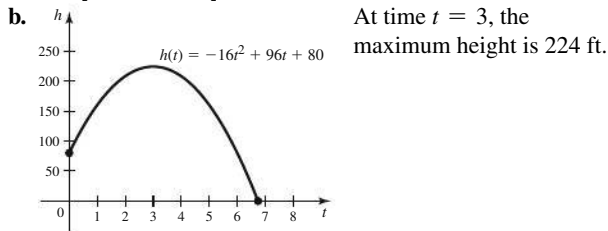


Answers

CHAPTER 1

Section 1.1 Exercises, pp. 9–13

1. A function is a rule that assigns to each value of the independent variable in the domain a unique value of the dependent variable in the range. 3. B 5. The first statement 7. $D = \mathbb{R}$, $R = [-10, \infty)$ 9. The independent variable is h ; the dependent variable is V ; $D = [0, 50]$. 11. -3 ; $1/8$; $1/(2x)$ 13. The domain of $f \circ g$ consists of all x in the domain of g such that $g(x)$ is in the domain of f . 15. a. 4 b. 1 c. 3 d. 3 e. 8 f. 1 17. 15.4 ft/s; radiosonde rises at an average rate of 15.4 ft/s during the first 5 seconds of its flight. 19. 2; 2; 2; -2 21. A is even, B is odd, and C is even. 23. $D = \{x: x \neq 2\}$; $R = \{y: y \neq -1\}$ 25. $D = [-\sqrt{7}, \sqrt{7}]$; $R = [0, \sqrt{7}]$ 27. $D = \mathbb{R}$ 29. $D = [-3, 3]$ 31. a. $[0, 3 + \sqrt{14}]$

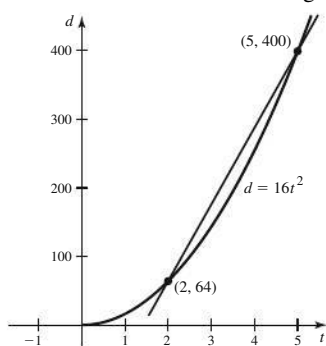


33. $1/z^3$ 35. $1/(y^3 - 3)$ 37. $(u^2 - 4)^3$ 39. $\frac{x-3}{10-3x}$ 41. x 43. $g(x) = x^3 - 5$, $f(x) = x^{10}$ 45. $g(x) = x^4 + 2$, $f(x) = \sqrt{x}$ 47. $|x^2 - 4|$; $D = \mathbb{R}$ 49. $\frac{1}{|x-2|}$; $D = \{x: x \neq 2\}$ 51. $\frac{1}{x^2 - 6}$; $D = \{x: x \neq \sqrt{6}, -\sqrt{6}\}$ 53. $x^4 - 8x^2 + 12$; $D = \mathbb{R}$ 55. $f(x) = x - 3$ 57. $f(x) = x^2$ 59. $f(x) = x^2$ 61. a. True b. False c. True d. False e. False f. True g. True h. False i. True 63. 3 65. $2x + h$ 67. $-\frac{2}{x(x+h)}$ 69. $x + a + 1$ 71. $x^2 + ax + a^2 - 2$ 73. $\frac{4(x+a)}{a^2x^2}$ 75. a. 864 ft/hr;

the hiker's elevation increases at an average rate of 864 ft/hr.

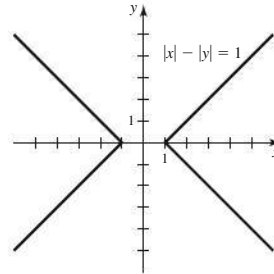
- b. -487 ft/hr; the hiker's elevation decreases at an average rate of 487 ft/hr. c. The hiker might have stopped to rest during this interval of time and/or the trail was level during this portion of the hike.

77. a.



- b. $m_{\text{sec}} = 112$ ft/s; the object falls at an average rate of 112 ft/s.

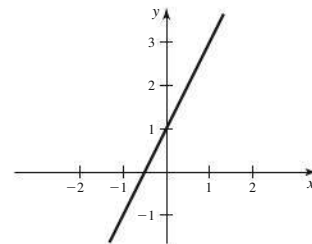
79. y-axis 81. No symmetry 83. x-axis, y-axis, origin 85. Origin 87. a. 4 b. 1 c. 3 d. -2 e. -1 f. 7 89.



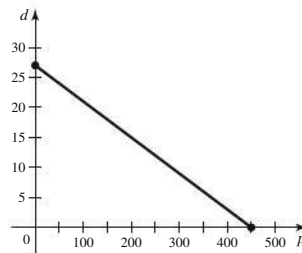
91. The equation $y = 2 - \sqrt{-x^2 + 6x + 16}$ can be rewritten as $(x - 3)^2 + (y - 2)^2 = 5^2$. Because $y \leq 2$, the function is the lower half of a circle of radius 5 centered at $(3, 2)$. $D = [-2, 8]$; $R = [-3, 2]$ 93. $f(x) = 3x - 2$ or $f(x) = -3x + 4$ 95. $f(x) = x^2 - 6$ 97. $\frac{1}{\sqrt{x+h} + \sqrt{x}}$; $\frac{1}{\sqrt{x} + \sqrt{a}}$ 99. $\frac{3}{\sqrt{x}(x+h) + x\sqrt{x+h}}$; $\frac{3}{x\sqrt{a} + a\sqrt{x}}$ 101. None 103. y-axis

Section 1.2 Exercises, pp. 22–27

1. A formula, a graph, a table, words 3. $y = -\frac{2}{3}x - 1$ 5. The set of all real numbers for which the denominator does not equal 0 7. $y = \begin{cases} x + 3 & \text{if } x < 0 \\ -\frac{1}{2}x + 3 & \text{if } x \geq 0 \end{cases}$ 9. Shift the graph to the left 2 units. 11. Compress the graph horizontally by a factor of $\frac{1}{3}$. 13. $f(x) = |x - 2| + 3$; $g(x) = -|x + 2| - 1$ 15. $f(x) = 2x + 1$

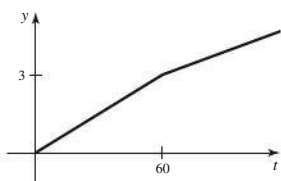


17. $f(x) = 3x - 7$ 19. $C_s = 5.71$; 856.5 million 21. $d = -3p/50 + 27$; $D = [0, 450]$

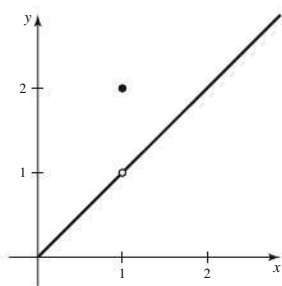


23. a. $p(t) = 328.3t + 1875$ b. 4830 25. $f(x) = \begin{cases} 3 & \text{if } x \leq 3 \\ 2x - 3 & \text{if } x > 3 \end{cases}$

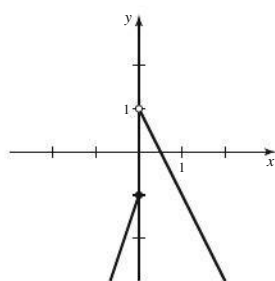
$$27. c(t) = \begin{cases} 0.05t & \text{if } 0 \leq t \leq 60 \\ 1.2 + 0.03t & \text{if } 60 < t \leq 120 \end{cases}$$



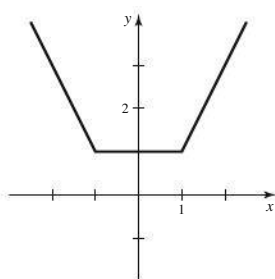
29.



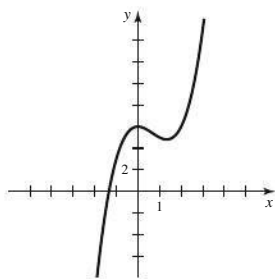
31.



33.

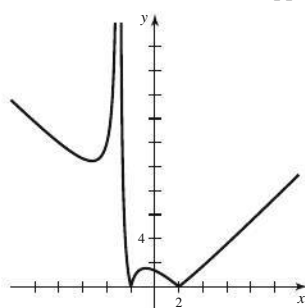


35. a.



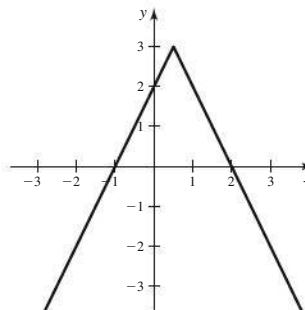
b. $D = \mathbb{R}$ c. One peak near $x = 0$; one valley near $x = 4/3$; x -intercept approx. $(-1.3, 0)$, y -intercept $(0, 6)$

37. a.



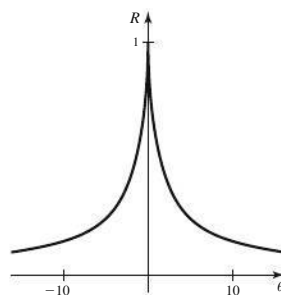
b. $D = \{x: x \neq -3\}$ c. Undefined at $x = -3$; a valley near $x = -5.2$; x -intercepts (and valleys) at $(-2, 0)$ and $(2, 0)$; a peak near $x = -0.8$; y -intercept $(0, \frac{4}{3})$

39. a.



b. $D = \mathbb{R}$ c. One peak at $x = \frac{1}{2}$; x -intercepts $(-1, 0)$ and $(2, 0)$; y -intercept $(0, 2)$ 41. a. A, D, F, I
b. E c. B, H d. I e. A

43. a.



b. $\theta = 0$; vision is sharpest when we look straight ahead.

c. $|\theta| \leq 0.19^\circ$ (less than $\frac{1}{5}$ of a degree) 45. $S(x) = 2$

$$47. S(x) = \begin{cases} 1 & \text{if } x < 0 \\ -\frac{1}{2} & \text{if } x > 0 \end{cases}$$

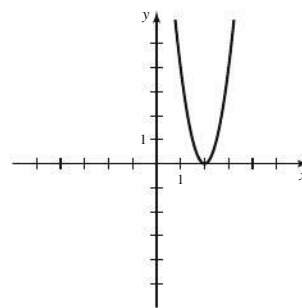
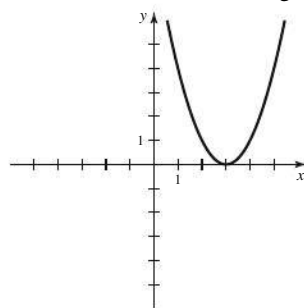
49. a. 12 b. 36 c. $A(x) = 6x$ 51. a. 12 b. 21

$$c. A(x) = \begin{cases} 8x - x^2 & \text{if } 0 \leq x \leq 3 \\ 2x + 9 & \text{if } x > 3 \end{cases}$$

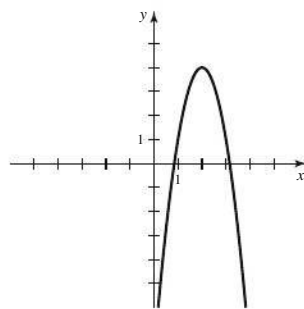
53. a. True b. False c. True d. False

55. a. Shift 3 units to the right.

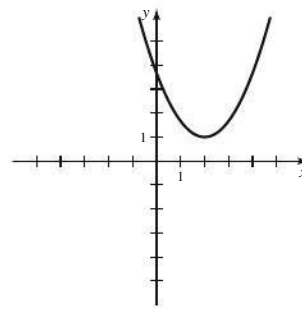
b. Horizontal compression by a factor of $\frac{1}{2}$, then shift 2 units to the right.



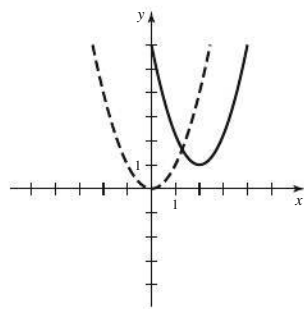
c. Shift to the right 2 units, vertically stretch by a factor of 3, reflect across the x -axis, and shift up 4 units.



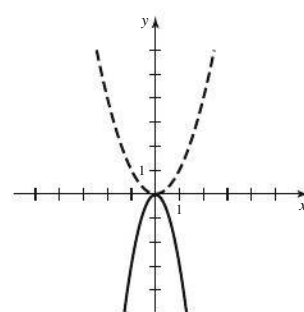
d. Horizontal stretch by a factor of 3, horizontal shift right 2 units, vertical stretch by a factor of 6, and vertical shift up 1 unit.



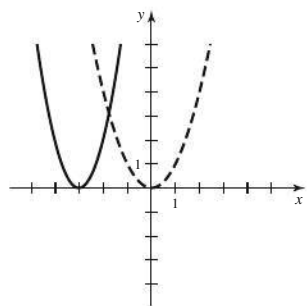
57. Shift the graph of $y = x^2$ right 2 units and up 1 unit.



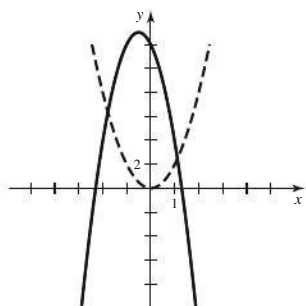
59. Stretch the graph of $y = x^2$ vertically by a factor of 3 and reflect across the x -axis.



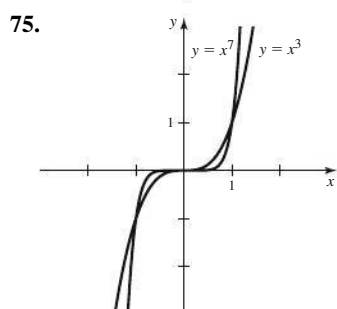
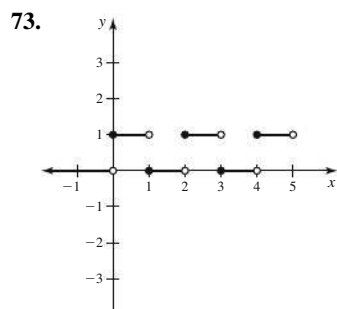
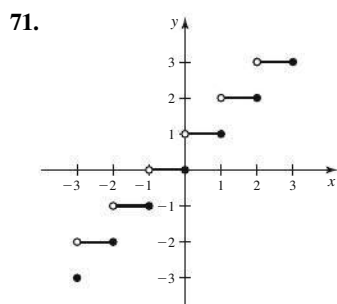
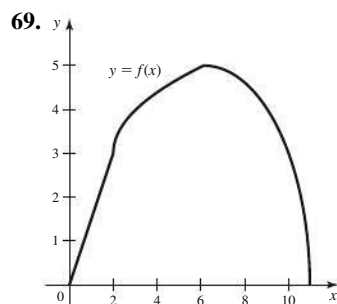
61. Shift the graph of $y = x^2$ left 3 units and stretch vertically by a factor of 2.



63. Shift the graph of $y = x^2$ to the left $\frac{1}{2}$ unit, stretch vertically by a factor of 4, reflect across the x -axis, and then shift up 13 units.

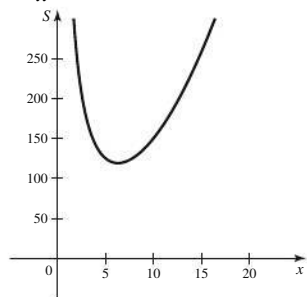


65. $(0, 0); (2, 8)$ 67. $(0, 0); (4, 16)$



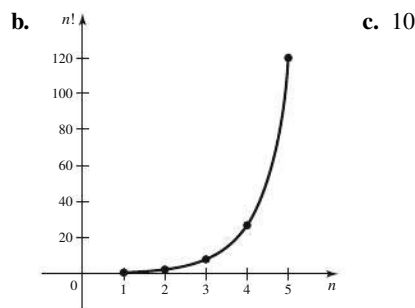
77. a. 0.9; 90% chance that server will win from deuce given that such servers win 75% of their service points b. 0.1; 10% chance that server will win from deuce given that such servers win 25% of their service points 79. a. $f(m) = 350m + 1200$ b. Buy

81. a. $S(x) = x^2 + \frac{500}{x}$ b. Approximately 6.3 ft



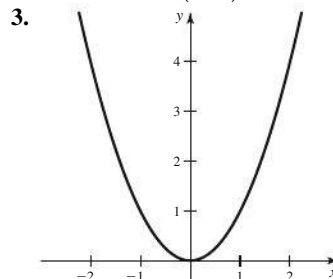
85. a.

n	1	2	3	4	5
$n!$	1	2	6	24	120



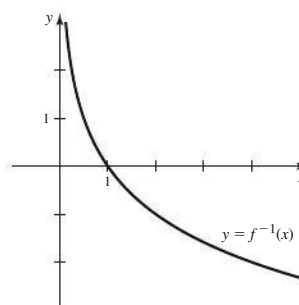
Section 1.3 Exercises, pp. 35–39

1. $D = \mathbb{R}; R = (0, \infty)$



5. $(-\infty, -1], [-1, 1], [1, \infty)$ 7. If a function f is not one-to-one, then there are domain values, x_1 and x_2 , such that $x_1 \neq x_2$ but $f(x_1) = f(x_2)$. If f^{-1} exists, then by definition, $f^{-1}(f(x_1)) = x_1$ and $f^{-1}(f(x_2)) = x_2$, so f^{-1} assigns two different range values to the single domain value of $f(x_1)$.

9. $f^{-1}(x) = \frac{1}{2}x$ 11.



