(y_{p1}'+y_{p2}')+q(y_{p1}+y_{p2})$
$= y_{p1}''+y_{p2}''+py_{p1}'+py_{p2}'+qy_{p1}+qy_{p2}$
$= (y_{p1}''+py_{p1}'+qy_{p1})+(y_{p2}''+py_{p2}'+qy_{p2})$
$= f(t)+g(t)$

Section D2.4, pp. D2-47–D2-52

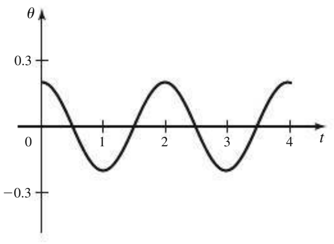
1. A system is damped if the object in motion encounters resistance to the movement; otherwise the motion is undamped. The motion is free if no external forces act on the object; otherwise, the motion is forced.
3. Beats occur when the natural frequency of the oscillator ω_0 is close in value to the forcing frequency ω.
5. Find a solution y_h to the homogeneous equation. Find a particular solution y_p, form the general solution $y_h + y_p$, and evaluate constants using the initial conditions.
7. $y = 0.75\cos 1.2t$; period $= 5\pi/3$



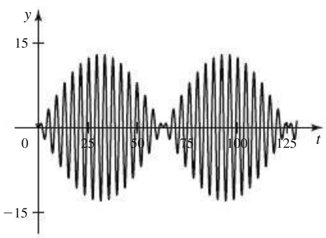
9. $y = -0.3\cos 2t$; period $= \pi$



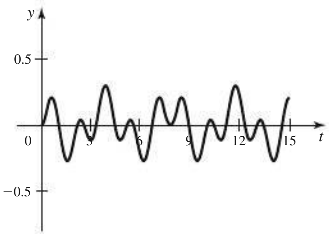
11. $\theta = 0.2\cos\sqrt{10}\,t$; period $= 2\pi/\sqrt{10}$



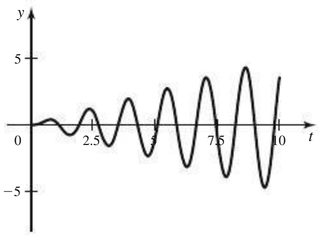
13. a. $\omega = 1.5$: $y = \frac{200}{31}\cos\frac{3}{2}t - \frac{200}{31}\cos\frac{8}{5}t$. This solution has beats.



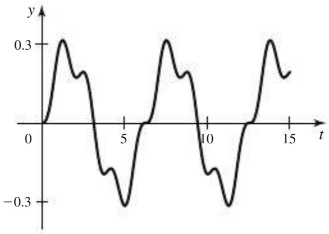
b. $\omega = 4$: $y = \frac{25}{168}\cos\frac{8}{5}t - \frac{25}{168}\cos 4t$



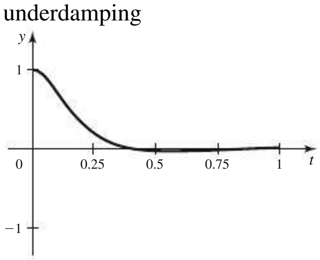
15. a. $\omega = 4$: $y = \frac{1}{8}\sin 4t - \frac{1}{2}t\cos 4t$; resonance case



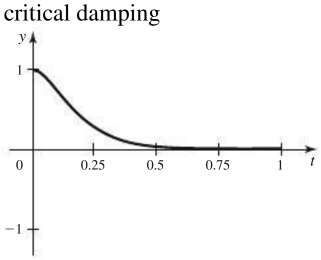
b. $\omega = 1$: $y = \frac{4}{15}\sin t - \frac{1}{15}\sin 4t$



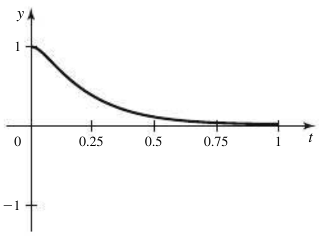
17. a. $y = \frac{4}{3}e^{-8t}\sin 6t + e^{-8t}\cos 6t$; underdamping



b. $y = e^{-10t} + 10te^{-10t}$; critical damping

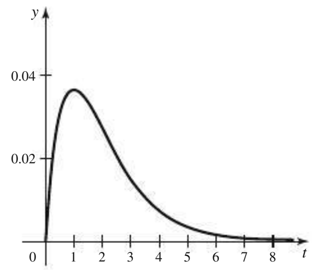


c. $y = -\frac{1}{3}e^{-20t} + \frac{4}{3}e^{-5t}$; overdamping



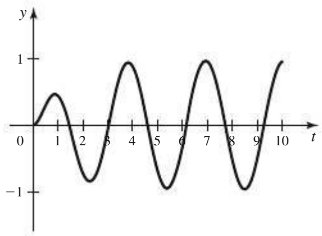
19. a. $k = 250$ N/m **b.** $y = \frac{1}{10}te^{-t}$

c.



The maximum displacement is $\frac{1}{10}e^{-1} \approx 0.037$ m. **d.** With an increase in k of 50%, the motion becomes underdamped, maximum displacement decreases to 0.034 m. With an increase in k of 100%, the motion becomes underdamped; maximum displacement decreases to 0.032 m. **e.** With a decrease in k of 50%, the motion becomes overdamped; maximum displacement increases to 0.041 m.

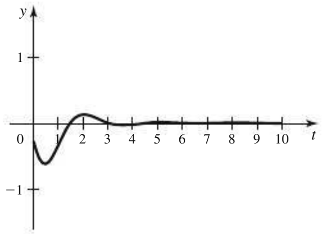
21. a. $y = -\frac{18}{17}e^{-t}\sin 2t - \frac{4}{17}e^{-t}\cos 2t + \frac{16}{17}\sin 2t + \frac{4}{17}\cos 2t$



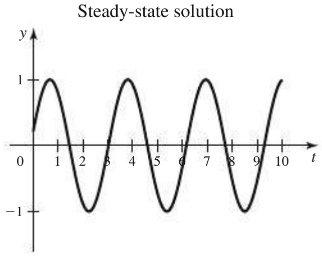
b. transient solution: $y = -\frac{18}{17}e^{-t}\sin 2t - \frac{4}{17}e^{-t}\cos 2t$

steady-state: $y = \frac{16}{17}\sin 2t + \frac{4}{17}\cos 2t$

Transient solution



Steady-state solution



c. After approximately 3 seconds.