13. $g_1(x) = x^2 + 1; D = [0, \infty); R = [1, \infty);$

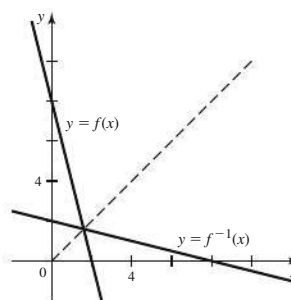
$g_1^{-1}(x) = \sqrt{x - 1}; D = [1, \infty); R = [0, \infty)$

15. The expression $\log_b x$ represents the power to which b must be raised to obtain x . 17. $D = (0, \infty); R = \mathbb{R}$

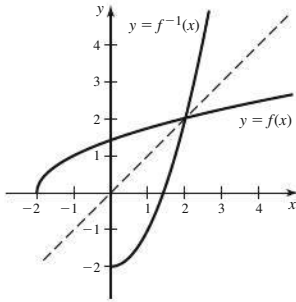
19. a. 3 b. 4 c. -2 d. 3 e. $1/2$ 21. $(-\infty, \infty)$

23. $(-\infty, 5) \cup (5, \infty)$ 25. $(-\infty, 0), (0, \infty)$

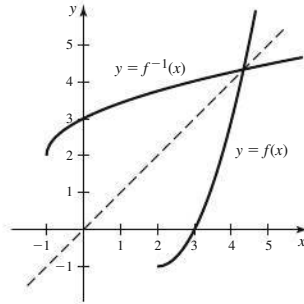
27. $f^{-1}(x) = -\frac{1}{4}x + 2$



29. $f^{-1}(x) = x^2 - 2$



31. $f^{-1}(x) = 2 + \sqrt{x+1}$



33. $f^{-1}(x) = \sqrt{\frac{2}{x}} - 1$ 35. $f^{-1}(x) = \frac{1}{2} \ln x - 3$

37. $f^{-1}(x) = \frac{e^x - 1}{3}$ 39. $f^{-1}(x) = -\frac{1}{2} \log_{10} x$

41. $f^{-1}(x) = \ln\left(\frac{2x}{1-x}\right)$

43. a. $f_1(x) = \sqrt{1-x^2}$; $0 \leq x \leq 1$
 $f_2(x) = \sqrt{1-x^2}$; $-1 \leq x \leq 0$
 $f_3(x) = -\sqrt{1-x^2}$; $-1 \leq x \leq 0$
 $f_4(x) = -\sqrt{1-x^2}$; $0 \leq x \leq 1$

b. $f_1^{-1}(x) = \sqrt{1-x^2}$; $0 \leq x \leq 1$
 $f_2^{-1}(x) = -\sqrt{1-x^2}$; $0 \leq x \leq 1$
 $f_3^{-1}(x) = -\sqrt{1-x^2}$; $-1 \leq x \leq 0$
 $f_4^{-1}(x) = \sqrt{1-x^2}$; $-1 \leq x \leq 0$

45. -0.2 47. 1.19 49. -0.096 51. 1000 53. 2 55. $1/e$

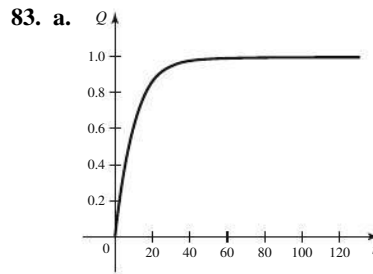
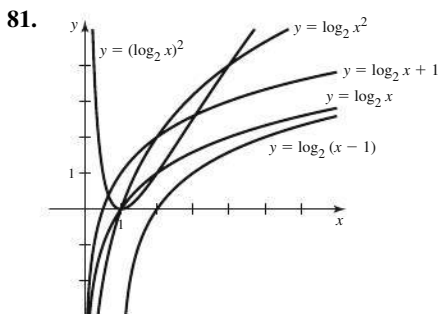
57. $\ln 21 / \ln 7$ 59. $\ln 5 / (3 \ln 3) + 5/3$ 61. 451 years

63. 9.53 years 65. a. No b. $f^{-1}(h) = 2 - \frac{1}{4} \sqrt{64-h}$

c. $f^{-1}(h) = 2 + \frac{1}{4} \sqrt{64-h}$ d. 0.542 s e. 3.837 s

67. $\frac{\ln 15}{\ln 2} \approx 3.9069$ 69. $\frac{\ln 40}{\ln 4} \approx 2.6610$ 71. $e^{x \ln 2}$

73. $\log_5 |x| / \log_5 e$ 75. e 77. a. False b. False c. False
d. True e. False f. False g. True 79. A is $y = \log_2 x$; B is
 $y = \log_4 x$; C is $y = \log_{10} x$.

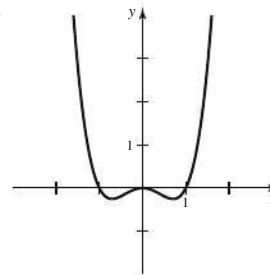


b. Vertical scaling; steady state equals a . c. Horizontal scaling; steady state remains constant. d. a

85. $f^{-1}(x) = \sqrt{x-5} + 1$, $x \geq 5$ 87. $f^{-1}(x) = \sqrt[3]{x} - 1$, $D = \mathbb{R}$

89. $f_1^{-1}(x) = \sqrt{2/x-2}$, $D_1 = (0, 1]$; $f_2^{-1}(x) = -\sqrt{2/x-2}$, $D_2 = (0, 1]$

95. a. f is one-to-one on the intervals $(-\infty, -1/\sqrt{2}]$, $[-1/\sqrt{2}, 0]$, $[0, 1/\sqrt{2}]$, and $[1/\sqrt{2}, \infty)$.



b. $x = \sqrt{\frac{1 \pm \sqrt{4y+1}}{2}}$, $-\sqrt{\frac{1 \pm \sqrt{4y+1}}{2}}$

Section 1.4 Exercises, pp. 48–51

1. $\sin \theta = \text{opp/hyp}$; $\cos \theta = \text{adj/hyp}$; $\tan \theta = \text{opp/adj}$;
 $\cot \theta = \text{adj/opp}$; $\sec \theta = \text{hyp/adj}$; $\csc \theta = \text{hyp/opp}$ 3. 3 s

5. The radian measure of an angle θ is the length s of an arc on the unit circle associated with θ . 7. $\sin^2 \theta + \cos^2 \theta = 1$,
 $1 + \cot^2 \theta = \csc^2 \theta$, $\tan^2 \theta + 1 = \sec^2 \theta$ 9. $\theta = 3\pi/2$

11. $\{x: x \text{ is an odd multiple of } \pi/2\}$

13. Sine is not one-to-one on its domain. 15. $3\pi/4$

17. Horizontal asymptotes at $y = \pi/2$ and $y = -\pi/2$

19. $-1/2$ 21. 1 23. $-1/\sqrt{3}$ 25. $1/\sqrt{3}$ 27. 1 29. -1

31. Undefined 33. $\frac{\sqrt{2+\sqrt{3}}}{2}$ or $\frac{\sqrt{6}+\sqrt{2}}{4}$

35. $\pi/4 + n\pi$, $n = 0, \pm 1, \pm 2, \dots$

37. $\pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

39. $\pi/4 + 2n\pi, 3\pi/4 + 2n\pi$, $n = 0, \pm 1, \pm 2, \dots$

41. $0, \pi/2, \pi, 3\pi/2$

43. $\pi/12, 5\pi/12, 3\pi/4, 13\pi/12, 17\pi/12, 7\pi/4$

45. 0.1007; 1.4701 47. 17.3° ; 72.7° 49. $\pi/2$ 51. $-\pi/6$

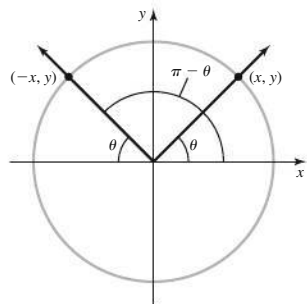
53. $\pi/3$ 55. $2\pi/3$ 57. -1 59. $\sin \theta = \frac{12}{13}$; $\tan \theta = \frac{12}{5}$

61. $\sqrt{1-x^2}$ 63. $\frac{\sqrt{4-x^2}}{2}$ 65. $2x\sqrt{1-x^2}$

67. $\sec \theta = \frac{r}{x} = \frac{1}{x/r} = \frac{1}{\cos \theta}$

69. Dividing both sides of $\cos^2 \theta + \sin^2 \theta = 1$ by $\cos^2 \theta$ gives
 $1 + \tan^2 \theta = \sec^2 \theta$. 71. Because $\cos(\pi/2 - \theta) = \sin \theta$,
for all θ , $1/\cos(\pi/2 - \theta) = 1/\sin \theta$, excluding integer
multiples of π , and $\sec(\pi/2 - \theta) = \csc \theta$.

73. $\cos^{-1} x + \cos^{-1}(-x) = \theta + (\pi - \theta) = \pi$



75. $\pi/3$ 77. $\pi/3$ 79. $\pi/4$ 81. $\pi/2 - 2$

83. $\frac{1}{\sqrt{x^2 + 1}}$ 85. $1/x$ 87. $x/\sqrt{x^2 + 16}$

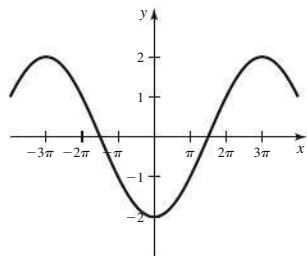
89. $\theta = \sin^{-1} \frac{x}{6} = \tan^{-1} \left(\frac{x}{\sqrt{36 - x^2}} \right) = \sec^{-1} \left(\frac{6}{\sqrt{36 - x^2}} \right)$

91. a. False b. False c. False d. False e. True f. False
g. True h. False 93. $\sin \theta = \frac{12}{13}$; $\tan \theta = \frac{12}{5}$; $\sec \theta = \frac{13}{5}$;
 $\csc \theta = \frac{13}{12}$; $\cot \theta = \frac{5}{12}$ 95. $\sin \theta = \frac{12}{13}$; $\cos \theta = \frac{5}{13}$; $\tan \theta = \frac{12}{5}$;
 $\sec \theta = \frac{13}{5}$; $\cot \theta = \frac{5}{12}$ 97. Amp = 3; period = 6π

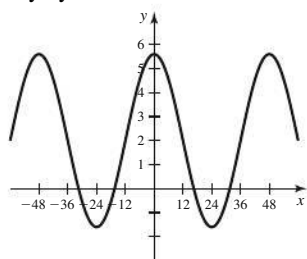
99. Amp = 3.6; period = 48 103. Area of circle is πr^2 ; $\theta/(2\pi)$ represents the proportion of area swept out by a central angle θ .

Therefore, the area of such a sector is $(\theta/2\pi)\pi r^2 = r^2\theta/2$.

105. Stretch the graph of $y = \cos x$ horizontally by a factor of 3, stretch vertically by a factor of 2, and reflect across the x -axis.



107. Stretch the graph of $y = \cos x$ horizontally by a factor of $24/\pi$; then stretch it vertically by a factor of 3.6 and shift it up 2 units.



109. $y = 3 \sin(\pi x/12 - 3\pi/4) + 13$ 111. About 6 ft

113. $d(t) = 10 \cos(4\pi t/3)$ 115. h

Chapter 1 Review Exercises, pp. 51–55

1. a. True b. False c. False d. True e. False f. False
g. True 3. f is one-to-one but not g .

5. $D = \{w: w \neq 2\}$; $R = \{y: y \neq 5\}$

7. $D = (-\infty, -1] \cup [3, \infty)$; $R = [0, \infty)$ 9. Yes; no 11. 8

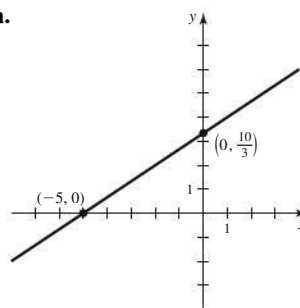
13. 7 15. 8 17. -2 19. a. 1 b. $\sqrt{x^3}$ c. $\sin^3 \sqrt{x}$

d. $\mathbb{R}$ e. $[-1, 1]$ 21. $2x + h - 2$; $x + a - 2$

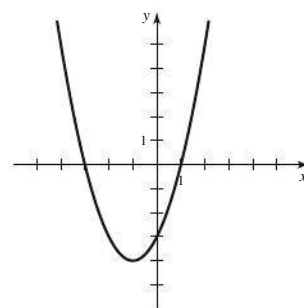
23. $3x^2 + 3xh + h^2$; $x^2 + ax + a^2$ 25. a. $y = \frac{5}{2}x - 8$

b. $y = \frac{3}{4}x + 3$ c. $y = \frac{1}{2}x - 2$ 27. $B = -\frac{1}{500}a + 212$

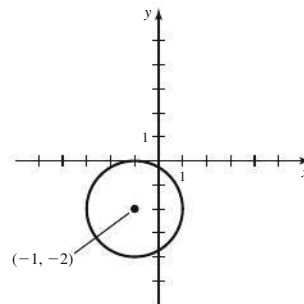
29. a.



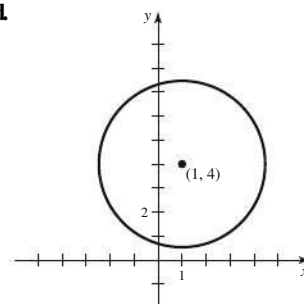
b.



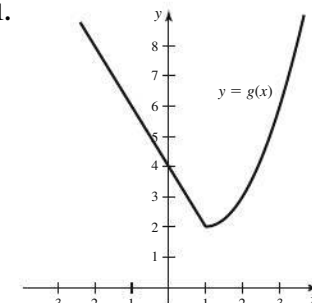
c.



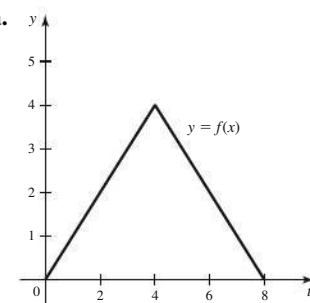
d.



31.

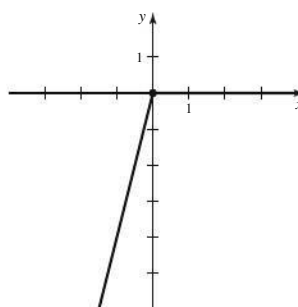


33. a.



b. 2; 14 c. $A(x) = \begin{cases} x^2/2 & \text{if } 0 \leq x \leq 4 \\ -x^2/2 + 8x - 16 & \text{if } 4 < x \leq 8 \end{cases}$

35. $f(x) = \begin{cases} 4x & \text{if } x < 0 \\ 0 & \text{if } x \geq 0 \end{cases}$



37. $D_f = \mathbb{R}$, $R_f = \mathbb{R}$; $D_g = [0, \infty)$, $R_g = [0, \infty)$

39. Shift $y = x^2$ left 3 units and down 12 units.

41. a. y -axis b. y -axis c. x -axis, y -axis, origin 43. $x = 2$

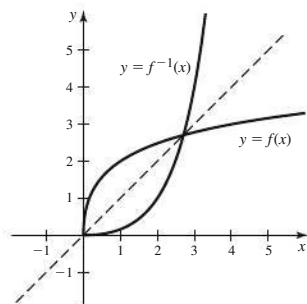
45. $t = \frac{e^4 - 4}{5}$ 47. $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ 49. $\theta = \pm \frac{\pi}{12}, \pm \frac{5\pi}{12}$

51. Approx. 35 years 53. $(-\infty, 0], [0, 2]$, and $[2, \infty)$

55. $f^{-1}(x) = -\frac{1}{4}x + \frac{3}{2}$ 57. $f^{-1}(x) = 2 + \sqrt{x-1}$

59. $f^{-1}(x) = -\sqrt{\frac{x-1}{3}}$ 61. $f^{-1}(x) = \sqrt{\ln x - 1}$

63. $f^{-1}(x) = \frac{4x^2}{(6-x)^2}$, for $0 \leq x < 6$



65. a. $f(t) = -2 \cos \frac{\pi t}{3}$ b. $f(t) = 5 \sin \frac{\pi t}{12} + 15$

67. a. F b. E c. D d. B e. C f. A

69. $(7\pi/6, -1/2)$; $(11\pi/6, -1/2)$ 71. $-\frac{\sqrt{2+\sqrt{2}}}{2}$

73. $\pi/6$ 75. $-\pi/2$ 77. x , provided $-1 \leq x \leq 1$

79. $\cos \theta = \frac{5}{13}$; $\tan \theta = \frac{12}{5}$; $\cot \theta = \frac{5}{12}$; $\sec \theta = \frac{13}{5}$; $\csc \theta = \frac{13}{12}$

81. $\frac{\sqrt{16-x^2}}{4}$ 83. $\pi/2 - \theta$ 85. 0

87. $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$
 $= \frac{2 \sin \theta \cos \theta / \cos^2 \theta}{(\cos^2 \theta - \sin^2 \theta) / \cos^2 \theta} = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

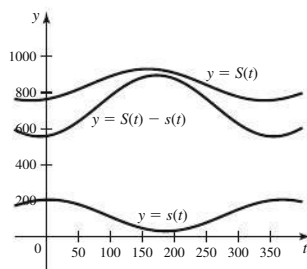
89. a.

n	1	2	3	4	5	6	7	8	9	10
$T(n)$	1	5	14	30	55	91	140	204	285	385

b. $D = \{n: n \text{ is a positive integer}\}$ c. 14

91. $s(t) = 117.5 - 87.5 \sin\left(\frac{\pi}{182.5}(t - 95)\right)$

$S(t) = 844.5 + 87.5 \sin\left(\frac{\pi}{182.5}(t - 67)\right)$



CHAPTER 2

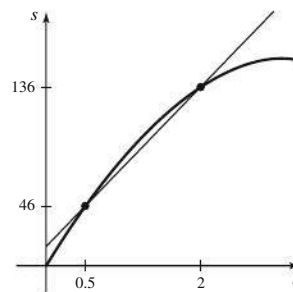
Section 2.1 Exercises, pp. 61–62

1. $\frac{s(b) - s(a)}{b - a}$ 3. 20 5. a. 36 b. 44 c. 52 d. 60

7. 47.84, 47.984, 47.9984; instantaneous velocity appears to be 48

9. $\frac{f(b) - f(a)}{b - a}$ 11. The instantaneous velocity at $t = a$ is the slope of the line tangent to the position curve at $t = a$. 13. a. 48

b. 64 c. 80 d. $16(6 - h)$ 15. $m_{\text{sec}} = 60$; the slope is the average velocity of the object over the interval $[0.5, 2]$.



17.

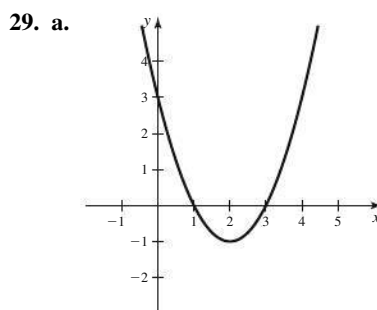
Time interval	Average velocity
$[1, 2]$	80
$[1, 1.5]$	88
$[1, 1.1]$	94.4
$[1, 1.01]$	95.84
$[1, 1.001]$	95.984
$v_{\text{inst}} = 96$	

21.

Time interval	Average velocity
$[3, 3.5]$	-24
$[3, 3.1]$	-17.6
$[3, 3.01]$	-16.16
$[3, 3.001]$	-16.016
$[3, 3.0001]$	-16.002
$v_{\text{inst}} = -16$	

25.

Interval	Slope of secant line
$[1, 2]$	6
$[1.5, 2]$	7
$[1.9, 2]$	7.8
$[1.99, 2]$	7.98
$[1.999, 2]$	7.998
$m_{\text{tan}} = 8$	



c.

Interval	Slope of secant line
$[2, 2.5]$	0.5
$[2, 2.1]$	0.1
$[2, 2.01]$	0.01
$[2, 2.001]$	0.001
$[2, 2.0001]$	0.0001
$m_{\text{tan}} = 0$	

19.

Time interval	Average velocity
$[2, 3]$	20
$[2.9, 3]$	5.60
$[2.99, 3]$	4.16
$[2.999, 3]$	4.016
$[2.9999, 3]$	4.002
$v_{\text{inst}} = 4$	

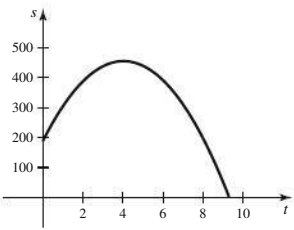
23.

Time interval	Average velocity
$[0, 1]$	36.372
$[0, 0.5]$	67.318
$[0, 0.1]$	79.468
$[0, 0.01]$	79.995
$[0, 0.001]$	80.000
$v_{\text{inst}} = 80$	

27.

Interval	Slope of secant line
$[0, 1]$	1.718
$[0, 0.5]$	1.297
$[0, 0.1]$	1.052
$[0, 0.01]$	1.005
$[0, 0.001]$	1.001
$m_{\text{tan}} = 1$	

b. $(2, -1)$

31. a.  b. $t = 4$

c.

Interval	Average velocity
$[4, 4.5]$	-8
$[4, 4.1]$	-1.6
$[4, 4.01]$	-0.16
$[4, 4.001]$	-0.016
$[4, 4.0001]$	-0.0016
$v_{\text{inst}} = 0$	

- d. $0 \leq t < 4$ e. $4 < t \leq 9$ 33. 0.6366, 0.9589, 0.9996, 1

Section 2.2 Exercises, pp. 67–71

1. As x approaches a from either side, the values of $f(x)$ approach L .

3. a. 5 b. 3 c. Does not exist d. 1 e. 2

5. a. -1 b. 1 c. 2 d. 2

7. a.

x	$f(x)$	x	$f(x)$
1.9	3.9	2.1	4.1
1.99	3.99	2.01	4.01
1.999	3.999	2.001	4.001
1.9999	3.9999	2.0001	4.0001

 b. 4

9. a.

t	$g(t)$	t	$g(t)$
8.9	5.983287	9.1	6.016621
8.99	5.998333	9.01	6.001666
8.999	5.999833	9.001	6.000167

 b. 6

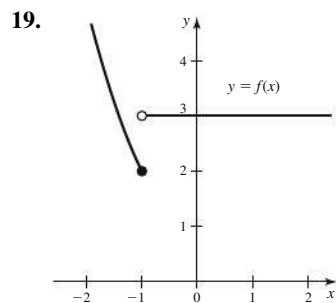
11. As x approaches a from the right, the values of $f(x)$ approach L .

13. $L = M$ 15. a. 0 b. 1 c. 0

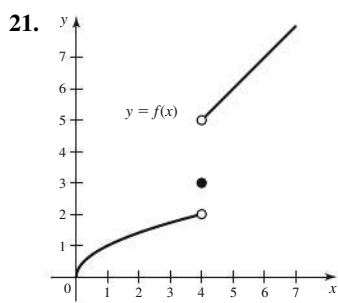
- d. Does not exist; $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$ 17. a. 3 b. 2

- c. 2 d. 2 e. 2 f. 4 g. 1 h. Does not exist

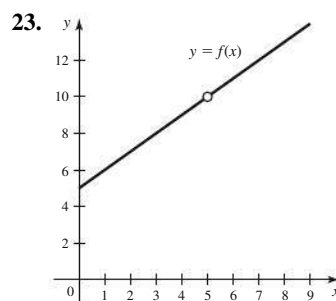
- i. 3 j. 3 k. 3 l. 3



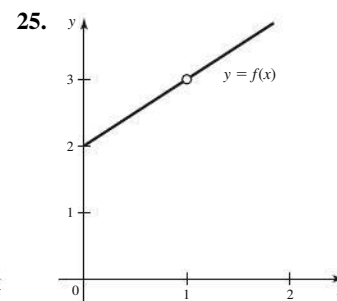
2; 2; 3; does not exist



3; 2; 5; does not exist

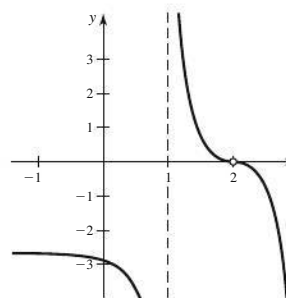


undefined; 10; 10; 10



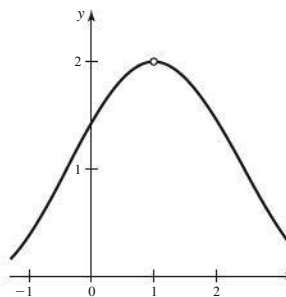
undefined; 3; 3; 3

27. From the graph and table, the limit appears to be 0.



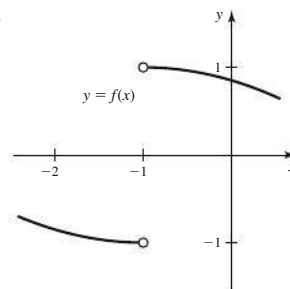
x	1.99	1.999	1.9999
$f(x)$	0.0021715	0.00014476	0.000010857
x	2.0001	2.001	2.01
$f(x)$	-0.000010857	-0.00014476	-0.0021715

29. From the graph and table, the limit appears to be 2.



x	0.9	0.99	0.999
$f(x)$	1.993342	1.999933	1.999999
x	1.001	1.01	1.1
$f(x)$	1.999999	1.999933	1.993342

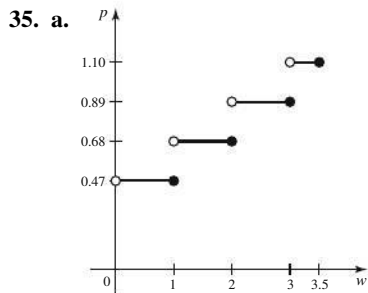
- 31.



x	-1.1	-1.01	-1.001
$g(x)$	-0.9983342	-0.9999833	-0.9999998
x	-0.999	-0.99	-0.9
$g(x)$	0.9999998	0.9999833	0.9983342

From the table and the graph, it appears that the limit does not exist.

33. a. False b. False c. False d. False e. True



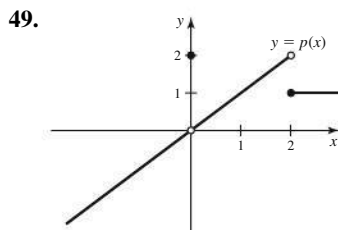
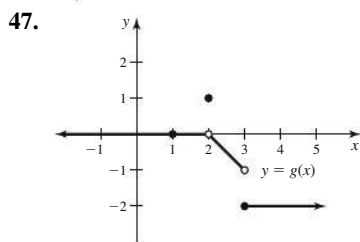
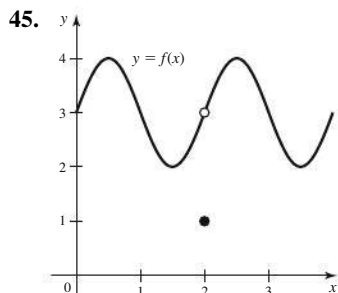
b. 0.89 c. Because $\lim_{w \rightarrow 3^-} f(w) = 0.89$ and $\lim_{w \rightarrow 3^+} f(w) = 1.1$, we know that $\lim_{w \rightarrow 3^-} f(w) \neq \lim_{w \rightarrow 3^+} f(w)$. So $\lim_{w \rightarrow 3} f(w)$ does not exist.

37. 3 39. 16 41. 1

43. a. The function values alternate between 1 and -1 .

x	$\sin(1/x)$
$2/\pi$	1
$2/(3\pi)$	-1
$2/(5\pi)$	1
$2/(7\pi)$	-1
$2/(9\pi)$	1
$2/(11\pi)$	-1

b. The function values alternate between 1 and -1 infinitely many times on the interval $(0, h)$ no matter how small $h > 0$ becomes. c. Does not exist



51. a. $-2, -1, 1, 2$ b. $2, 2, 2$

c. $\lim_{x \rightarrow a^-} \lfloor x \rfloor = a - 1$ and $\lim_{x \rightarrow a^+} \lfloor x \rfloor = a$, if a is an integer

d. $\lim_{x \rightarrow a^-} \lfloor x \rfloor = \lfloor a \rfloor$ and $\lim_{x \rightarrow a^+} \lfloor x \rfloor = \lfloor a \rfloor$, if a is not an integer

e. Limit exists provided a is not an integer

53. a. 8 b. 5 55. a. $2; 3; 4$ b. p 57. p/q

Section 2.3 Exercises, pp. 79–82

1. $\lim_{x \rightarrow a} p(x) = p(a)$ 3. Those values of a for which the denominator

is not zero 5. $\frac{x^2 - 7x + 12}{x - 3} = x - 4$, for $x \neq 3; -1$

7. 32; Constant Multiple Law 9. 5; Difference Law

11. 8; Quotient and Difference Laws 13. 4; Root and Power

Laws 15. 5; 1 17. 20 19. 5 21. -45 23. 8 25. 3

27. 3 29. -5 31. 8 33. 2 35. -8 37. -1 39. -12 41. $\frac{1}{6}$

43. $-\frac{1}{36}$ 45. $2\sqrt{a}$ 47. $\frac{1}{8}$ 49. $-\frac{1}{16}$ 51. 5 53. 10 55. 2

57. -54 59. 0 61. 1 63. 1 65. $1/2$ 67. Does not exist

69. Does not exist 71. a. False b. False c. False d. False

e. False 73. a. 2 b. 0 c. Does not exist 75. a. 0

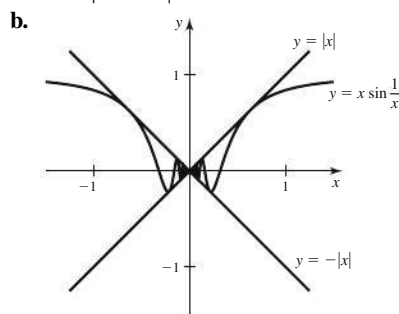
b. $\sqrt{x - 2}$ is undefined for $x < 2$. 77. 0.0435 N/C

79. $\lim_{S \rightarrow 0^+} r(S) = 0$; the radius of the cylinder approaches 0 as the surface area of the cylinder approaches 0.

81. a. Because $\left| \sin \frac{1}{x} \right| \leq 1$, for all $x \neq 0$, we have that

$$|x| \left| \sin \frac{1}{x} \right| \leq |x|.$$

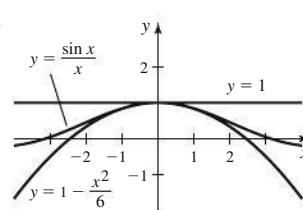
That is, $|x \sin \frac{1}{x}| \leq |x|$, so $-|x| \leq x \sin \frac{1}{x} \leq |x|$, for all $x \neq 0$.



c. $\lim_{x \rightarrow 0} -|x| = 0$ and $\lim_{x \rightarrow 0} |x| = 0$; by part (a) and the Squeeze

Theorem, $\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$.

83. a.

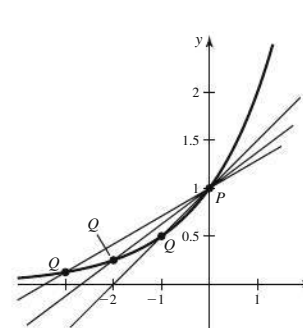


b. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

85. Because $\lim_{x \rightarrow 0^-} |x| = \lim_{x \rightarrow 0^-} (-x) = 0$ and $\lim_{x \rightarrow 0^+} |x| = \lim_{x \rightarrow 0^+} x = 0$, we know that $\lim_{x \rightarrow 0} |x| = 0$. 87. 1 89. $a = -13$; $\lim_{x \rightarrow -1} g(x) = 6$

91. 6 93. $5a^4$

95. a.



b. $\frac{2^x - 1}{x}$

c.

x	$\frac{2^x - 1}{x}$
-1	0.5
-0.1	0.6697
-0.01	0.6908
-0.001	0.6929
-0.0001	0.6931
-0.00001	0.6931
Limit ≈ 0.693	

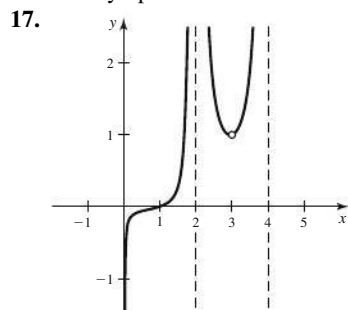
97. 6; 5 99. $\frac{1}{3}$ 101. $f(x) = x - 1, g(x) = \frac{5}{x - 1}$
 103. $b = 2$ and $c = -8$; yes 105. 6; 4

Section 2.4 Exercises, pp. 88–91

1. As x approaches a from the right, the values of $f(x)$ are negative and become arbitrarily large in magnitude.
 3. A vertical asymptote for a function f is a vertical line $x = a$, where one (or more) of the following is true:

$$\lim_{x \rightarrow a^-} f(x) = \pm \infty; \lim_{x \rightarrow a^+} f(x) = \pm \infty.$$

5. ∞ 7. a. ∞ b. ∞ c. ∞ d. ∞ e. $-\infty$ f. Does not exist
 9. a. $-\infty$ b. $-\infty$ c. $-\infty$ d. ∞ e. $-\infty$ f. Does not exist
 11. a. ∞ b. $-\infty$ c. $-\infty$ d. ∞ 13. $-\infty$ 15. No; there is a vertical asymptote at $x = 2$ but not at $x = 1$.



19. a and b are correct. 21. a. ∞ b. $-\infty$ c. Does not exist
 23. a. $-\infty$ b. $-\infty$ c. $-\infty$ 25. a. ∞ b. $-\infty$ c. Does not exist
 27. a. $-\infty$ b. $-\infty$ c. $-\infty$ 29. a. ∞ b. Does not exist
 c. Does not exist 31. a. ∞ b. $1/54$ c. Does not exist
 33. -5 35. ∞ 37. $-\infty$ 39. ∞ 41. $-\infty$ 43. ∞
 45. a. $1/10$ b. $-\infty$ c. ∞ ; vertical asymptote: $x = -5$
 47. $x = 3$; $\lim_{x \rightarrow 3^+} f(x) = -\infty$; $\lim_{x \rightarrow 3^-} f(x) = \infty$; $\lim_{x \rightarrow 3} f(x)$ does not exist 49. $x = 0$ and $x = 2$; $\lim_{x \rightarrow 0^+} f(x) = \infty$;
 $\lim_{x \rightarrow 0^-} f(x) = -\infty$; $\lim_{x \rightarrow 0} f(x)$ does not exist; $\lim_{x \rightarrow 2^-} f(x) = \infty$;
 $\lim_{x \rightarrow 2^+} f(x) = \infty$; $\lim_{x \rightarrow 2} f(x) = \infty$

51. a. $-\infty$ b. ∞ c. $-\infty$ d. ∞ 53. a. False b. True
 c. False 55. $r(x) = \frac{(x-1)^2}{(x-1)(x-2)^2}$ 57. $f(x) = \frac{1}{x-6}$

59. $x = 0$ 61. $x = -1$ 63. $\theta = 10k + 5$, for any integer k
 65. $x = 0$ 67. a. $a = 4$ or $a = 3$ b. Either $a > 4$ or $a < 3$

- c. $3 < a < 4$ 69. a. $\frac{1}{\sqrt[3]{h}}$ regardless of the sign of h

- b. $\lim_{h \rightarrow 0^+} \frac{1}{\sqrt[3]{h}} = \infty$; $\lim_{h \rightarrow 0^-} \frac{1}{\sqrt[3]{h}} = -\infty$; the tangent line at $(0, 0)$ is vertical.