23. a. $y = c_1e^{-t/2}\sin t + c_2e^{-t/2}\cos t + \frac{32\omega}{16\omega^4 - 24\omega^2 + 25}\sin\omega t - \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}\cos\omega t$

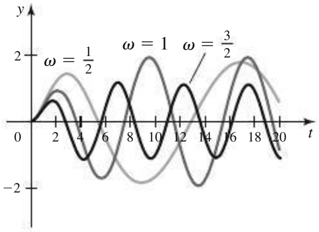
b. $y = -\frac{16\omega^2 + 20}{16\omega^4 - 24\omega^2 + 25}e^{-t/2}\sin t + \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}e^{-t/2}\cos t + \frac{32\omega}{16\omega^4 - 24\omega^2 + 25}\sin\omega t - \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}\cos\omega t$

c. transient solution:

$$y = -\frac{16\omega^2 + 20}{16\omega^4 - 24\omega^2 + 25}e^{-t/2}\sin t + \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}e^{-t/2}\cos t$$

steady-state solution:

$$\frac{32\omega}{16\omega^4 - 24\omega^2 + 25}\sin\omega t - \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25}\cos\omega t$$



25. When $L = 0$, the equation $LQ'' + RQ' + \frac{1}{C}Q = E(t)$; becomes $RQ' + \frac{1}{C}Q = E(t)$. Solution is $Q = -\frac{1}{20}e^{-20t} + \frac{1}{20}$ and it approaches a steady-state value of $\frac{1}{20}$. **27. a.** $y = 50e^{-40t} - 50e^{-60t}$ **b.** transient current: $y = 50e^{-40t} - 50e^{-60t}$; steady-state current: $y = 0$

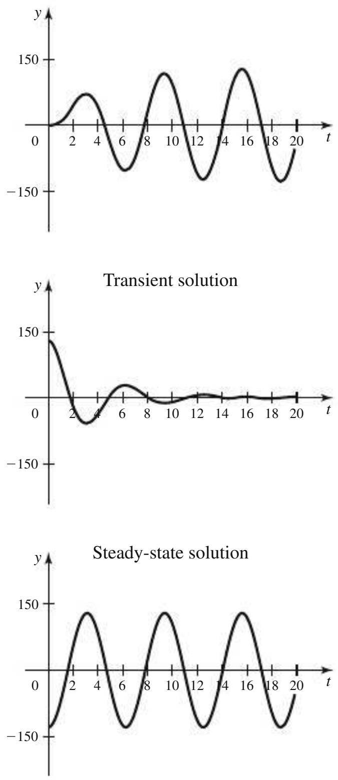
29. a. $y = \frac{40\sqrt{15}}{3}e^{-80t}\sin 2\sqrt{15}t$ **b.** transient current: $y = \frac{40\sqrt{15}}{3}e^{-80t}\sin 2\sqrt{15}t$; steady-state current: $y = 0$

31. $Q = \frac{12}{5} - \frac{4}{5}e^{-4t}(3\cos 3t + 4\sin 3t)$; $I = 20e^{-4t}\sin 3t$

33. a. True **b.** True **c.** True **d.** False **e.** False **f.** True

35. a. $y = 16e^{-t/4}\sin t + 128e^{-t/4}\cos t + 16\sin t - 128\cos t$
transient solution: $y = 16e^{-t/4}\sin t + 128e^{-t/4}\cos t$
steady-state solution: $y = 16\sin t - 128\cos t$

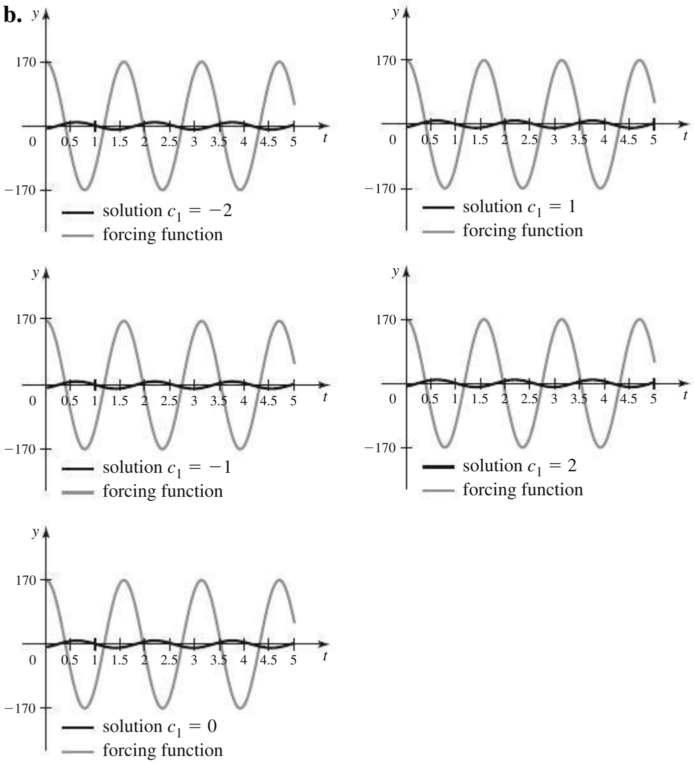
b.



c. Around $t = 20$; $\lim_{t\to\infty}(16e^{-t/4}\sin t + 128e^{-t/4}\cos t) = 0$ so the solution approaches the steady-state solution.

37. a. $y = c_1e^{-2t} + c_2e^{-t} + 6\sin 4t - 7\cos 4t$
transient solution: $y = c_1e^{-2t} + c_2e^{-t}$
steady-state solution: $y = 6\sin 4t - 7\cos 4t$

b.

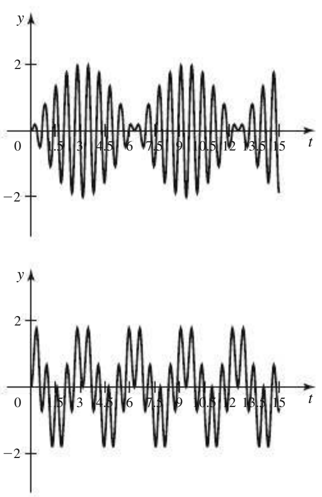


c. In all cases, the solution approaches the steady state solution quickly. The steady-state solution has smaller magnitude than the external force function and is shifted.

39. a.

$$\cos\omega t - \cos\omega_0 t = 2\sin\left(\frac{\omega_0 t - \omega t}{2}\right)\sin\left(\frac{\omega_0 t + \omega t}{2}\right) = 2\sin\frac{(\omega_0 - \omega)t}{2}\sin\frac{(\omega_0 + \omega)t}{2}$$

b.