Section 2.5 Exercises, pp. 100–102

1. As $x < 0$ becomes arbitrarily large in magnitude, the corresponding values of f approach 10. 3. ∞ 5. 0 7. 0 9. 3 11. 0
 13. ∞ ; 0; 0 15. 3; 3 17. 0 19. 0 21. ∞ 23. $-\infty$
 25. $2/3$ 27. $-\infty$ 29. 5 31. -4 33. $-3/2$ 35. 0

37. $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = \frac{1}{5}$; $y = \frac{1}{5}$

39. $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 2$; $y = 2$

41. $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$; $y = 0$

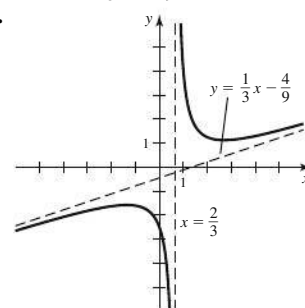
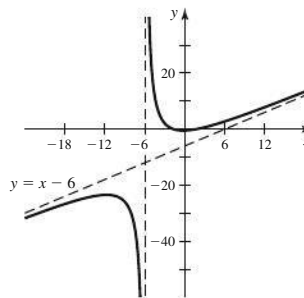
43. $\lim_{x \rightarrow \infty} f(x) = \infty$; $\lim_{x \rightarrow -\infty} f(x) = -\infty$; none

45. $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = \frac{4}{9}$; $y = \frac{4}{9}$

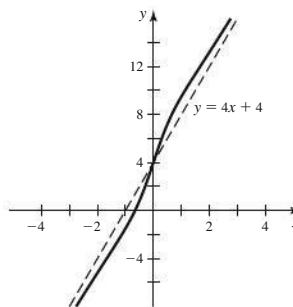
47. $\lim_{x \rightarrow \infty} f(x) = \frac{2}{3}$; $\lim_{x \rightarrow -\infty} f(x) = -2$; $y = \frac{2}{3}$; $y = -2$

49. $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = \frac{1}{4 + \sqrt{3}}$; $y = \frac{1}{4 + \sqrt{3}}$

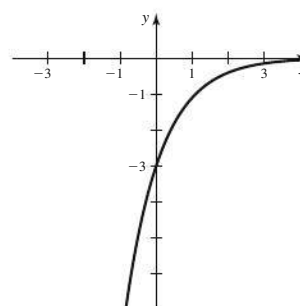
51. a. $y = x - 6$ b. $x = -6$ 53. a. $y = \frac{1}{3}x - \frac{4}{9}$ b. $x = \frac{2}{3}$
 c.



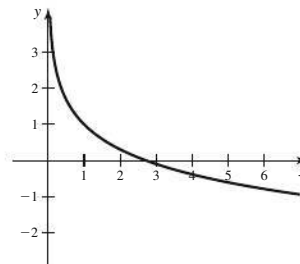
55. a. $y = 4x + 4$ b. No vertical asymptote
 c.



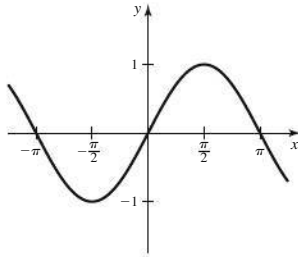
57. $\lim_{x \rightarrow \infty} (-3e^{-x}) = 0$;
 $\lim_{x \rightarrow -\infty} (-3e^{-x}) = -\infty$



59. $\lim_{x \rightarrow \infty} (1 - \ln x) = -\infty$;
 $\lim_{x \rightarrow 0^+} (1 - \ln x) = \infty$



61. $\lim_{x \rightarrow \infty} \sin x$ does not exist; $\lim_{x \rightarrow -\infty} \sin x$ does not exist



63. a. False b. False c. True d. False 65. 3500

67. No steady state 69. 2

71. a. $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 2$; $y = 2$

- b. $x = 0$; $\lim_{x \rightarrow 0^+} f(x) = \infty$; $\lim_{x \rightarrow 0^-} f(x) = -\infty$

73. a. $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 3$; $y = 3$

- b. $x = -3$ and $x = 4$; $\lim_{x \rightarrow -3^-} f(x) = \infty$; $\lim_{x \rightarrow -3^+} f(x) = -\infty$;
 $\lim_{x \rightarrow 4^-} f(x) = -\infty$; $\lim_{x \rightarrow 4^+} f(x) = \infty$

75. a. $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 1$; $y = 1$

- b. $x = 0$; $\lim_{x \rightarrow 0^+} f(x) = \infty$; $\lim_{x \rightarrow 0^-} f(x) = -\infty$

77. a. $\lim_{x \rightarrow \infty} f(x) = 1$; $\lim_{x \rightarrow -\infty} f(x) = -1$; $y = 1$ and $y = -1$

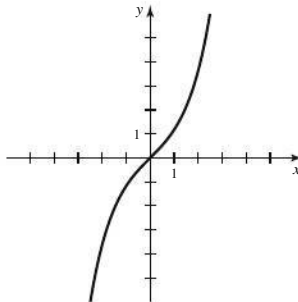
- b. No vertical asymptote

79. a. $\lim_{x \rightarrow \infty} f(x) = 0$; $\lim_{x \rightarrow -\infty} f(x) = 0$; $y = 0$

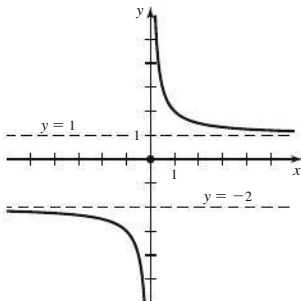
- b. No vertical asymptote 81. a. $\lim_{x \rightarrow \infty} f(x) = 2$; $\lim_{x \rightarrow -\infty} f(x)$ does not exist; $y = 2$ b. $x = 0$; $\lim_{x \rightarrow 0^+} f(x) = \infty$; $\lim_{x \rightarrow 0^-} f(x)$ does not exist

83. a. $\frac{\pi}{2}$ b. $\frac{\pi}{2}$ 85. a. $\lim_{x \rightarrow \infty} \sinh x = \infty$; $\lim_{x \rightarrow -\infty} \sinh x = -\infty$

- b. $\sinh 0 = 0$

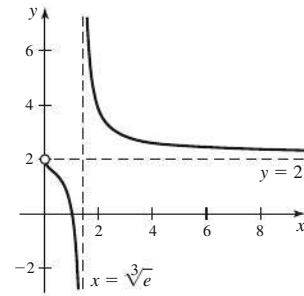


87.



89. 1 91. 0 93. a. No; f has a horizontal asymptote if $m = n$, and it has a slant asymptote if $m = n + 1$. b. Yes;
 $f(x) = x^4/\sqrt{x^6 + 1}$ 95. $y = 3$ and $y = 2$

97. $y = 2$; $x = \sqrt[3]{e}$



Section 2.6 Exercises, pp. 112–115

1. a, c 3. A function is continuous on an interval if it is continuous at each point of the interval. If the interval contains endpoints, then the function must be right- or left-continuous at those points. 5. $a = 1$, item 1; $a = 2$, item 3; $a = 3$, item 2 7. $a = 1$, item 1; $a = 2$, item 2; $a = 3$, item 1 9. a. $\lim_{x \rightarrow a^-} f(x) = f(a)$ b. $\lim_{x \rightarrow a^+} f(x) = f(a)$

11. $(0, 1)$, $(1, 2)$, $(2, 3)$, $(3, 5)$; left-continuous at 3

13. $(0, 1)$, $(1, 2)$, $[2, 3)$, $(3, 5)$; right-continuous at 2

15. $\{x: x \neq 0\}$, $\{x: x \neq 0\}$ 17. No; $f(-5)$ is undefined.

19. No; $f(1)$ is undefined. 21. No; $\lim_{x \rightarrow 1} f(x) = 2$ but $f(1) = 3$.

23. No; $f(4)$ is undefined. 25. $(-\infty, \infty)$

27. $(-\infty, -3)$, $(-3, 3)$, $(3, \infty)$ 29. $(-\infty, -2)$, $(-2, 2)$, $(2, \infty)$

31. 1 33. $2\sqrt{6}$ 35. 16 37. $\ln 2$ 39. a. $\lim_{x \rightarrow 1} f(x)$ does not exist.

- b. Continuous from the right c. $(-\infty, 1)$, $[1, \infty)$ 41. $(-\infty, 5]$;

- left-continuous at 5 43. $(-\infty, -2\sqrt{2})$, $[2\sqrt{2}, \infty)$; left-continuous at $-2\sqrt{2}$; right-continuous at $2\sqrt{2}$ 45. $(-\infty, \infty)$ 47. $(-\infty, \infty)$

49. 3 51. 1 53. 4 55. 2 57. $-\frac{1}{2}$ 59. 4

61. $(n\pi, (n+1)\pi)$, where n is an integer; $\sqrt{2}$; $-\infty$

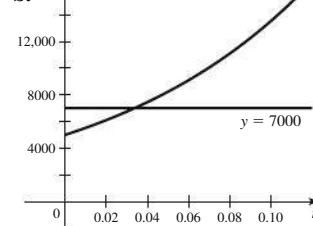
63. $\left(\frac{n\pi}{2}, \left(\frac{n}{2} + 1\right)\frac{\pi}{2}\right)$, where n is an odd integer; ∞ ; $\sqrt{3} - 2$

65. $(-\infty, 0)$, $(0, \infty)$; ∞ ; $-\infty$ 67. b. $x \approx 0.835$

69. b. $x \approx -0.285$; $x \approx 0.778$; $x \approx 4.507$ 71. b. $x \approx -0.567$

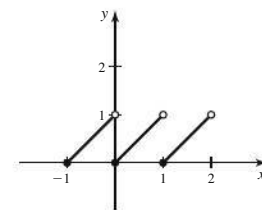
73. a. True b. True c. False d. False 75. a. $A(r)$ is continuous on $[0, 0.08]$, and 7000 is between $A(0) = 5000$ and $A(0.08) = 11,098.20$. By the Intermediate Value Theorem, there is at least one c in $(0, 0.08)$ such that $A(c) = 7000$.

- b. $c \approx 0.034$ or 3.4%

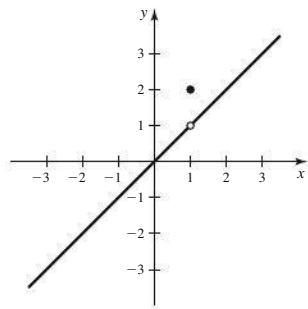
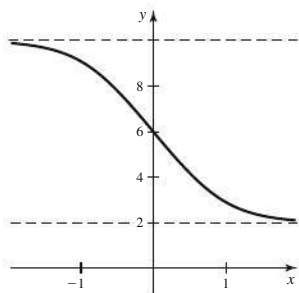


77. $[0, \pi/2]$; 0.45 79. $(-\infty, \infty)$ 81. $[0, 16)$, $(16, \infty)$

83. The vertical line segments should not appear.



85. a, b.

87. a. 2 b. 8 c. No; $\lim_{x \rightarrow 1^-} g(x) = 2$ and $\lim_{x \rightarrow 1^+} g(x) = 8$.89. $\lim_{x \rightarrow 0} f(x) = 6$, $\lim_{x \rightarrow -\infty} f(x) = 10$, and $\lim_{x \rightarrow \infty} f(x) = 2$; no vertical asymptote; $y = 2$ and $y = 10$ are the horizontal asymptotes.91. $x_1 = \frac{1}{7}$; $x_2 = \frac{1}{2}$; $x_3 = \frac{3}{5}$ 93. Yes. Imagine there is a clone of the monk who walks down the path at the same time the monk walks up the path. The monk and his clone must cross paths at some time between dawn and dusk. 95. No; f cannot be made continuous at $x = a$ by redefining $f(a)$. 97. $\lim_{x \rightarrow 2} f(x) = -3$; define $f(2)$ to be -3 .99. $a = 0$ removable discontinuity; $a = 1$ infinite discontinuity101. a. Yes b. No 103. a. For example, $f(x) = 1/(x-1)$, $g(x) = x+1$ b. For continuity, g must be continuous at 0, and f must be continuous at $g(0)$.**Section 2.7 Exercises, pp. 124–128**

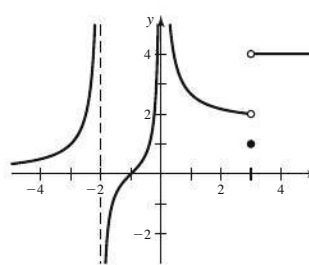
1. 1 3. c 5. Given any $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$. 7. $0 < \delta \leq 2$
 9. a. $\delta = 1$ b. $\delta = \frac{1}{2}$ 11. a. $\delta = 2$ b. $\delta = \frac{1}{2}$
 13. a. $0 < \delta \leq 1$ b. $0 < \delta \leq 0.79$ 15. a. $0 < \delta \leq 1$
 b. $0 < \delta \leq \frac{1}{2}$ c. $0 < \delta \leq \varepsilon$ 17. a. $0 < \delta \leq 1$ b. $0 < \delta \leq \frac{1}{2}$
 c. $0 < \delta \leq \frac{\varepsilon}{2}$ 19. $\delta = \varepsilon/8$ 21. $\delta = \varepsilon$ 23. $\delta = \varepsilon$
 25. $\delta = \varepsilon/3$ 27. $\delta = \sqrt{\varepsilon}$ 29. $\delta = \min\{1, \varepsilon/8\}$ 31. $\delta = \varepsilon/2$
 33. $\delta = \min\{1, 6\varepsilon\}$ 35. $\delta = \min\{1/20, \varepsilon/200\}$
 37. $\delta = \min\{1, \sqrt{\varepsilon/2}\}$ 39. $\delta = \varepsilon/|m|$ if $m \neq 0$; use any $\delta > 0$ if $m = 0$ 41. $\delta = \min\{1, 8\varepsilon/15\}$ 45. $\delta = 1/\sqrt{N}$
 47. $\delta = 1/\sqrt{N-1}$ 49. a. False b. False c. True d. True
 51. For $x > a$, $|x - a| = x - a$. 53. a. $\delta = \varepsilon/2$ b. $\delta = \varepsilon/3$
 c. Because $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = -4$, $\lim_{x \rightarrow 0} f(x) = -4$.
 55. $\delta = \varepsilon^2$ 57. a. For each $N > 0$ there exists $\delta > 0$ such that $f(x) > N$ whenever $0 < x - a < \delta$. b. For each $N < 0$ there exists $\delta > 0$ such that $f(x) < N$ whenever $0 < a - x < \delta$.
 c. For each $N > 0$ there exists $\delta > 0$ such that $f(x) > N$ whenever $0 < a - x < \delta$. 59. $\delta = 1/N$ 61. $\delta = (-10/M)^{1/4}$
 65. $N = 1/\varepsilon$ 67. $N = M - 1$

Chapter 2 Review Exercises, pp. 128–130

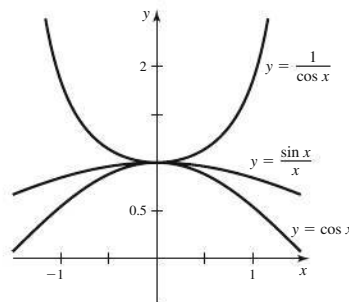
1. a. False b. False c. False d. True e. False f. False
 g. False h. True 3. 12 ft/s 5. $x = -1$; $\lim_{x \rightarrow -1} f(x)$ does not exist; $x = 1$; $\lim_{x \rightarrow 1} f(x) \neq f(1)$; $x = 3$; $f(3)$ is undefined.

7. a. 1.414 b. $\sqrt{2}$

9.



11. $\sqrt{11}$ 13. 13 15. 2 17. $\frac{1}{3}$ 19. $-\frac{1}{16}$ 21. 108 23. $\frac{1}{108}$
 25. 0 27. $-\infty$ 29. ∞ 31. 4 33. $-\infty$ 35. $\frac{1}{2}$ 37. $-3/\sqrt{a}$
 39. $2/(1-a)$ 41. $3\pi/2 + 2$ 43. 1; ∞ 45. $2/3$
 47. $-1/3$; $2/7$ 49. 5 51. $-\infty$
 53. a.



$$\text{b. } \lim_{x \rightarrow 0} \cos x \leq \lim_{x \rightarrow 0} \frac{\sin x}{x} \leq \lim_{x \rightarrow 0} \frac{1}{\cos x};$$

$$1 \leq \lim_{x \rightarrow 0} \frac{\sin x}{x} \leq 1;$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

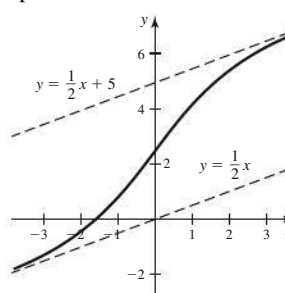
$$55. \lim_{x \rightarrow \infty} f(x) = -4; \lim_{x \rightarrow -\infty} f(x) = -4$$

$$57. \lim_{x \rightarrow \infty} f(x) = 1; \lim_{x \rightarrow -\infty} f(x) = -\infty$$

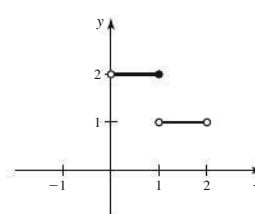
$$59. \lim_{x \rightarrow \infty} f(x) = 2; \lim_{x \rightarrow -\infty} f(x) = 5 \quad 61. \text{ a. } \infty; -\infty$$

b. $y = 3x + 2$ is the slant asymptote.63. a. $-\infty$; ∞ b. $y = -x - 2$ is the slant asymptote.65. a. ∞ , $-\infty$ b. $y = 4x + 5$ is the slant asymptote.67. Horizontal asymptotes at $y = 2/\pi$ and $y = -2/\pi$; vertical asymptote at $x = 0$

69.

71. No; $f(5)$ does not exist. 73. Yes; $h(5) = \lim_{x \rightarrow 5} h(x) = 4$ 75. $(-\infty, -\sqrt{5}]$ and $[\sqrt{5}, \infty)$; left-continuous at $-\sqrt{5}$ and right-continuous at $\sqrt{5}$ 77. $(-\infty, -5)$, $(-5, 0)$, $(0, 5)$, and $(5, \infty)$ 79. $a = 3$, $b = 0$

81.

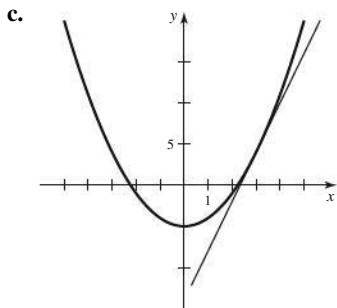


83. a. Let $f(x) = x - \cos x$; $f(0) < 0 < f\left(\frac{\pi}{2}\right)$ b. $x \approx 0.739$
 85. a. $m(0) < 30 < m(5)$ and $m(5) > 30 > m(15)$
 b. $m = 30$ when $t \approx 2.4$ hr and $t \approx 10.8$ hr c. No; the maximum amount is approximately $m(5.5) \approx 38.5$ 87. $\delta = \varepsilon$
 89. $\delta = \min\left\{1, \frac{\varepsilon}{15}\right\}$ 91. $\delta = 1/\sqrt[4]{N}$

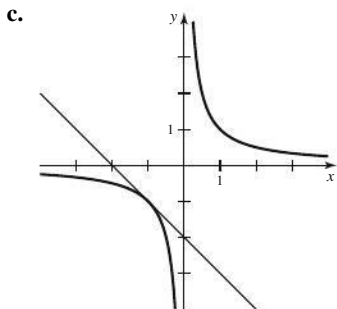
CHAPTER 3

Section 3.1 Exercises, pp. 137–140

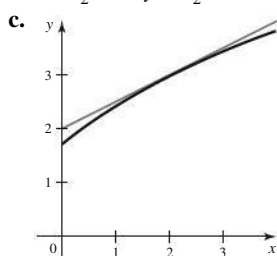
1. Given the point $(a, f(a))$ and any point $(x, f(x))$ near $(a, f(a))$, the slope of the secant line joining these points is $\frac{f(x) - f(a)}{x - a}$. The limit of this quotient as x approaches a is the slope of the tangent line at the point. 3. The average rate of change over the interval $[a, x]$ is $\frac{f(x) - f(a)}{x - a}$. The value of $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ is the slope of the tangent line; it is also the limit of average rates of change, which is the instantaneous rate of change at $x = a$. 5. $f'(a)$ is the slope of the tangent line at $(a, f(a))$ or the instantaneous rate of change in f at a .
 7. $f(2) = 7$; $f'(2) = 4$ 9. $y = 3x - 1$ 11. -5 13. 68 ft/s
 15. a. 6 b. $y = 6x - 14$



17. a. -1 b. $y = -x - 2$

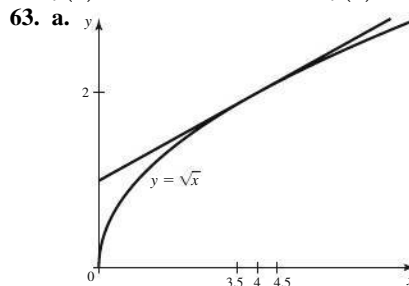


19. a. $\frac{1}{2}$ b. $y = \frac{1}{2}x + 2$



21. a. 2 b. $y = 2x + 1$ 23. a. 2 b. $y = 2x - 3$
 25. a. 4 b. $y = 4x - 8$ 27. a. 3 b. $y = 3x - 2$
 29. a. $\frac{2}{25}$ b. $y = \frac{2}{25}x + \frac{7}{25}$ 31. a. $\frac{1}{4}$ b. $y = \frac{1}{4}x + \frac{7}{4}$
 33. a. 8 b. $y = 8x$ 35. a. -14 b. $y = -14x - 16$
 37. a. -4 b. $y = -4x + 3$ 39. a. $\frac{1}{3}$ b. $y = \frac{1}{3}x + \frac{5}{3}$
 41. a. $-\frac{1}{100}$ b. $y = -\frac{x}{100} + \frac{3}{20}$ 43. $-\frac{1}{4}$ 45. $\frac{1}{5}$ 47. a. True
 b. False c. True 49. $d'(4) = 128$ ft/s; the object falls with an instantaneous speed of 128 ft/s four seconds after being dropped.
 51. $v'(3) = -4$ m/s per second; the instantaneous rate of change in the car's speed is -4 m/s² at $t = 3$ s.
 53. a. $L'(1.5) \approx 4.3$ mm/week; the talon is growing at a rate of approximately 4.3 mm/week at $t = 1.5$ weeks (answers will vary). b. $L'(a) \approx 0$, for $a \geq 4$; the talon stops growing at $t = 4$ weeks. 55. $D'(60) \approx 0.05$ hr/day; the number of

daylight hours is increasing at about 0.05 hr/day, 60 days after Jan 1. $D'(170) \approx 0$ hr/day; the number of daylight hours is neither increasing nor decreasing 170 days after Jan 1. 57. $f(x) = 5x^2$; $a = 2$; 20
 59. $f(x) = x^4$; $a = 2$; 32 61. $f(x) = |x|$; $a = -1$; -1



b.

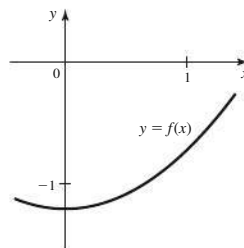
h	Approximation	Error
0.1	0.25002	2.0×10^{-5}
0.01	0.25000	2.0×10^{-7}
0.001	0.25000	2.0×10^{-9}

- c. Values of x on both sides of 4 are used in the formula.
 d. The centered difference approximations are more accurate than the forward and backward difference approximations. 65. a. 0.39470, 0.41545 b. 0.02, 0.0003

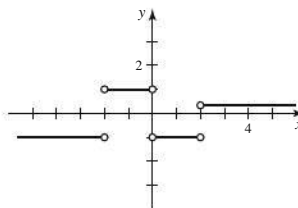
Section 3.2 Exercises, pp. 148–152

1. f' is the slope function of f . 3. $\frac{dy}{dx}$ is the limit of $\frac{\Delta y}{\Delta x}$ as $\Delta x \rightarrow 0$.

5. 7. Yes



9. A line with a y-intercept of 1 and a slope of 3
 11. $f'(x) = 7$ 13. $\frac{dy}{dx} = 2x$; $\frac{dy}{dx}\bigg|_{x=3} = 6$; $\frac{dy}{dx}\bigg|_{x=-2} = -4$
 15. a-C; b-C; c-A; d-B
 17.



19. a. Not continuous at $x = 1$ b. Not differentiable at $x = 0, 1$
 21. a. $f'(x) = 5$ b. $f'(1) = 5$; $f'(2) = 5$
 23. a. $f'(x) = 8x$ b. $f'(2) = 16$; $f'(4) = 32$
 25. a. $f'(x) = -\frac{1}{(x+1)^2}$ b. $f'\left(-\frac{1}{2}\right) = -4$; $f'(5) = -\frac{1}{36}$
 27. a. $f'(t) = -\frac{1}{2t^{3/2}}$ b. $f'(9) = -\frac{1}{54}$; $f'\left(\frac{1}{4}\right) = -4$
 29. a. $f'(s) = 12s^2 + 3$ b. $f'(-3) = 111$; $f'(-1) = 15$
 31. a. $v(t) = -32t + 100$ b. $v(1) = 68$ ft/s; $v(2) = 36$ ft/s
 33. $\frac{dy}{dx} = \frac{1}{(x+2)^2}$; $\frac{dy}{dx}\bigg|_{x=2} = \frac{1}{16}$ 35. a. $6x + 2$
 b. $y = 8x - 13$ 37. a. $\frac{3}{2\sqrt{3x+1}}$ b. $y = 3x/10 + 13/5$

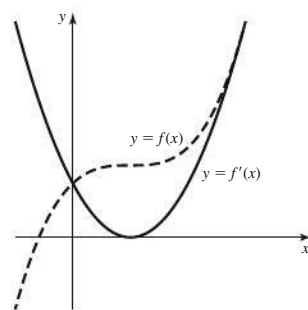
39. a. $\frac{6}{(3x+1)^2}$ b. $y = -3x/2 - 5/2$

41. a. Approximately 10 kW; approximately -5 kW

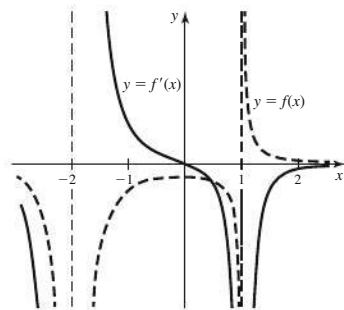
b. $t = 6, 18$ c. $t = 12$ 43. a. $2ax + b$ b. $8x - 3$ c. 5

45. a. C, D b. A, B, E c. A, B, E, D, C 47. a-D; b-C; c-B; d-A

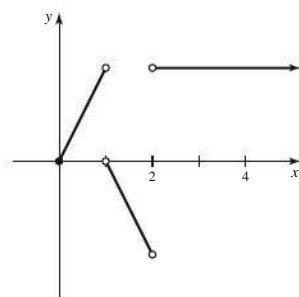
49.



51.

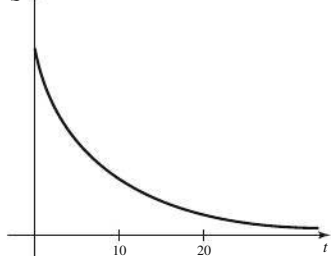


53. a. $x = 1$ b. $x = 1, x = 2$ c.



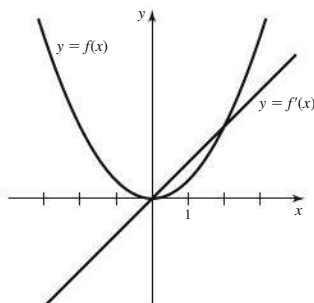
55. a. $t = 0$ b. Positive c. Decreasing

d. Q'



57. a. True b. True c. False 59. $a = 4$

61. Yes

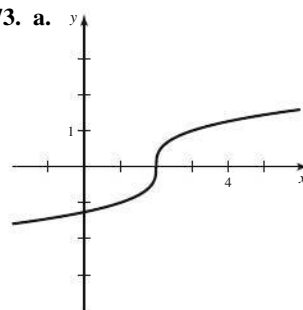


63. $y = -\frac{x}{3} - \frac{2}{3}$ 65. $y = \frac{x}{2} + \frac{3}{2}$ 67. (1, 2), (5, 26)

69. (1, 1), $(-\frac{1}{2}, -2)$ 71. b. $f'_+(2) = 1$; $f'_-(2) = -1$

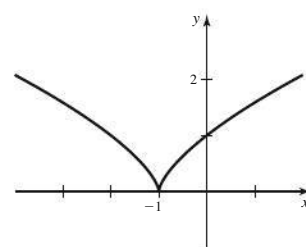
c. f is continuous but not differentiable at $x = 2$.

73. a.



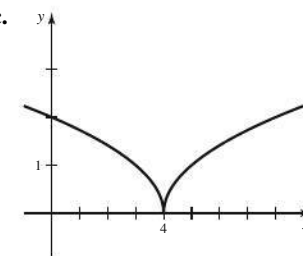
Vertical tangent line $x = 2$

b.



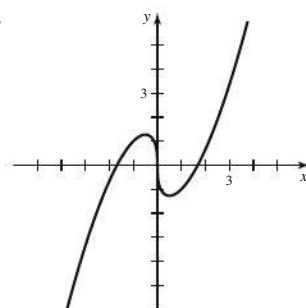
Vertical tangent line $x = -1$

c.



Vertical tangent line $x = 4$

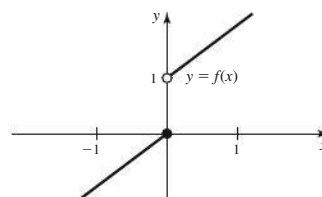
d.



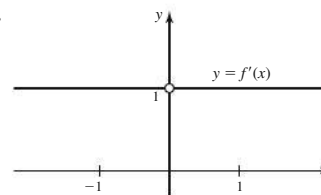
Vertical tangent line $x = 0$

75. $f'(x) = \frac{1}{3}x^{-2/3}$ and $\lim_{x \rightarrow 0^-} |f'(x)| = \lim_{x \rightarrow 0^+} |f'(x)| = \infty$

77. a.



b. 1 c. 1 d.



e. f is not differentiable at 0 because it is not continuous at 0.

Section 3.3 Exercises, pp. 159–162

1. Using the definition can be tedious. 3. $f(x) = e^x$ 5. Take the product of the constant and the derivative of the function. 7. 4

9. $-\frac{1}{2}$ 11. -2 13. 7.5 15. $10t^9$; $90t^8$; $720t^7$ 17. $\frac{2}{5}$ 19. $5x^4$

21. 0 23. $15x^2$ 25. t 27. 8 29. $200t$ 31. $12x^3 + 7$

33. $40x^3 - 32$ 35. $6w^2 + 6w + 10$ 37. $3e^x + 5$

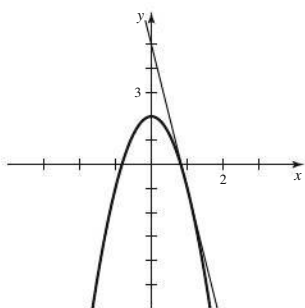
39. $\begin{cases} 2x & \text{if } x < 0 \\ 4x + 1 & \text{if } x > 0 \end{cases}$ 41. a. $d'(t) = 32t$; ft/s; the velocity of the stone b. 576 ft; approx. 131 mi/hr 43. a. $A'(t) = -\frac{1}{25}t + 2$ measures the rate at which the city grows in mi^2/yr . b. $1.6 \text{ mi}^2/\text{yr}$ c. 1200 people/yr

45. $w'(x) = \begin{cases} 0.4 & \text{if } 19 < x < 21 \\ 0.8 & \text{if } 21 < x < 32 \\ 1.5 & \text{if } x > 32 \end{cases}$ 47. $18x^2 + 6x + 4$

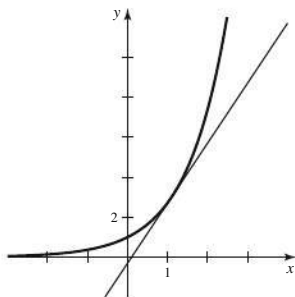
49. $2w$, for $w \neq 0$ 51. $4x^3 + 4x$ 53. 1, for $x \neq 1$

55. $\frac{1}{2\sqrt{x}}$, for $x \neq a$ 57. e^w

59. a. $y = -6x + 5$ b.



61. a. $y = 3x + 3 - 3 \ln 3$ b.



63. a. $x = 3$ b. $x = 4$

65. a. $(-1, 11), (2, -16)$ b. $(-3, -41), (4, 36)$

67. a. $(4, 4)$ b. $(16, 0)$ 69. $f'(x) = 20x^3 + 30x^2 + 3$;
 $f''(x) = 60x^2 + 60x$; $f'''(x) = 120x + 60$

71. $f'(x) = 1$; $f''(x) = f'''(x) = 0$, for $x \neq -1$

73. a. False b. True c. False d. False e. False

75. a. $y = 7x - 1$ b. $y = -2x + 5$ c. $y = 16x + 4$

77. $b = 2, c = 3$ 79. -10 81. 4 83. a. $f(x) = x + e^x$; $a = 0$

b. 2 85. a. $f(x) = \sqrt{x}$; $a = 9$ b. $\frac{1}{6}$ 87. a. $f(x) = e^x$; $a = 3$

b. e^3 89. 3 91. 1 95. d. $\frac{n}{2}x^{n/2-1}$ 97. c. $2e^{2x}$

Section 3.4 Exercises, pp. 168–170

1. $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$ 3. $6x + 5$

5. $\frac{5}{(3x+2)^2}$ 7. a. $2x - 1$ 9. a. $6x + 1$ 11. a. $2w$, for $w \neq 0$

13. 1, for $x \neq a$ 15. $23; \frac{7}{4}$ 17. $\frac{2}{27}; \frac{3}{8}$ 19. $36x^5 - 12x^3$

21. $\frac{1}{(x+1)^2}$ 23. $e^t t^{2/3} \left(t + \frac{5}{3}\right)$ 25. $\frac{e^x}{(e^x + 1)^2}$ 27. $e^{-x}(1-x)$

29. $-\frac{1}{(t-1)^2}$ 31. $4x^3$ 33. $e^w(w^3 + 3w^2 - 1)$ 35. $t^2 e^t$

37. $\frac{e^x(x^2 - 2x - 1)}{(x^2 - 1)^2}$ 39. $-27x^{-10}$ 41. $6t - 42/t^8$

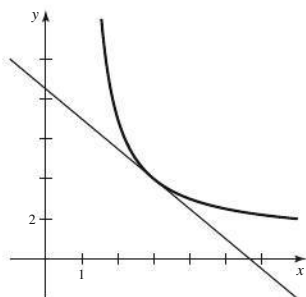
43. $-3/t^2 - 2/t^3$ 45. $\frac{e^x(x^2 - x - 5)}{(x-2)^2}$

47. $\frac{e^x(x^2 + x + 1)}{(x+1)^2}$ 49. $\frac{\sqrt{w}}{(\sqrt{w} - w)^2}$ 51. $\frac{5w^{2/3}}{3(w^{5/3} + 1)^2}$

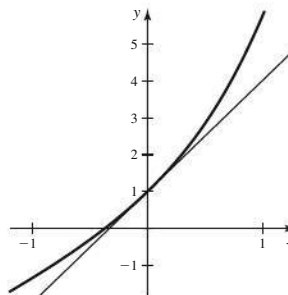
53. $8x - \frac{2}{(5x+1)^2}$ 55. $\frac{r - 6\sqrt{r} - 1}{2\sqrt{r}(r+1)^2}$

57. $300x^9 + 135x^8 + 105x^6 + 120x^3 + 45x^2 + 15$ 59. $e^x + 8x$

61. a. $y = -3x/2 + 17/2$ b.