41. $C^* = \dfrac{F_0}{\sqrt{c^2\omega^2 + m^2(\omega_0^2 - \omega^2)^2}} = \dfrac{\omega E_0}{\sqrt{R^2\omega^2 + L^2\left(\dfrac{1}{CL} - \omega^2\right)^2}}$

$= \dfrac{\omega E_0}{\sqrt{R^2\omega^2 + \omega^2\dfrac{L^2}{\omega^2}\left(\dfrac{1}{CL} - \omega^2\right)^2}}$

$= \dfrac{\omega E_0}{\sqrt{R^2\omega^2 + \omega^2\left(\dfrac{L}{\omega}\left(\dfrac{1}{CL} - \omega^2\right)\right)^2}} = \dfrac{\omega E_0}{\omega\sqrt{R^2 + \left(\dfrac{L}{\omega}\left(\dfrac{1}{CL} - \omega^2\right)\right)^2}}$

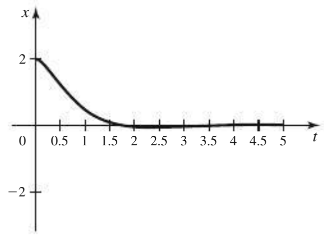
$= \dfrac{E_0}{\sqrt{R^2 + \left(\dfrac{1}{C\omega} - L\omega\right)^2}}$

$= \dfrac{E_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

$R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2$ is smallest when $\omega L = \dfrac{1}{\omega C}$ or $\omega^2 = \dfrac{1}{CL}$.

43. a. $x = \dfrac{4\sqrt{7}}{7}e^{-3t/2}\sin\dfrac{\sqrt{7}}{2}t + 2e^{-3t/2}\cos\dfrac{\sqrt{7}}{2}t$

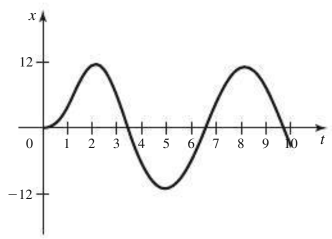
b.



c. The motion is underdamped.

45. a. $x = -\dfrac{22}{5}e^{-t/2}\sin 2t + \dfrac{16}{5}e^{-t/2}\cos 2t + \dfrac{52}{5}\sin t - \dfrac{16}{5}\cos t$

b.



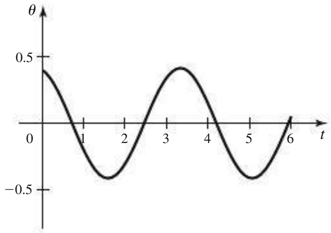
c. The motion approaches a steady-state solution of $x = \dfrac{52}{5}\sin t - \dfrac{16}{5}\cos t$.

47. a. There are two forces acting: inertia, given by $ma = ms''(t) = m\ell\theta''(t)$ and weight of the bob. The component of weight in the direction of motion is $mg\sin\theta(t)$. By Newton's law, the sum of the forces is 0 or $m\ell\theta''(t) + mg\sin\theta(t) = 0 \Rightarrow m\ell\theta''(t) = -mg\sin\theta(t)$.

b. $m\ell\theta''(t) = -mg\sin\theta(t) \Rightarrow \theta''(t) = -\dfrac{g}{\ell}\sin\theta(t)$
$= -\omega_0^2\sin\theta(t)$ or $\theta'' + \omega_0^2\sin\theta = 0$

c. Using $\sin\theta \approx \theta$ in the equation from (c) gives $\theta'' + \omega_0^2\theta = 0$.

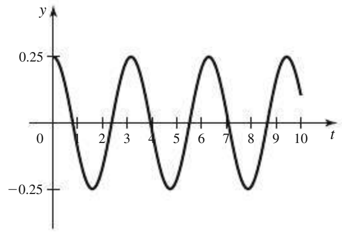
d. The frequency is given by $\omega_0^2 = \dfrac{\ell}{g}$ or $\omega_0 = \sqrt{\dfrac{g}{\ell}}$. The period is $T = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{\sqrt{\dfrac{g}{\ell}}} = 2\pi\sqrt{\dfrac{\ell}{g}}$. Doubling the length increases the period by a factor of $\sqrt{2}$. **49. a.** $\theta(t) = -\dfrac{\sqrt{15}}{35}\sin\dfrac{7\sqrt{15}}{15}t + \dfrac{2}{5}\cos\dfrac{7\sqrt{15}}{15}t$



b. $\theta(t) = \dfrac{\sqrt{211}}{35}\sin\left(\dfrac{7\sqrt{15}}{15}t + \varphi\right)$ where $\tan\varphi = -\dfrac{14\sqrt{15}}{15}$
or $\theta(t) \approx 0.415\sin(1.807t + 1.841)$.

c. $T = 2\pi\sqrt{\dfrac{\ell}{g}} = 2\pi\sqrt{\dfrac{3}{9.8}} = 3.476$ seconds.

51. a. $y = 0.25\cos 2t$



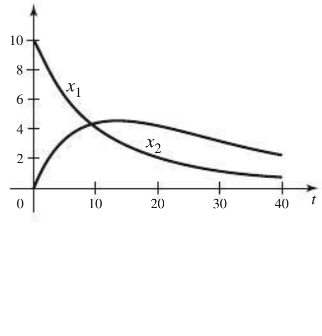
b. $y = 0.25\cos 2t$ **c.** $T = \pi$

53. a. $x_1' = -k_1x_1 + k_2x_2 + f(t) \Rightarrow x_1'' = -k_1x_1' + k_2x_2' + f'(t)$ $\Rightarrow k_2x_2' = x_1'' + k_1x_1' - f'(t)$. Eliminate x_2 from the given first-order system by multiplying the equation for x_1' by $(k_2 + k_3)$ and the equation for x_2' by k_2 and adding. This results in $(k_2 + k_3)x_1' + k_2x_2' = -k_1k_3x_1 + (k_2 + k_3)f(t)$. Substitute $k_2x_2' = x_1'' + k_1x_1' - f'(t)$ and solve for x_1'' to get $x_1'' = -(k_1 + k_2 + k_3)x_1' - k_1k_3x_1 + (k_2 + k_3)f(t) + f'(t)$. Replace x_1 with x. **b.** Because x is another name for x_1, $x_1(0) = A$ is the same as $x(0) = A$. Using the first DE gives $x'(0) = x_1'(0)$ $= -k_1x_1(0) + k_2x_2(0) + f(0) = -k_1x_1(0) + f(0)$.

55. a. $x_1(t) = 6.59e^{-0.1322t} + 3.41e^{-0.0378t}$

b. $x_2(t) = -10.60e^{-0.1322t} + 10.60e^{-0.0378t}$

c.

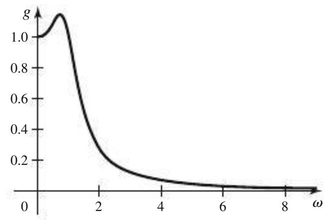


Section D2.5, pp. D2-61–D2-63

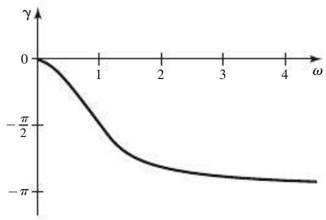
1. $H(\omega) = g(\omega)e^{i\gamma(\omega)}$ **3.** The phase lag function gives the phase of the output relative to the input. The output lags the input.

5. a. $g(\omega) = \dfrac{1}{\sqrt{(1-\omega^2)^2+\omega^2}}$;

$\tan\gamma(\omega) = \dfrac{\omega}{\omega^2-1}$



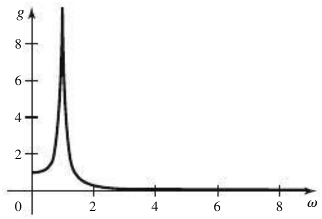
b. Local maximum of the gain function at $\omega = \dfrac{\sqrt{2}}{2}$. Weak damping.

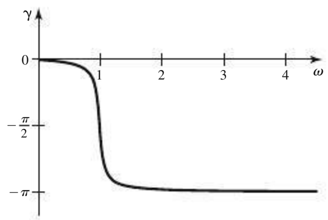


7. a. $g(\omega) = \dfrac{1}{\sqrt{(1-\omega^2)^2+\dfrac{\omega^2}{100}}}$;

$\tan\gamma(\omega) = \dfrac{\omega}{10(\omega^2-1)}$

b. Local maximum of the gain function at $\omega = \dfrac{\sqrt{398}}{20}$. Weak damping.

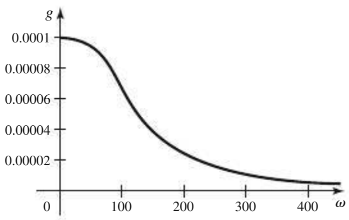


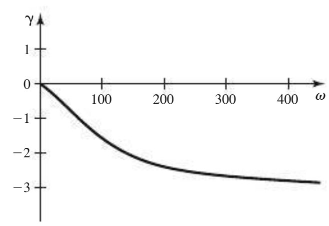


9. a. $g(\omega) = \dfrac{1}{\sqrt{(10{,}000-\omega^2)^2+22{,}500\omega^2}}$;

$\tan\gamma(\omega) = \dfrac{150\omega}{\omega^2-10{,}000}$

b. Local maximum of the gain function at $\omega = 0$. Strong damping.

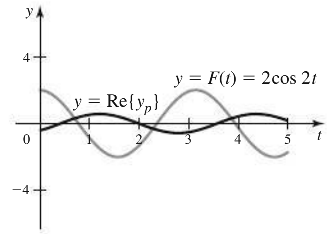




11. a. See Exercise 5a.

b. $\text{Re}\{y_p\} = \dfrac{2}{\sqrt{13}}\cos\left(2t + \tan^{-1}\dfrac{2}{3} - \pi\right)$
$\approx 0.555\cos(2t - 2.554)$

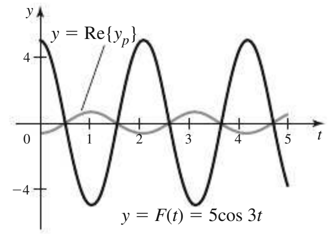
c.



13. a. See Exercise 7a.

b. $\text{Re}\{y_p\} = \dfrac{50}{\sqrt{6409}}\cos\left(3t + \tan^{-1}\dfrac{3}{80} - \pi\right)$
$\approx 0.625\cos(3t - 3.104)$

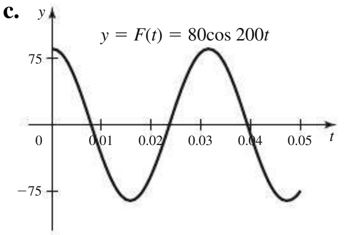
c.

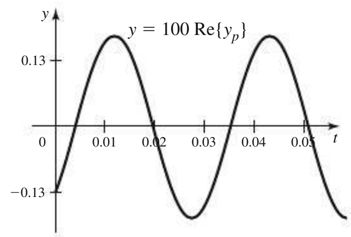


15. a. See Exercise 9a.

b. $\text{Re}\{y_p\} = \dfrac{\sqrt{2}}{750}\cos(200t + \tan^{-1}1 - \pi)$
$\approx 0.00189\cos(200t - 2.356)$

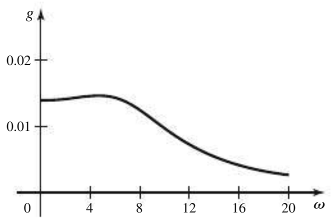
c.



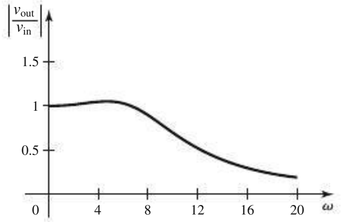


17. a. $g(\omega) = \dfrac{1}{\sqrt{(72-\omega^2)^2+100\omega^2}}$

b. Local maximum of the gain function at $\omega \approx \sqrt{22}$. **c.** $(\sqrt{22}, \infty)$

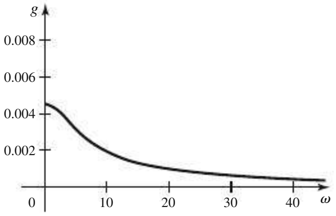


d. $\left|\dfrac{v_{\text{out}}}{v_{\text{in}}}\right| = \dfrac{72}{\sqrt{(72 - \omega^2)^2 + 100\omega^2}}$

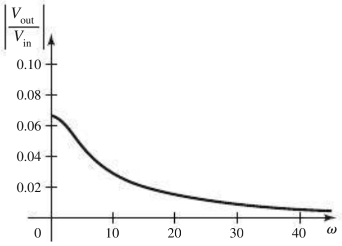


19. a. $g(\omega) = \dfrac{1}{\sqrt{(15 - \omega^2)^2 + 2500\omega^2}}$

b. Local maximum of the gain function at $\omega = 0$. **c.** $[0, \infty)$



d. $\left|\dfrac{v_{\text{out}}}{v_{\text{in}}}\right| = \dfrac{15}{\sqrt{(15 - \omega^2)^2 + 2500\omega^2}}$



21. a. True **b.** True

23. a. With 1 in the numerator, the smaller the denominator, the larger the value of $g(\omega)$. **b.** A square root will be smallest when the quantity under the square root is smallest.