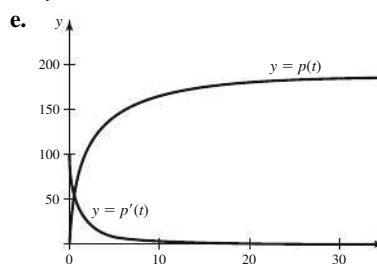


63. a. $y = 3x + 1$ b.



65. a. $p'(t) = \left(\frac{20}{t+2}\right)^2$ b. $p'(5) \approx 8.16$ c. $t = 0$

d. $\lim_{t \rightarrow \infty} p(t) = 200$; the population approaches a steady state.



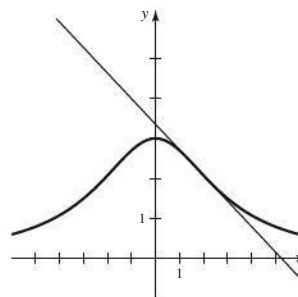
67. a. $F'(x) = -\frac{1.8 \times 10^{10} Qq}{x^3} \text{ N/m}$ b. $-1.8 \times 10^{19} \text{ N/m}$

c. $|F'(x)|$ decreases as x increases. 69. a. False b. False

c. False d. False 71. $4x - \frac{1}{x^2}; 2\left(\frac{1}{x^3} + 2\right); -\frac{6}{x^4}$

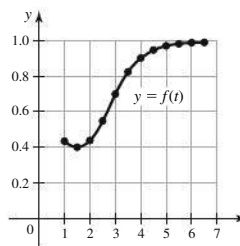
73. $\frac{x^2 + 2x - 7}{(x+1)^2}; \frac{16}{(x+1)^3}$

75. a. $y = -\frac{108}{169}x + \frac{567}{169}$ b.



77. $-\frac{3}{2}$ 79. $\frac{1}{9}$ 81. $\frac{7}{8}$

83. a. b. $t \approx 3$



c. $f'(3) \approx 0.28 \frac{\text{mm/g}}{\text{week}}$; at a young age, the bird's wings

are growing quickly relative to its weight.

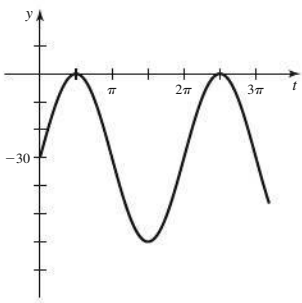
d. $f'(6.5) \approx 0.003 \frac{\text{mm/g}}{\text{week}}$; the rate of change of the ratio of wing chord length to mass is nearly 0. 85. $\frac{15}{2}$ 87. $-\frac{5}{2}$ 89. 1

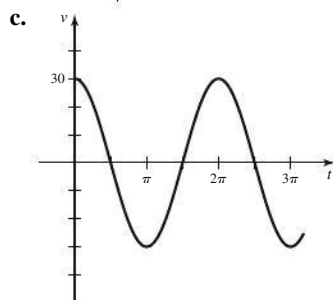
91. a. $y = -2x + 16$ b. $y = -\frac{5}{9}x + \frac{23}{9}$

93. -90 97. $f''g + 2f'g' + fg''$ 99. a. $f'gh + fg'h + fgh'$
b. $e^x(x^2 + 4x - 1)$

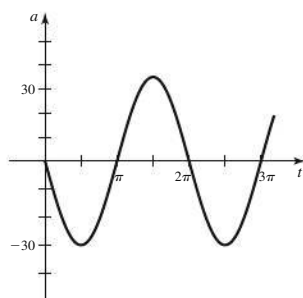
Section 3.5 Exercises, pp. 175–178

1. $\frac{\sin x}{x}$ is undefined at $x = 0$. 3. The tangent and cotangent functions are defined as ratios of the sine and cosine functions. 5. -1 7. $y = x$ 9. $-\sin x - \cos x$ 11. 3 13. $\frac{7}{3}$ 15. 5 17. 7 19. $\frac{1}{4}$ 21. a/b 23. $\cos x - \sin x$ 25. $e^{-x}(\cos x - \sin x)$ 27. $\sin x + x \cos x$ 29. $-\frac{1}{1 + \sin x}$ 31. $\cos^2 x - \sin^2 x = \cos 2x$ 33. $-2 \sin x \cos x = -\sin 2x$ 35. $w^2 \cos w$ 37. $x \cos 2x + \frac{1}{2} \sin 2x$ 39. $\frac{1}{1 + \cos x}$ 41. $\frac{2 \sin x}{(1 + \cos x)^2}$ 43. $\sec x \tan x - \csc x \cot x$ 45. $e^x \csc x(1 - \cot x)$ 47. $-\frac{\csc x}{1 + \csc x}$ 49. $\cos^2 z - \sin^2 z = \cos 2z$ 51. $2 \sin^2 x$

55. a.  b. $v(t) = 30 \cos t$

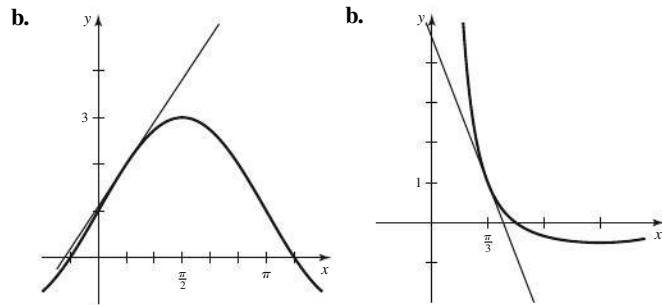


- d. $v(t) = 0$, for $t = (2k + 1)\frac{\pi}{2}$, where k is any nonnegative integer; the position is $y\left((2k + 1)\frac{\pi}{2}\right) = 0$ if k is even or $y\left((2k + 1)\frac{\pi}{2}\right) = -60$ if k is odd. e. $v(t)$ has a maximum at $t = 2k\pi$, where k is a nonnegative integer; the position is $y(2k\pi) = -30$. f. $a(t) = -30 \sin t$



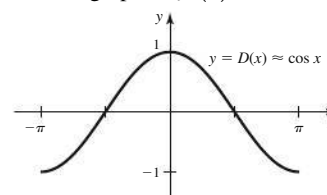
57. $2 \cos x - x \sin x$ 59. $2e^x \cos x$ 61. $2 \csc^2 x \cot x$ 63. $2(\sec^2 x \tan x + \csc^2 x \cot x)$ 65. a. False b. False c. True d. True 67. 2 69. $-\frac{1}{2}$ 71. $\frac{4}{3}$

73. a. $y = \sqrt{3}x + 2 - \frac{\pi\sqrt{3}}{6}$ 75. a. $y = -2\sqrt{3}x + \frac{2\sqrt{3}\pi}{3} + 1$



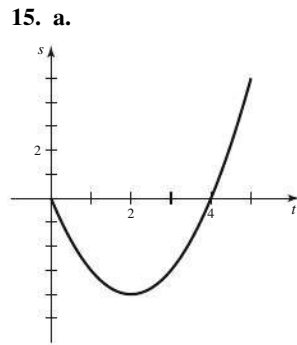
77. $x = 7\pi/6 + 2k\pi$ and $x = 11\pi/6 + 2k\pi$, where k is an integer 85. $a = 0$ 87. a. $2 \sin x \cos x$ b. $3 \sin^2 x \cos x$ c. $4 \sin^3 x \cos x$ d. $n \sin^{n-1} x \cos x$; the conjecture is true for $n = 1$. If it holds for $n = k$, then when $n = k + 1$, we have $\frac{d}{dx}(\sin^{k+1} x) = \frac{d}{dx}(\sin^k x \cdot \sin x) = \sin^k x \cos x + \sin x \cdot k \sin^{k-1} x \cos x = (k + 1) \sin^k x \cos x$.

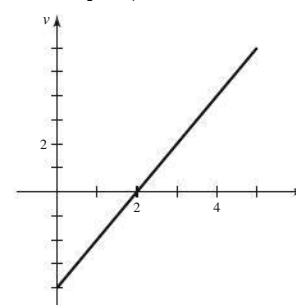
89. Because D is a difference quotient for f (and $h = 0.01$ is small), D is a good approximation to f' . Therefore, the graph of D is nearly indistinguishable from the graph of $f'(x) = \cos x$.



Section 3.6 Exercises, pp. 186–191

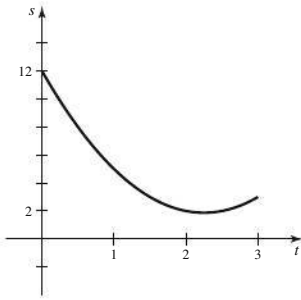
1. The average rate of change is $\frac{f(x + \Delta x) - f(x)}{\Delta x}$, whereas the instantaneous rate of change is the limit as Δx goes to zero in this quotient. 3. Small 5. At 15 weeks, the puppy grows at a rate of 1.75 lb/week. 7. If the position of the object at time t is $s(t)$, then the acceleration at time t is $a(t) = d^2s/dt^2$. 9. $v'(T) = 0.6$; the speed of sound increases by approximately 0.6 m/s for each increase of 1°C . 11. a. 40 mi/hr b. 40 mi/hr; yes c. -60 mi/hr; -60 mi/hr; south d. The police car drives away from the police station going north until about 10:08, when it turns around and heads south, toward the police station. It continues south until it passes the police station at about 11:02 and keeps going south until about 11:40, when it turns around and heads north. 13. The first 200 stoves cost, on average, \$70 to produce. When 200 stoves have already been produced, the 201st stove costs \$65 to produce.

15. a.  b. $v(t) = 2t - 4$; stationary at $t = 2$, to the right on $(2, 5]$, to the left on $[0, 2)$

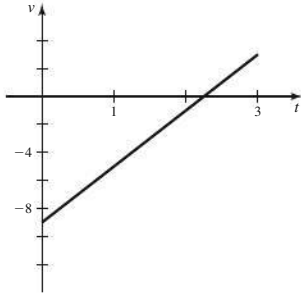


c. $v(1) = -2 \text{ ft/s}$; $a(1) = 2 \text{ ft/s}^2$ d. $a(2) = 2 \text{ ft/s}^2$ e. $(2, 5]$

17. a.



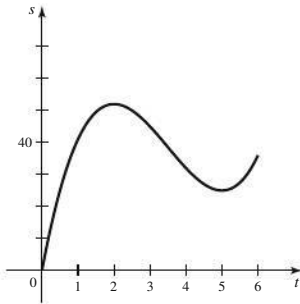
b. $v(t) = 4t - 9$; stationary at $t = \frac{9}{4}$, to the right on $(\frac{9}{4}, 3]$, to the left on $[0, \frac{9}{4})$



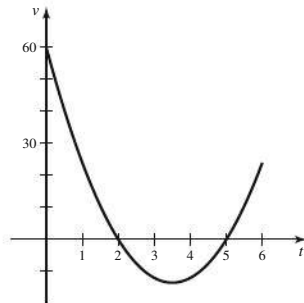
c. $v(1) = -5 \text{ ft/s}$; $a(1) = 4 \text{ ft/s}^2$

d. $a(\frac{9}{4}) = 4 \text{ ft/s}^2$ e. $(\frac{9}{4}, 3]$

19. a.



b. $v(t) = 6t^2 - 42t + 60$; stationary at $t = 2$ and $t = 5$, to the right on $[0, 2)$ and $(5, 6]$, to the left on $(2, 5)$



c. $v(1) = 24 \text{ ft/s}$; $a(1) = -30 \text{ ft/s}^2$ d. $a(2) = -18 \text{ ft/s}^2$; $a(5) = 18 \text{ ft/s}^2$ e. $(2, \frac{7}{2}), (5, 6]$ 21. -64 ft/s ; 64 ft/s

23. a. $v(t) = -32t + 32$ b. At $t = 1 \text{ s}$ c. 64 ft d. At $t = 3 \text{ s}$

e. -64 ft/s f. $(1, 3)$ 25. a. $v(t) = -32t + 64$ b. At $t = 2$

c. 96 ft d. At $2 + \sqrt{6}$ e. $-32\sqrt{6} \text{ ft/s}$ f. $(2, 2 + \sqrt{6})$

27. Approx. 90.5 ft/s 29. a. $\bar{C}(x) = \frac{1000}{x} + 0.1$; $C'(x) = 0.1$

b. $\bar{C}(2000) = \$0.60/\text{item}$; $C'(2000) = \$0.10/\text{item}$

c. The average cost per item when 2000 items are produced is $\$0.60/\text{item}$. The cost of producing the 2001st item is $\$0.10$.

31. a. $\bar{C}(x) = -0.01x + 40 + 100/x$; $C'(x) = -0.02x + 40$

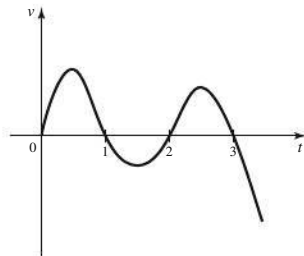
b. $\bar{C}(1000) = \$30.10/\text{item}$; $C'(1000) = \$20/\text{item}$

c. The average cost per item is about $\$30.10$ when 1000 items are produced. The cost of producing the 1001st item is $\$20$.

33. a. 20 b. $\$20$ c. $E(p) = \frac{p}{p-20}$ d. Elastic for $p > 10$; inelastic for $0 < p < 10$ e. 2.5% f. 2.5% 35. a. False b. True c. False

d. True 37. 240 ft 39. 64 ft/s 41. a. $t = 1, 2, 3$ b. It is moving in the positive direction for t in $(0, 1)$ and $(2, 3)$; it is moving in the negative direction for t in $(1, 2)$ and $t > 3$.

c. d. $(0, \frac{1}{2}), (1, \frac{3}{2}), (2, \frac{5}{2}), (3, \infty)$



43. a. $P(x) = 0.02x^2 + 50x - 100$

b. $\frac{P(x)}{x} = 0.02x + 50 - \frac{100}{x}$; $\frac{dP}{dx} = 0.04x + 50$

c. $\frac{P(500)}{500} = 59.8$; $\frac{dP}{dx}(500) = 70$

d. The profit, on average, for each of the first 500 items produced is 59.8; the profit for the 501st item produced is 70.

45. a. $P(x) = 0.04x^2 + 100x - 800$

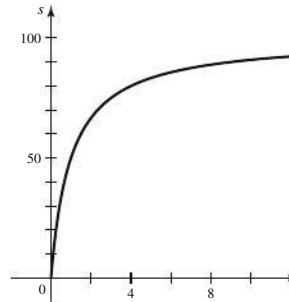
b. $\frac{P(x)}{x} = 0.04x + 100 - \frac{800}{x}$; $\frac{dP}{dx} = 0.08x + 100$

c. $\frac{P(1000)}{1000} = 139.2$; $p'(1000) = 180$

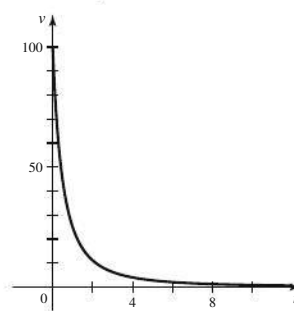
d. The average profit per item for each of the first 1000 items produced is $\$139.20$. The profit for the 1001st item produced is $\$180$.

47. About 1935; approximately 890,000 people/yr (answers will vary)

49. a. b. $v = \frac{100}{(t+1)^2}$



c.

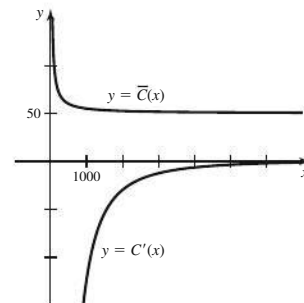


The marble moves fastest at the beginning and slows considerably over the first 5 s. It continues to slow but never actually stops.

d. $t = 4 \text{ s}$ e. $t = -1 + \sqrt{2} \approx 0.414 \text{ s}$

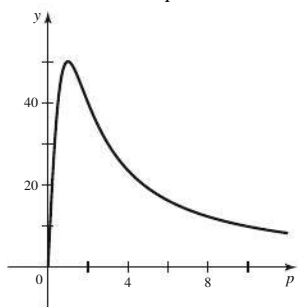
51. a. $C'(x) = -\frac{125,000,000}{x^2} + 1.5$;

$\bar{C}(x) = \frac{C(x)}{25,000} = 50 + \frac{5000}{x} + 0.00006x$

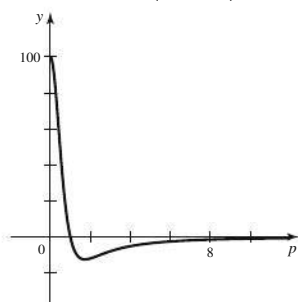


b. $C'(5000) = -3.5$; $\bar{C}(5000) = 51.3$ c. Marginal cost: If the batch size is increased from 5000 to 5001, then the cost of producing 25,000 gadgets will *decrease* by about $\$3.50$. Average cost: When batch size is 5000, it costs $\$51.30$ per item to produce all 25,000 gadgets.