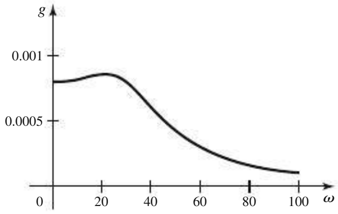
c. $h'(\omega) = \dfrac{d}{d\omega}\left[(\omega_0^2 - \omega^2)^2 + b^2\omega^2\right] = 2(\omega_0^2 - \omega^2)(-2\omega) + 2b\omega$
$= 2\omega(-2(\omega_0^2 - \omega^2) + b) = 2\omega(b - 2\omega_0^2 + 2\omega^2)$

d. From part (c), $h'(\omega) = 2\omega(b^2 - 2\omega_0^2 + 2\omega^2)$. If $b < \sqrt{2}\omega_0$, then $h'(\omega) = 0$ has a real solution: $b^2 - 2\omega_0^2 + 2\omega^2 = 0$
$\Rightarrow 2\omega^2 = 2\omega_0^2 - b^2 \Rightarrow \omega = \sqrt{\dfrac{2\omega_0^2 - b^2}{2}} = \sqrt{\omega_0^2 - \dfrac{b^2}{2}}$.
That this gives a local minimum for *h,* and hence a local maximum for *g*, can be verified by the First or Second Derivative Tests. The maximum value of *g* is $\dfrac{\sqrt{2}}{\sqrt{b^2 + b^2(2\omega_0^2 - b^2)}}$.

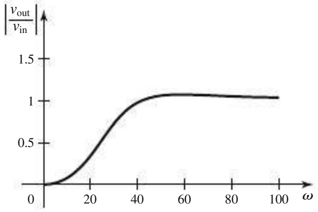
e. If $b \geq \sqrt{2}\omega_0$, then $b^2 \geq 2\omega_0^2$ and $b^2 - 2\omega_0^2 + 2\omega^2 \geq 0$. Hence $h'(\omega) = 2\omega(b^2 - 2\omega_0^2 + 2\omega^2) \geq 0$ for $\omega \geq 0$ and h is an increasing function. It follows that $g = \dfrac{1}{\sqrt{h}}$ is decreasing for $\omega \geq 0$ and so has a local maximum at $\omega = 0$. **f.** Part (e); the gain function is maximized at 0.

25. a. $g(\omega) = \dfrac{1}{\sqrt{(\omega^2 - 1250)^2 + 1600\omega^2}}$

b. Increases until $\omega = 15\sqrt{2}$ then decreases.

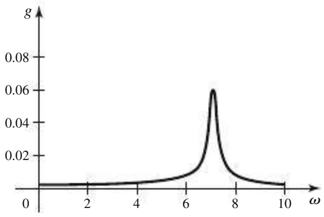


c. $\left|\dfrac{v_{\text{out}}}{v_{\text{in}}}\right| = \dfrac{\omega^2}{\sqrt{(\omega^2 - 1250)^2 + 1600\omega^2}}$

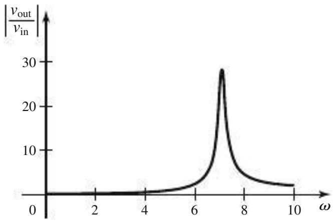


27. a. $g(\omega) = \dfrac{4}{\sqrt{16(\omega^2 - 50)^2 + \omega^2}}$

b. Increases until $\omega = \dfrac{\sqrt{3198}}{8} \approx 7.069$ then decreases.



c. $\left|\dfrac{v_{\text{out}}}{v_{\text{in}}}\right| = \dfrac{4\omega^2}{\sqrt{16(\omega^2 - 50)^2 + \omega^2}}$



Chapter D2 Review Exercises, pp. D2-63–D2-64

1. a. True **b.** False **c.** False **d.** False **e.** True

3. $y = c_1e^{-2t} + c_2e^{4t}$ **5.** $y = c_1 \sin 6t + c_2 \cos 6t$

7. $y = c_1e^{-2t} + c_2te^{-2t}$ **9.** $y = c_1e^t \sin 2t + c_2e^t \cos 2t$

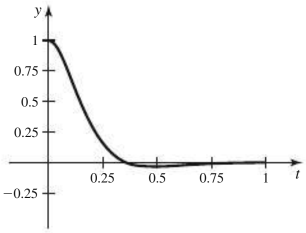
11. $y_p = \dfrac{1}{7}\cos 2t$ **13.** $y_p = -\dfrac{1}{6}t - \dfrac{5}{36} - \dfrac{1}{5}e^{-t}$

15. $y_p = \dfrac{1}{4}t \sin 4t$ **17.** $y = c_1 \sin 2t + c_2 \cos 2t - \dfrac{3}{5}\sin 3t$

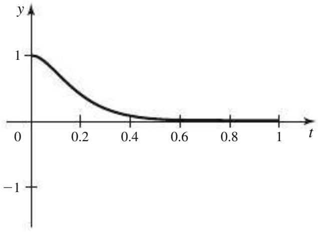
19. $y = c_1e^{-2t}\sin t + c_2e^{-2t}\cos t + \dfrac{1}{4}\sin t + \dfrac{1}{4}\cos t$

21. $y = c_1e^{-t} + c_2e^{t} + te^{t}$

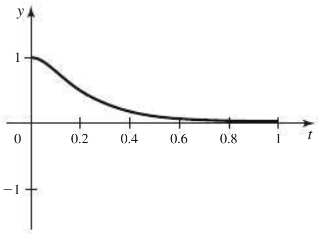
23. a. $y = \frac{3\sqrt{7}}{7}e^{-15t/2}\sin\frac{5\sqrt{7}}{2}t + e^{-15t/2}\cos\frac{5\sqrt{7}}{2}t$; underdamping



b. $y = e^{-10t} + 10te^{-10t}$; critical damping



c. $y = -\frac{1}{3}e^{-20t} + \frac{4}{3}e^{-5t}$; overdamping



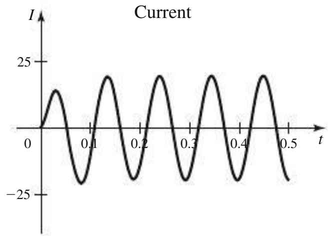
25. a. $I = 50e^{-60t} - \frac{600}{13}e^{-40t} + \frac{250}{13}\sin 60t - \frac{50}{13}\cos 60t$

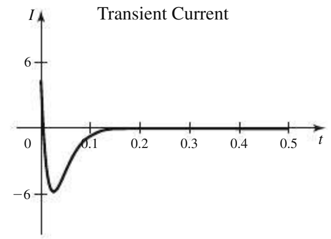
b. Transient current:

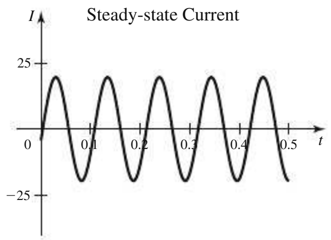
$I = 50e^{-60t} - \frac{600}{13}e^{-40t}$

c. Steady-state current:

$I = \frac{250}{13}\sin 60t - \frac{50}{13}\cos 60t$







d. The solutions are the same.

Index

Note:

- Italics indicate figures or margin notes.
- "t" indicates a table.
- "e" indicates an exercise.
- GP indicates a Guided Project (located online in MyLab Math).
- AP-, B-, and C- indicate Appendices A, B, and C respectively (Appendices B and C are online at bit.ly/2y3Nck3 and bit.ly/2Rvf08V).

E

G

H

J

K

L

M

N

O

T

W

X

Y

Z

Index of Applications

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Economics and Business

Engineering and Physical Sciences

General

Life Sciences

Social and Behavioral Sciences

TABLE OF INTEGRALS

Substitution Rule	**Integration by Parts**
$\int f(g(x))g'(x)\,dx = \int f(u)\,du \quad (u = g(x))$	$\int u\,dv = uv - \int v\,du$
$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$	$\int_a^b uv'\,dx = uv\Big\|_a^b - \int_a^b vu'\,dx$

Basic Integrals

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C;\ n \neq -1$

2. $\int \frac{dx}{x} = \ln|x| + C$

3. $\int \cos ax\,dx = \frac{1}{a}\sin ax + C$

4. $\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$

5. $\int \tan x\,dx = \ln|\sec x| + C$

6. $\int \cot x\,dx = \ln|\sin x| + C$

7. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

8. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

9. $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

10. $\int b^{ax}\,dx = \frac{1}{a\ln b}b^{ax} + C;\ b > 0, b \neq 1$

11. $\int \ln x\,dx = x\ln x - x + C$

12. $\int \log_b x\,dx = \frac{1}{\ln b}(x\ln x - x) + C$

13. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

14. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C, a > 0$

15. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C, a > 0$

16. $\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1 - x^2} + C$

17. $\int \cos^{-1}x\,dx = x\cos^{-1}x - \sqrt{1 - x^2} + C$

18. $\int \tan^{-1}x\,dx = x\tan^{-1}x - \frac{1}{2}\ln(1 + x^2) + C$

19. $\int \sec^{-1}x\,dx = x\sec^{-1}x - \ln(x + \sqrt{x^2 - 1}) + C$

20. $\int \sinh x\,dx = \cosh x + C$

21. $\int \cosh x\,dx = \sinh x + C$

22. $\int \operatorname{sech}^2 x\,dx = \tanh x + C$

23. $\int \operatorname{csch}^2 x\,dx = -\coth x + C$

24. $\int \operatorname{sech} x\tanh x\,dx = -\operatorname{sech} x + C$

25. $\int \operatorname{csch} x\coth x\,dx = -\operatorname{csch} x + C$

26. $\int \tanh x\,dx = \ln\cosh x + C$

27. $\int \coth x\,dx = \ln|\sinh x| + C$

28. $\int \operatorname{sech} x\,dx = \tan^{-1}\sinh x + C = \sin^{-1}\tanh x + C$

29. $\int \operatorname{csch} x\,dx = \ln|\tanh(x/2)| + C$

Trigonometric Integrals

30. $\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$

31. $\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$

32. $\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$

33. $\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$

34. $\int \tan^2 x\,dx = \tan x - x + C$

35. $\int \cot^2 x\,dx = -\cot x - x + C$

36. $\int \cos^3 x\,dx = -\frac{1}{3}\sin^3 x + \sin x + C$

37. $\int \sin^3 x\,dx = \frac{1}{3}\cos^3 x - \cos x + C$

38. $\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$

39. $\int \csc^3 x\,dx = -\frac{1}{2}\csc x \cot x - \frac{1}{2}\ln|\csc x + \cot x| + C$

40. $\int \tan^3 x\,dx = \frac{1}{2}\tan^2 x - \ln|\sec x| + C$

41. $\int \cot^3 x\,dx = -\frac{1}{2}\cot^2 x - \ln|\sin x| + C$

42. $\int \sec^n ax \tan ax\,dx = \frac{1}{na}\sec^n ax + C;\ n \neq 0$

43. $\int \csc^n ax \cot ax\,dx = -\frac{1}{na}\csc^n ax + C;\ n \neq 0$

44. $\int \frac{dx}{1 + \sin ax} = -\frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) + C$

45. $\int \frac{dx}{1 - \sin ax} = \frac{1}{a}\tan\left(\frac{\pi}{4} + \frac{ax}{2}\right) + C$

46. $\int \frac{dx}{1 + \cos ax} = \frac{1}{a}\tan\frac{ax}{2} + C$

47. $\int \frac{dx}{1 - \cos ax} = -\frac{1}{a}\cot\frac{ax}{2} + C$

48. $\int \sin mx \cos nx\,dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C;\ m^2 \neq n^2$

49. $\int \sin mx \sin nx\,dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C;\ m^2 \neq n^2$

50. $\int \cos mx \cos nx\,dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C;\ m^2 \neq n^2$

Reduction Formulas for Trigonometric Functions

51. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$

52. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$

53. $\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx;\ n \neq 1$

54. $\int \cot^n x\,dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\,dx;\ n \neq 1$

55. $\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx;\ n \neq 1$

56. $\int \csc^n x\,dx = -\frac{\csc^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} x\,dx;\ n \neq 1$

57. $\int \sin^m x \cos^n x\,dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x\,dx;\ m \neq -n$

58. $\int \sin^m x \cos^n x\,dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x\,dx;\ m \neq -n$

59. $\int x^n \sin ax\,dx = -\frac{x^n \cos ax}{a} + \frac{n}{a}\int x^{n-1}\cos ax\,dx;\ a \neq 0$

60. $\int x^n \cos ax\,dx = \frac{x^n \sin ax}{a} - \frac{n}{a}\int x^{n-1}\sin ax\,dx;\ a \neq 0$

Integrals Involving $a^2 - x^2$; $a > 0$

61. $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

62. $\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right| + C$

63. $\int \frac{dx}{x^2\sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$

64. $\int x^2\sqrt{a^2 - x^2}\,dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2 - x^2} + \frac{a^4}{8}\sin^{-1}\frac{x}{a} + C$

65. $\int \frac{\sqrt{a^2 - x^2}}{x^2}\,dx = -\frac{1}{x}\sqrt{a^2 - x^2} - \sin^{-1}\frac{x}{a} + C$

66. $\int \frac{x^2}{\sqrt{a^2 - x^2}}\,dx = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

67. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{x + a}{x - a}\right| + C$

Integrals Involving $x^2 - a^2$; $a > 0$

68. $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + C$

69. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left|x + \sqrt{x^2 - a^2}\right| + C$

70. $\int \frac{dx}{x^2\sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$

71. $\int x^2\sqrt{x^2 - a^2}\,dx = \frac{x}{8}(2x^2 - a^2)\sqrt{x^2 - a^2} - \frac{a^4}{8}\ln\left|x + \sqrt{x^2 - a^2}\right| + C$