53. a. $R(p) = \frac{100p}{p^2 + 1}$

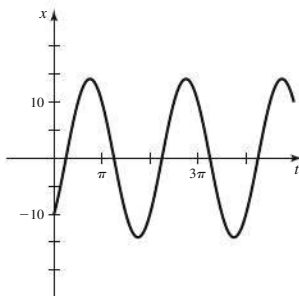


b. $R'(p) = \frac{100(1 - p^2)}{(p^2 + 1)^2}$



c. $p = 1$

55. a.



b. $dx/dt = 10 \cos t + 10 \sin t$

c. $t = 3\pi/4 + k\pi$, where k is any positive integer

d. The graph implies that the spring never stops oscillating.

In reality, the weight would eventually come to rest.

57. a. Juan starts faster than Jean and opens up a big lead. Then Juan slows down while Jean speeds up. Jean catches up, and the race finishes in a tie. b. Same average velocity c. Tie d. At $t = 2$, $\theta'(2) = \pi/2$ rad/min; $\theta'(4) = \pi$ = Jean's greatest velocity

e. At $t = 2$, $\varphi'(2) = \pi/2$ rad/min; $\varphi'(0) = \pi$ = Juan's greatest velocity 59. a. $v(t) = -15e^{-t}(\sin t + \cos t)$; $v(1) \approx -7.6$ m/s, $v(3) \approx 0.63$ m/s b. Down (0, 2.4) and (5.5, 8.6); up (2.4, 5.5) and (8.6, 10) c. ≈ 0.65 m/s 61. a. $-T'(1) = -80$, $-T'(3) = 80$ b. $-T'(x) < 0$ for $0 \leq x < 2$; $-T'(x) > 0$ for $2 < x \leq 4$ c. Near $x = 0$, with $x > 0$, $-T'(x) < 0$, so heat flows toward the end of the rod. Similarly, near $x = 4$, with $x < 4$, $-T'(x) > 0$.

Section 3.7 Exercises, pp. 196–200

1. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$; $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

3. $u = x^3 + x + 1$; $y = u^4$; $4(x^3 + x + 1)^3(3x^2 + 1)$

5. $u = \cos x$, $y = u^3$, $dy/dx = -3 \cos^2 x \sin x$

$u = x^3$, $y = \cos u$, $dy/dx = -3x^2 \sin x^3$ 7. $g(x)$, x 9. $\frac{2}{\sqrt{4x+1}}$

11. 50 13. ke^{kx} 15. $u = 3x + 7$; $f(u) = u^{10}$; $30(3x + 7)^9$

17. $u = \sin x$; $f(u) = u^5$; $5 \sin^4 x \cos x$

19. $u = x^2 + 1$; $f(u) = \sqrt{u}$; $\frac{x}{\sqrt{x^2 + 1}}$

21. $u = 4x^2 + 1$; $f(u) = e^u$; $8xe^{4x^2+1}$

23. $u = 5x^2$; $f(u) = \tan u$; $10x \sec^2 5x^2$ 25. a. 100 b. -100

c. -16 d. 40 e. 40 27. $10(6x + 7)(3x^2 + 7x)^9$

29. $\frac{5}{\sqrt{10x+1}}$ 31. $-\frac{315x^2}{(7x^3+1)^4}$ 33. $3 \sec(3x+1) \tan(3x+1)$

35. $e^x \sec^2 e^x$ 37. $(12x^2 + 3) \cos(4x^3 + 3x + 1)$

39. $\frac{10}{3(5x+1)^{1/3}}$ 41. $-\frac{3}{27/4 x^{3/4}(4x-3)^{5/4}}$

43. $5 \sec x (\sec x + \tan x)^5$ 45. $25(12x^5 - 9x^2)(2x^6 - 3x^3 + 3)^{24}$

47. $9(1 + 2 \tan u)^{3.5} \sec^2 u$ 49. $-\frac{\cot x \csc^2 x}{\sqrt{1 + \cot^2 x}}$

51. $\frac{2}{3}e^x - e^{-x}$ 53. $e^x \cos(\sin e^x) \cos e^x$

55. $-15 \sin^4(\cos 3x) (\sin 3x) (\cos(\cos 3x))$

57. $\frac{2e^{2t}}{(1 + e^{2t})^2}$ 59. $\frac{1}{2\sqrt{x + \sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)$

61. $f'(g(x^2))g'(x^2) 2x$ 63. $\frac{5x^4}{(x+1)^6}$

65. $xe^{x^2+1}(2 \sin x^3 + 3x \cos x^3)$ 67. $\theta(2 + 5\theta \tan 5\theta) \sec 5\theta$

69. $4((x+2)(x^2+1))^3(3x+1)(x+1)$ 71. $\frac{4x^3 - 2 \sin 2x}{5(x^4 + \cos 2x)^{4/5}}$

73. $2(p+3)(\sin p^2 + p(p+3) \cos p^2)$

75. $f'(x)/(2\sqrt{f(x)})$ 77. a. True b. True c. True

d. False 79. -0.297 hPa/min 81. Approx. 0.33 g/day; mass is increasing by 0.33 g/day 65 days after the diet switch.

83. a. \$297.77 b. \$11.85/yr c. $y = 11.85t + 179.27$

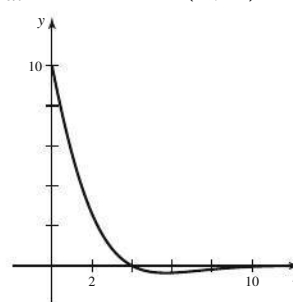
85. a. $x = -\frac{1}{2}$ b. The line tangent to the graph of $f(x)$ at $x = -\frac{1}{2}$ is horizontal. 87. $2 \cos x^2 - 4x^2 \sin x^2$ 89. $4e^{-2x^2}(4x^2 - 1)$

91. $y = 6x - 1$ 93. a. $h(4) = 9$, $h'(4) = -6$ b. $y = -6x + 33$

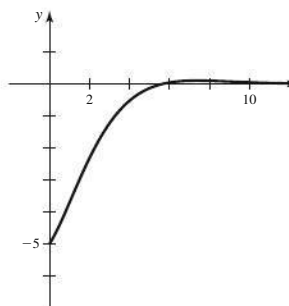
95. $y = 6x + 3 - 3 \ln 3$ 97. a. -3π b. -5π

99. a. $\frac{d^2 y}{dt^2} = -\frac{y_0 k}{m} \cos\left(t\sqrt{\frac{k}{m}}\right)$

101. a.



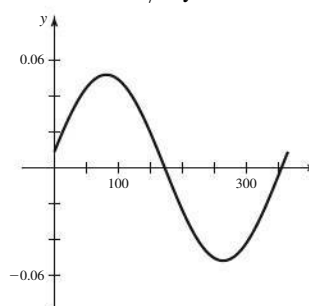
b. $v(t) = -5e^{-t/2}\left(\frac{\pi}{4} \sin \frac{\pi t}{8} + \cos \frac{\pi t}{8}\right)$



103. a. 10.88 hr b. $D'(t) = \frac{6\pi}{365} \sin\left(\frac{2\pi(t+10)}{365}\right)$

c. 2.87 min/day; on March 1, the length of day is increasing at a rate of about 2.87 min/day.

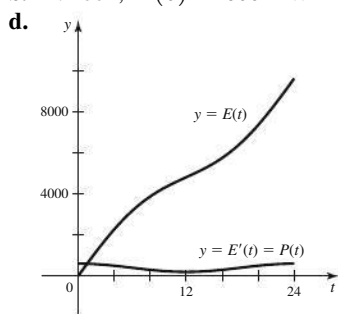
d.



e. Most rapidly: approximately March 22 and September 22; least rapidly: approximately December 21 and June 21

105. a. $E'(t) = 400 + 200 \cos \frac{\pi t}{12}$ MW

b. At noon; $E'(0) = 600$ MW c. At midnight; $E'(12) = 200$ MW



109. a. $g(x) = (x^2 - 3)^5$; $a = 2$ b. 20

111. a. $g(x) = \sin x^2$; $a = \pi/2$ b. $\pi \cos(\pi^2/4)$ 113. $10 f'(25)$

Section 3.8 Exercises, pp. 205–208

1. There may be more than one expression for y or y' .

3. When derived implicitly, dy/dx is usually given in terms

of both x and y . 5. $\frac{1}{2y}$ 7. $\frac{1}{\cos y}$ 9. a. $(0, 0)$, $(0, -1)$, $(0, 1)$

c. Slope at $(0, 0)$ is 2; slope at $(0, -1)$ and $(1, 0)$ is -1 .

11. $\frac{d^2y}{dx^2} = -\frac{2}{9y^5}$ 13. a. $-\frac{x^3}{y^3}$ b. 1 15. a. $\frac{2}{y}$ b. 1

17. a. $\frac{20x^3}{\cos y}$ b. -20 19. a. $-\frac{1}{\sin y}$ b. -1 21. a. $-\frac{y}{x}$ b. -7

23. a. $-\frac{1}{4x^{2/3}y^{1/3}}$ b. $-\frac{1}{4}$ 25. a. $-\frac{3y}{x + 3y^{2/3}}$ b. $-\frac{24}{13}$

27. $\frac{\cos x}{1 - \cos y}$ 29. $-\frac{1}{1 + \sin y}$ 31. $\frac{1 - y \cos xy}{x \cos xy - 1}$ 33. $\frac{1}{2y \sin y^2 + e^y}$

35. $\frac{3x^2(x - y)^2 + 2y}{2x}$ 37. $\frac{13y - 18x^2}{21y^2 - 13x}$ 39. $\frac{5\sqrt{x^4 + y^2} - 2x^3}{y - 6y^2\sqrt{x^4 + y^2}}$

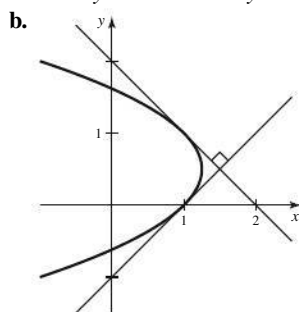
41. a. $\frac{dK}{dL} = -\frac{K}{2L}$ b. -4 43. $\frac{dr}{dh} = \frac{h - 2r}{h}$; -3

45. b. $y = -5x$ 47. b. $y = -5x/4 + 7/2$ 49. b. $y = \frac{x}{2}$

51. $-\frac{1}{4y^3}$ 53. $\frac{\sin y}{(\cos y - 1)^3}$ 55. $\frac{4e^{2y}}{(1 - 2e^{2y})^3}$ 57. a. False

b. True c. False d. False 59. a. $\frac{y(3\sqrt{x} + 2y^{3/2})}{x(\sqrt{x} - 2y^{3/2})}$ b. -5

61. a. $y = x - 1$ and $y = -x + 2$



63. a. $y' = -\frac{2xy}{x^2 + 4}$ b. $y = \frac{1}{2}x + 2$, $y = -\frac{1}{2}x + 2$

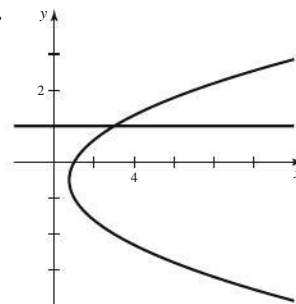
c. $-\frac{16x}{(x^2 + 4)^2}$ 65. a. $\left(\frac{5}{4}, \frac{1}{2}\right)$ b. No

67. Horizontal: $y = -6$, $y = 0$; vertical: $x = 1$, $x = 3$

69. a. $\frac{dy}{dx} = 0$ on the $y = 1$ branch; $\frac{dy}{dx} = \frac{1}{2y + 1}$ on the other two

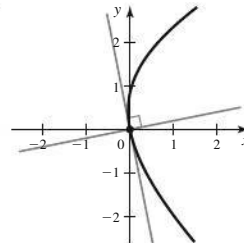
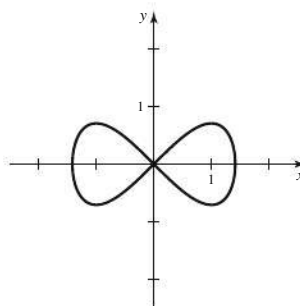
branches. b. $f_1(x) = 1$, $f_2(x) = \frac{-1 + \sqrt{4x - 3}}{2}$,

$f_3(x) = \frac{-1 - \sqrt{4x - 3}}{2}$ c.

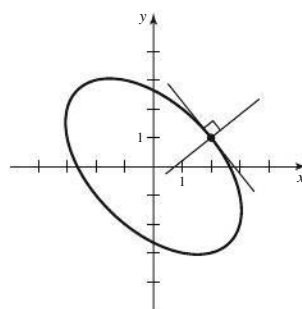


71. a. $\frac{dy}{dx} = \frac{x - x^3}{y}$ b. $f_1(x) = \sqrt{x^2 - \frac{x^4}{2}}$; $f_2(x) = -\sqrt{x^2 - \frac{x^4}{2}}$

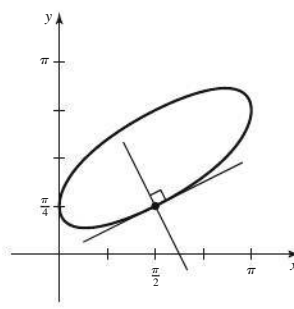
c. 73. $y = \frac{x}{5}$



75. $y = \frac{4x}{5} - \frac{3}{5}$

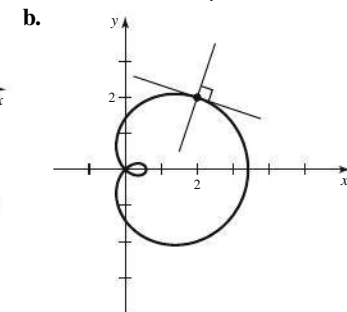
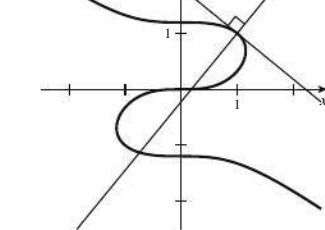


77. $y = -2x + \frac{5\pi}{4}$



79. a. Tangent line $y = -\frac{9x}{11} + \frac{20}{11}$; normal line $y = \frac{11x}{9} - \frac{2}{9}$

b. 81. a. Tangent line $y = -\frac{x}{3} + \frac{8}{3}$; normal line $y = 3x - 4$



83. For $y = mx$, $dy/dx = m$; for $x^2 + y^2 = a^2$, $dy/dx = -x/y$.

85. For $xy = a$, $dy/dx = -y/x$; for $x^2 - y^2 = b$, $dy/dx = x/y$. Because $(-y/x) \cdot (x/y) = -1$, the families of curves

form orthogonal trajectories. 87. $\frac{7y^2 - 3x^2 - 4xy^2 - 4x^3}{2y(2x^2 + 2y^2 - 7x)}$

89. $\frac{2y^2(5 + 8x\sqrt{y})}{(1 + 2x\sqrt{y})^3}$ 91. No horizontal tangent line; vertical tangent lines at $(2, 1)$, $(-2, 1)$ 93. No horizontal tangent line; vertical tangent lines at $(0, 0)$, $(\frac{3\sqrt{3}}{2}, \sqrt{3})$, $(-\frac{3\sqrt{3}}{2}, -\sqrt{3})$

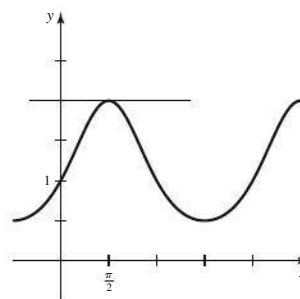
Section 3.9 Exercises, pp. 215–218

1. $x = e^y \Rightarrow 1 = e^y y'(x) \Rightarrow y'(x) = 1/e^y = 1/x$
 3. $\frac{d}{dx}(\ln kx) = \frac{d}{dx}(\ln k + \ln x) = \frac{d}{dx}(\ln x)$ 5. $f'(x) = \frac{1}{x \ln b}$;
 if $b = e$, then $f'(x) = \frac{1}{x}$. 7. $(x^2 + 1)^x$ 9. $\frac{x}{x^2 + 1}$
 11. $f(x) = e^{h(x) \ln g(x)}$ 13. $\frac{1+x}{x}$ 15. $\frac{1}{x}$ 17. $2/x$ 19. $\cot x$
 21. $\frac{4x^3}{x^4 + 1}$ 23. $2/(1 - x^2)$ 25. $(x^2 + 1)/x + 2x \ln x$
 27. $-2x \ln x^2$ or $-4x \ln x$ 29. $1/(x \ln x)$ 31. $\frac{1}{x(\ln x + 1)^2}$
 33. ex^{e-1} 35. $\pi(2^x + 1)^{\pi-1} 2^x \ln 2$ 37. $8^x \ln 8$ 39. $5 \cdot 4^x \ln 4$
 41. $2^{3+\sin x}(\ln 2)\cos x$ 43. $3^x \cdot x^2(x \ln 3 + 3)$
 45. $1000(1.045)^{4t} \ln 1.045$ 47. $\frac{2^x \ln 2}{(2^x + 1)^2}$
 49. $x^{\cos x - 1}(\cos x - x \ln x \sin x)$; $-\ln(\pi/2)$
 51. $x^{\sqrt{x}}\left(\frac{2 + \ln x}{2\sqrt{x}}\right)$; $4(2 + \ln 4)$
 53. $\frac{(\sin x)^{\ln x}(\ln(\sin x) + x(\ln x) \cot x)}{x}$; 0
 55. $(4 \sin x + 2)^{\cos x} \left(\frac{2 \cos^2 x}{2 \sin x + 1} - \sin x \ln(4 \sin x + 2) \right)$; 1
 57. a. Approx. 28.7 s b. $-46.512 \text{ s}/1000 \text{ ft}$
 c. $dT/da = -2.74 \cdot 2^{-0.274a} \ln 2$
 At $a = 8$, $\frac{dT}{da} = -0.4156 \text{ min}/1000 \text{ ft}$
 $= -24.938 \text{ s}/1000 \text{ ft}$.