72. $\int \frac{\sqrt{x^2 - a^2}}{x^2}\,dx = \ln\left|x + \sqrt{x^2 - a^2}\right| - \frac{\sqrt{x^2 - a^2}}{x} + C$

73. $\int \frac{x^2}{\sqrt{x^2 - a^2}}\,dx = \frac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + \frac{x}{2}\sqrt{x^2 - a^2} + C$

74. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x - a}{x + a}\right| + C$

75. $\int \frac{dx}{x(x^2 - a^2)} = \frac{1}{2a^2}\ln\left|\frac{x^2 - a^2}{x^2}\right| + C$

Integrals Involving $a^2 + x^2$; $a > 0$

76. $\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\ln\left(x + \sqrt{a^2 + x^2}\right) + C$

77. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln\left(x + \sqrt{a^2 + x^2}\right) + C$

78. $\int \frac{dx}{x\sqrt{a^2 + x^2}} = \frac{1}{a}\ln\left|\frac{a - \sqrt{a^2 + x^2}}{x}\right| + C$

79. $\int \frac{dx}{x^2\sqrt{a^2 + x^2}} = -\frac{\sqrt{a^2 + x^2}}{a^2 x} + C$

80. $\int x^2\sqrt{a^2 + x^2}\, dx = \frac{x}{8}(a^2 + 2x^2)\sqrt{a^2 + x^2} - \frac{a^4}{8}\ln\left(x + \sqrt{a^2 + x^2}\right) + C$

81. $\int \frac{\sqrt{a^2 + x^2}}{x^2}\, dx = \ln\left|x + \sqrt{a^2 + x^2}\right| - \frac{\sqrt{a^2 + x^2}}{x} + C$

82. $\int \frac{x^2}{\sqrt{a^2 + x^2}}\, dx = -\frac{a^2}{2}\ln\left(x + \sqrt{a^2 + x^2}\right) + \frac{x\sqrt{a^2 + x^2}}{2} + C$

83. $\int \frac{\sqrt{a^2 + x^2}}{x}\, dx = \sqrt{a^2 + x^2} - a\ln\left|\frac{a + \sqrt{a^2 + x^2}}{x}\right| + C$

84. $\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}} + C$

85. $\int \frac{dx}{x(a^2 + x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2 + x^2}\right) + C$

Integrals Involving $ax \pm b$; $a \neq 0, b > 0$

86. $\int (ax + b)^n\, dx = \frac{(ax + b)^{n+1}}{a(n + 1)} + C;\ n \neq -1$

87. $\int (\sqrt{ax + b})^n\, dx = \frac{2}{a}\frac{(\sqrt{ax + b})^{n+2}}{n + 2} + C;\ n \neq -2$

88. $\int \frac{dx}{x\sqrt{ax - b}} = \frac{2}{\sqrt{b}}\tan^{-1}\sqrt{\frac{ax - b}{b}} + C$

89. $\int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}}\right| + C$

90. $\int \frac{x}{ax + b}\, dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax + b| + C$

91. $\int \frac{x^2}{ax + b}\, dx = \frac{1}{2a^3}\left((ax + b)^2 - 4b(ax + b) + 2b^2\ln|ax + b|\right) + C$

92. $\int \frac{dx}{x^2(ax + b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax + b}{x}\right| + C$

93. $\int x\sqrt{ax + b}\, dx = \frac{2}{15a^2}(3ax - 2b)(ax + b)^{3/2} + C$

94. $\int \frac{x}{\sqrt{ax + b}}\, dx = \frac{2}{3a^2}(ax - 2b)\sqrt{ax + b} + C$

95. $\int x(ax + b)^n\, dx = \frac{(ax + b)^{n+1}}{a^2}\left(\frac{ax + b}{n + 2} - \frac{b}{n + 1}\right) + C;\ n \neq -1, -2$

96. $\int \frac{dx}{x(ax + b)} = \frac{1}{b}\ln\left|\frac{x}{ax + b}\right| + C$

Integrals with Exponential and Trigonometric Functions

97. $\int e^{ax}\sin bx\, dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2 + b^2} + C$

98. $\int e^{ax}\cos bx\, dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2 + b^2} + C$

Integrals with Exponential and Logarithmic Functions

99. $\int \frac{dx}{x\ln x} = \ln|\ln x| + C$

100. $\int x^n\ln x\, dx = \frac{x^{n+1}}{n + 1}\left(\ln x - \frac{1}{n + 1}\right) + C;\ n \neq -1$

101. $\int xe^x\, dx = xe^x - e^x + C$

102. $\int x^n e^{ax}\, dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\, dx;\ a \neq 0$

103. $\int \ln^n x\, dx = x\ln^n x - n\int \ln^{n-1} x\, dx$

Miscellaneous Formulas

104. $\int x^n\cos^{-1} x\, dx = \frac{1}{n + 1}\left(x^{n+1}\cos^{-1}x + \int \frac{x^{n+1}dx}{\sqrt{1 - x^2}}\right);\ n \neq -1$

105. $\int x^n\sin^{-1} x\, dx = \frac{1}{n + 1}\left(x^{n+1}\sin^{-1} x - \int \frac{x^{n+1}\, dx}{\sqrt{1 - x^2}}\right);\ n \neq -1$

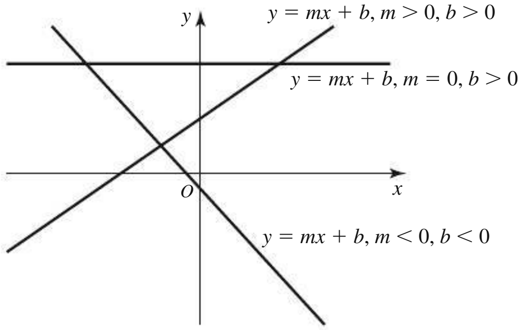
106. $\int x^n\tan^{-1} x\, dx = \frac{1}{n + 1}\left(x^{n+1}\tan^{-1} x - \int \frac{x^{n+1}\, dx}{x^2 + 1}\right);\ n \neq -1$

107. $\int \sqrt{2ax - x^2}\, dx = \frac{x - a}{2}\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x - a}{a}\right) + C;\ a > 0$

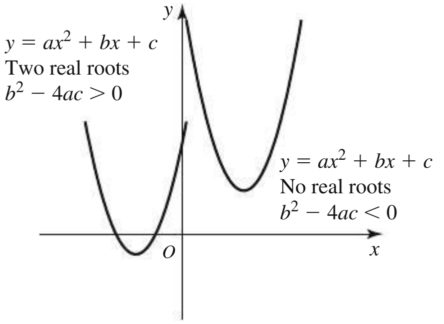
108. $\int \frac{dx}{\sqrt{2ax - x^2}} = \sin^{-1}\left(\frac{x - a}{a}\right) + C;\ a > 0$

GRAPHS OF ELEMENTARY FUNCTIONS

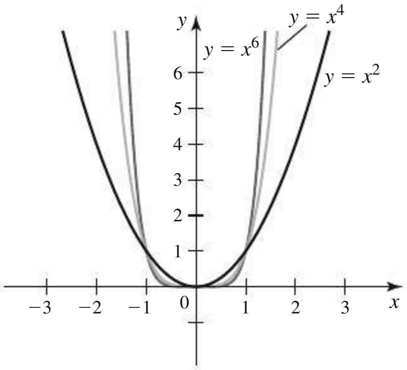
Linear functions



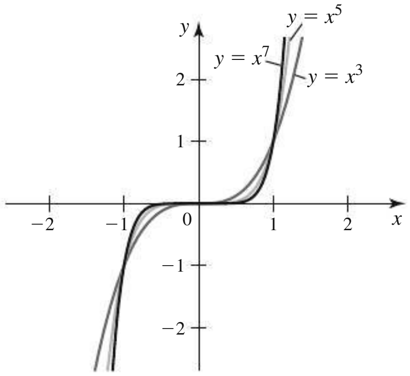
Quadratic functions



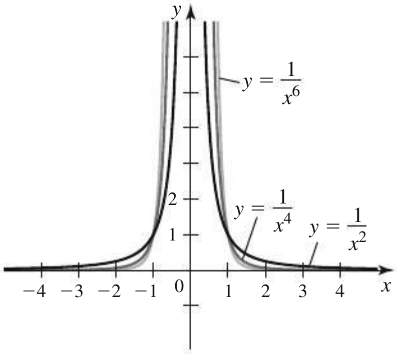
Positive even powers



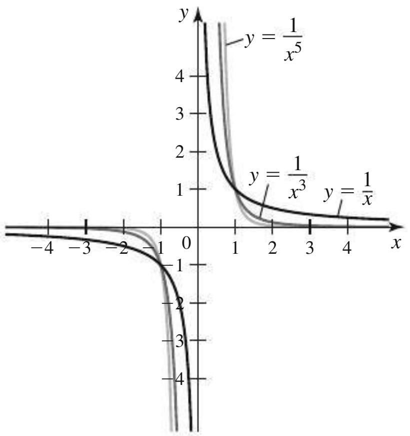
Positive odd powers



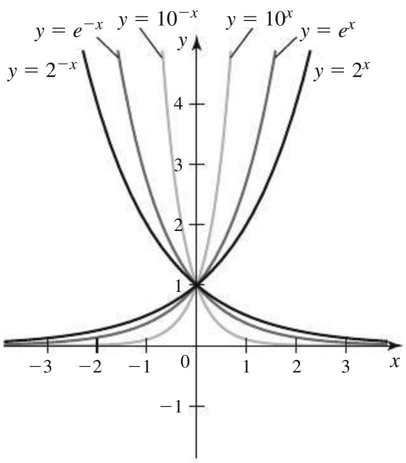
Negative even powers



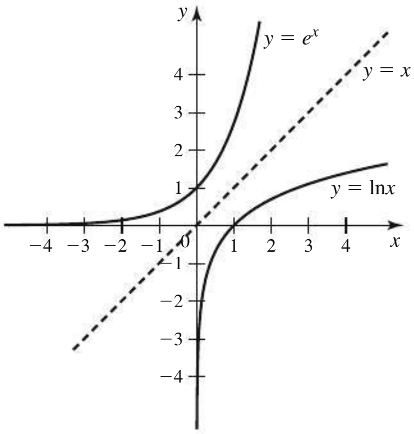
Negative odd powers



Exponential functions



Natural logarithmic and exponential functions



DERIVATIVES

General Formulas

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(cf(x)) = cf'(x)$

$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$

$\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$

$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{d}{dx}(x^n) = nx^{n-1}$, for real numbers n

$\frac{d}{dx}\big(f(g(x))\big) = f'(g(x)) \cdot g'(x)$

Trigonometric Functions

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

Inverse Trigonometric Functions

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$

$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1 + x^2}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2 - 1}}$

$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2 - 1}}$

Exponential and Logarithmic Functions

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(b^x) = b^x \ln b$

$\frac{d}{dx}(\ln |x|) = \frac{1}{x}$

$\frac{d}{dx}(\log_b |x|) = \frac{1}{x \ln b}$

Hyperbolic Functions

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

Inverse Hyperbolic Functions

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2 + 1}}$

$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}} \quad (x > 1)$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1 - x^2} \quad (|x| < 1)$

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1 - x^2} \quad (|x| > 1)$

$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1 - x^2}} \quad (0 < x < 1)$

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{1 + x^2}} \quad (x \neq 0)$

FORMS OF THE FUNDAMENTAL THEOREM OF CALCULUS

Fundamental Theorem of Calculus	$\int_a^b f'(x)\,dx = f(b) - f(a)$
Fundamental Theorem for Line Integrals	$\int_C \nabla f \cdot d\mathbf{r} = f(B) - f(A)$ (A and B are the initial and final points of C.)
Green's Theorem	$\iint_R (g_x - f_y)\,dA = \oint_C f\,dx + g\,dy$ $\iint_R (f_x + g_y)\,dA = \oint_C f\,dy - g\,dx$
Stokes' Theorem	$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\,dS = \oint_C \mathbf{F} \cdot d\mathbf{r}$
Divergence Theorem	$\iiint_D \nabla \cdot \mathbf{F}\,dV = \iint_S \mathbf{F} \cdot \mathbf{n}\,dS$

FORMULAS FROM VECTOR CALCULUS

Assume $\mathbf{F}(x, y, z) = f(x, y, z)\,\mathbf{i} + g(x, y, z)\,\mathbf{j} + h(x, y, z)\,\mathbf{k}$, where f, g, and h are differentiable on a region D of $\mathbb{R}^3$.

Gradient: $\nabla f(x, y, z) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

Divergence: $\nabla \cdot \mathbf{F}(x, y, z) = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$

Curl: $\nabla \times \mathbf{F}(x, y, z) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f & g & h \end{vmatrix}$

$\nabla \times (\nabla f) = \mathbf{0}$ $\quad$ $\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\mathbf{F}$ conservative on $D \Leftrightarrow \mathbf{F} = \nabla \varphi$ for some potential function φ

$\Leftrightarrow \oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ over closed paths C in D

$\Leftrightarrow \int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path for C in D

$\Leftrightarrow \nabla \times \mathbf{F} = \mathbf{0}$ on D