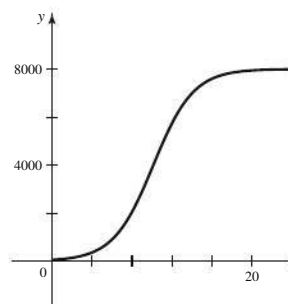
If a plane travels at 30,000 feet and increases its altitude by 1000 feet, the time of useful consciousness decreases by about 25 seconds.

59. $y = x \sin 1 + 1 - \sin 1$ 61. $y = e^{2/e}$ and $y = e^{-2/e}$
 63. $\frac{8x}{(x^2 - 1) \ln 3}$ 65. $-\sin x (\ln(\cos^2 x) + 2)$
 67. $-\frac{\ln 4}{x \ln^2 x}$ 69. $\frac{12}{3x + 1}$ 71. $\frac{1}{2x}$
 73. $\frac{2}{2x - 1} + \frac{3}{x + 2} + \frac{8}{1 - 4x}$ 75. $10x^{10x}(1 + \ln x)$
 77. $\frac{(x + 1)^{10}}{(2x - 4)^8} \left(\frac{10}{x + 1} - \frac{8}{x - 2} \right)$ 79. $2x^{\ln x - 1} \ln x$
 81. $\frac{(x + 1)^{3/2}(x - 4)^{5/2}}{(5x + 3)^{2/3}} \left(\frac{3}{2(x + 1)} + \frac{5}{2(x - 4)} - \frac{10}{3(5x + 3)} \right)$
 83. $(\sin x)^{\tan x}(1 + (\sec^2 x) \ln \sin x)$
 85. $\left(1 + \frac{1}{x}\right)^x \left(\ln \left(1 + \frac{1}{x}\right) - \frac{1}{x + 1} \right)$
 87. a. False b. False c. False d. False e. True f. True
 89. $-\frac{1}{x^2 \ln 10}$ 91. $\frac{2}{x}$ 93. $3^x \ln 3$

95. $y = 2$



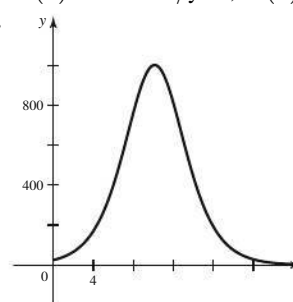
97. a.



b. $t = 2 \ln 265 \approx 11.2$ years; approx. 14.5 years

c. $P'(0) \approx 25$ fish/year; $P'(5) \approx 264$ fish/year

d.



The population is growing fastest after about 10 years.

99. b. $r(11) \approx 0.0133$; $r(21) \approx 0.0118$; the relative growth rate is decreasing. c. $\lim_{t \rightarrow \infty} r(t) = 0$; as the population gets close to carrying capacity, the relative growth rate approaches zero.

101. a. $A(5) = \$17,443$

$A(15) = \$72,705$

$A(25) = \$173,248$

$A(35) = \$356,178$

$\$5526.20/\text{year}$, $\$10,054.30/\text{year}$, $\$18,293/\text{year}$

b. $A(40) = \$497,873$

c. $\frac{dA}{dt} = 600,000 \ln(1.005)((1.005)^{12t})$
 $\approx (2992.5)(1.005)^{12t}$

A increases at an increasing rate.

103. $p = e^{1/e}$; (e, e) 105. $1/e$ 107. $27(1 + \ln 3)$

Section 3.10 Exercises, pp. 225–227

1. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$; $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$;
 $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x| \sqrt{x^2 - 1}}$ 3. $\frac{1}{5}$ 5. $\frac{1}{4}$ 7. a. $\frac{1}{2}$ b. $\frac{2}{3}$
 c. Cannot be determined d. $\frac{3}{2}$ 9. $y = \frac{1}{7}x + \frac{13}{7}$ 11. $\frac{2}{\sqrt{3}}$
 13. $\frac{2}{\sqrt{1 - 4x^2}}$ 15. $-\frac{4w}{\sqrt{1 - 4w^2}}$ 17. $-\frac{2e^{-2x}}{\sqrt{1 - e^{-4x}}}$
 19. $\frac{10}{100x^2 + 1}$ 21. $\frac{4y}{1 + (2y^2 - 4)^2}$ 23. $-\frac{1}{2\sqrt{z}(1 + z)}$

25. $6x^2 \cot^{-1} x$ 27. $\frac{2w^5}{1+w^4}$ 29. $\frac{1}{|x|\sqrt{x^2-1}}$

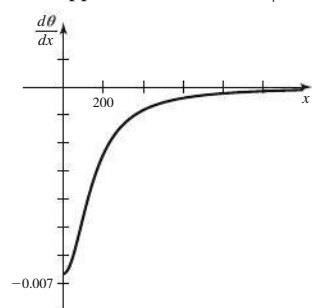
31. $-\frac{1}{|2u+1|\sqrt{u^2+u}}$ 33. $\frac{2y}{(y^2+1)^2+1}$

35. $\frac{1}{x|\ln x|\sqrt{(\ln x)^2-1}}$ 37. $-\frac{e^x \sec^2 e^x}{|\tan e^x|\sqrt{\tan^2 e^x-1}}$

39. $-\frac{e^s}{1+e^{2s}}$ 41. $y = x + \frac{\pi}{4} - \frac{1}{2}$ 43. $y = -\frac{4}{\sqrt{6}}x + \frac{\pi}{3} + \frac{2}{\sqrt{3}}$

45. a. Approx. -0.00055 rad/m

b.

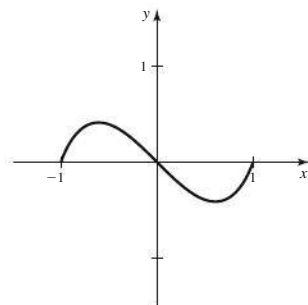


The magnitude of the change in angular size, $|d\theta/dx|$, is greatest when the boat is at the skyscraper (that is, at $x = 0$).

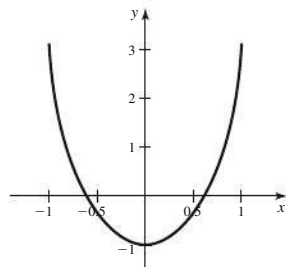
47. $\frac{1}{3}$ 49. $\frac{e}{5}$ 51. $\frac{1}{2}$ 53. 4 55. $\frac{1}{12}$ 57. $\frac{1}{4}$ 59. $\frac{5}{4}$ 61. a. True

b. False c. True d. True e. True

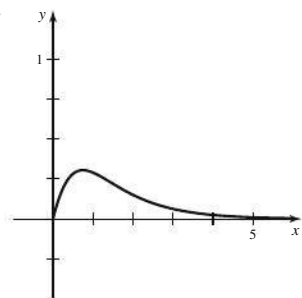
63. a.



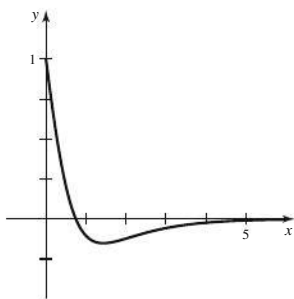
b. $f'(x) = 2x \sin^{-1} x + \frac{x^2 - 1}{\sqrt{1 - x^2}}$



65. a.



b. $f'(x) = \frac{e^{-x}}{1+x^2} - e^{-x} \tan^{-1} x$



67. $\frac{1}{3}$ 69. $1/(2\sqrt{x+4})$ 71. $\frac{1}{3x}$ 73. $\frac{1}{12x \ln 10}$ 75. $2x$

77. $-2/x^3$ 79. b. -0.0041 , -0.0289 , and -0.1984
c. $\lim_{\ell \rightarrow 10^+} d\theta/d\ell = -\infty$ d. The length ℓ is decreasing.

81. a. $1/\sqrt{D^2 - c^2}$ b. $1/D$ 85. Use the identity $\cot^{-1} x + \tan^{-1} x = \pi/2$.

Section 3.11 Exercises, pp. 231–236

1. As the side length s of a cube changes, the surface area $6s^2$ changes as well. 3. The other two opposite sides decrease in length.

5. a. $V = 200h$; $\frac{dV}{dt} = 200 \frac{dh}{dt}$ b. $50 \text{ ft}^3/\text{min}$

c. $\frac{1}{20} \text{ ft}/\text{min}$ 7. a. $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ b. $128\pi \text{ in}^3/\text{min}$

c. $\frac{1}{10\pi} \text{ in}/\text{min}$ 9. 59 11. a. $40 \text{ m}^2/\text{s}$ b. $80 \text{ m}^2/\text{s}$

13. a. $4 \text{ m}^2/\text{s}$ b. $\sqrt{2} \text{ m}^2/\text{s}$ c. $2\sqrt{2} \text{ m}/\text{s}$ 15. a. $\frac{1}{4\pi} \text{ cm}/\text{s}$

b. $\frac{1}{2} \text{ cm}/\text{s}$ 17. $-40\pi \text{ ft}^2/\text{min}$ 19. $\frac{3}{80\pi} \text{ in}/\text{min}$

23. $720.3 \text{ mi}/\text{hr}$ 25. $\frac{3\sqrt{5}}{2} \text{ ft}/\text{s}$ 27. $57.89 \text{ ft}/\text{s}$ 29. $4.66 \text{ in}/\text{s}$

31. $\frac{\pi}{2} \text{ ft}^3/\text{min}$ 33. $-75\pi \text{ cm}^3/\text{s}$ 35. $2592\pi \text{ cm}^3/\text{s}$

37. $9\pi \text{ ft}^3/\text{min}$ 39. $\frac{1}{25\pi} \text{ m}/\text{min}$ 41. $\frac{5}{24} \text{ ft}/\text{s}$

43. $-\frac{8}{3} \text{ ft}/\text{s}$, $-\frac{32}{3} \text{ ft}/\text{s}$ 45. $\frac{d\theta}{dt} = \frac{1}{5} \text{ rad}/\text{s}$, $\frac{d\theta}{dt} = \frac{1}{8} \text{ rad}/\text{s}$

47. $-0.0201 \text{ rad}/\text{s}$ 49. $10 \tan 20^\circ \text{ km}/\text{hr} \approx 3.6 \text{ km}/\text{hr}$

51. a. $187.5 \text{ ft}/\text{s}$ b. $0.938 \text{ rad}/\text{s}$ 53. a. $P = \frac{1}{2} v^2 \frac{dm}{dt}$

c. $17,388.7 \text{ W}$ d. 4347.2 W 55. $11.06 \text{ m}/\text{hr}$

57. $\frac{1}{500} \text{ m}/\text{min}$; 2000 min 59. $0.543 \text{ rad}/\text{hr}$

61. $\frac{d\theta}{dt} = 0 \text{ rad}/\text{s}$, for all $t \geq 0$ 63. a. $-\frac{\sqrt{3}}{10} \text{ m}/\text{hr}$ b. $-1 \text{ m}^2/\text{hr}$

Chapter 3 Review Exercises, pp. 236–240

1. a. False b. False c. False d. False e. True

3. $-\frac{2x}{(x^2+5)^2}$ 9. $2x^2 + 2\pi x + 7$ 11. $2^x \ln 2$

13. $2e^{2\theta}$ 15. $6x^3\sqrt{1+x^4}$ 17. $5t^2 \cos t + 10t \sin t$

19. $-x^2 e^{-x}$ 21. $\frac{2 \sec 2w \tan 2w}{(\sec 2w + 1)^2}$ 23. $3 \tan 3x$

25. $1000t(5t^2 + 10)^{99}$ 27. $3x^2 \cot x^3$ 29. $\frac{1}{t\sqrt{t^2-1}}$

31. $(8\theta + 12) \sec^2(\theta^2 + 3\theta + 2)$ 33. $\frac{1 - 5 \ln w}{w^6}$

35. $\frac{32u^2 + 8u + 1}{(8u + 1)^2}$ 37. $(\sec^2 \sin \theta) \cos \theta$

39. $-\frac{\cos \sqrt{\cos^2 x + 1} \cos x \sin x}{\sqrt{\cos^2 x + 1}}$ 41. $\frac{e^t}{2(e^t + 1)}$

43. $2 \tan^{-1}(\cot x)$ 45. $(2 + \ln x) \ln x$ 47. $(2x - 1) 2^{x^2-x} \ln 2$

49. $(x^2 + 1)^{\ln x} \left(\frac{\ln(x^2 + 1)}{x} + \frac{2x \ln x}{x^2 + 1} \right)$ 51. $-\frac{1}{|x|\sqrt{x^2-1}}$

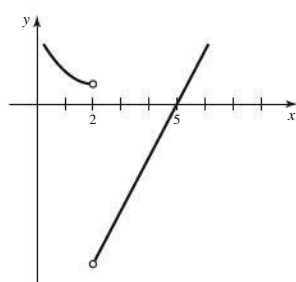
53. $6 \cot^{-1} 3x$ 55. $1 + \csc(x - y)$

57. $\frac{y \cos x}{e^y - 1 - \sin x}$ 59. $-\frac{xy}{x^2 + 2y^2}$

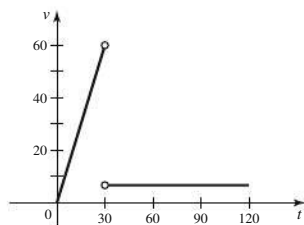
61. $\frac{(3x+5)^{10}\sqrt{x^2+5}}{(x^3+1)^{50}} \left(\frac{30}{3x+5} + \frac{x}{x^2+5} - \frac{150x^2}{x^3+1} \right)$
 63. $\sqrt{3} + \pi/6$ 65. 1 67. $2^x \ln 2(x \ln 2 + 2)$ 69. $\frac{6 \ln x - 5}{x^4}$
 71. $\frac{2(xy+y^2)}{(x+2y)^3} = \frac{2}{(x+2y)^3}$ 73. $y = x$ 75. $y = -\frac{4x}{5} + \frac{24}{5}$
 77. $x^2 f'(x) + 2x f(x)$ 79. $\frac{g(x)(x f'(x) + f(x)) - x f(x) g'(x)}{g^2(x)}$

81. a-D; b-C; c-B; d-A

83.



85. a. 27 b. $\frac{16}{27}$ c. 72 d. 1215 e. $\frac{1}{9}$ 87. $\frac{6}{13}$
 89. $(f^{-1})'(x) = -3/x^4$ 91. a. $\frac{1}{4}$ b. 1 c. $\frac{1}{3}$
 93. $y = 24x - 118$ 95. a. 84 ft/s b. 7 s c. 384 ft
 d. 96 ft/s 97. a. \$200,366; \$21,552/yr
 b. 14 yr; \$12,551/yr 99. a. 2.70 million people/yr
 b. The slope of the secant line through the two points is approximately equal to the slope of that tangent line at $t = 55$.
 c. 2.217 million people/yr 101. a. 40 m/s b. $20/3$ m/s
 c. 15 m/s d.



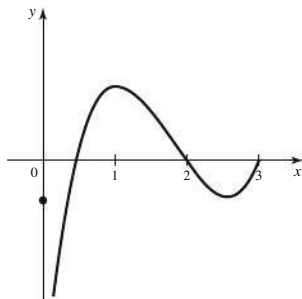
- e. The skydiver deployed the parachute. 103. $x = 4$; $x = 6$
 105. $f(x) = \tan(\pi\sqrt{3}x - 11)$, $a = 5$; $f'(5) = 3\pi/4$
 107. a. $C(3000) = \$341.67$; $C'(3000) = \$280$ b. The average cost of producing the first 3000 lawn mowers is \$341.67 per mower. The cost of producing the 3001st lawn mower is \$280.
 109. a. 6550 people/yr b. $p'(40) = 4800$ people/yr
 111. 50 mi/hr 113. $-5 \sin 65^\circ \text{ ft/s} \approx -4.5 \text{ ft/s}$
 115. -0.166 rad/s 117. 1.5 ft/s 119. a. $(f^{-1})'(1/\sqrt{2}) = \sqrt{2}$

CHAPTER 4

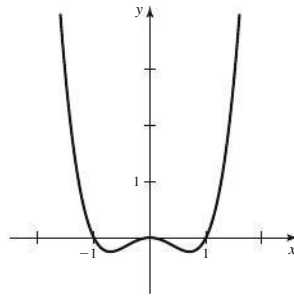
Section 4.1 Exercises, pp. 247–250

1. f has an absolute maximum at c in $[a, b]$ if $f(x) \leq f(c)$ for all x in $[a, b]$. f has an absolute minimum at c in $[a, b]$ if $f(x) \geq f(c)$ for all x in $[a, b]$. 3. The function must be continuous on a closed interval.

5.

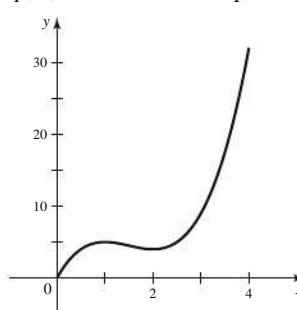


7.

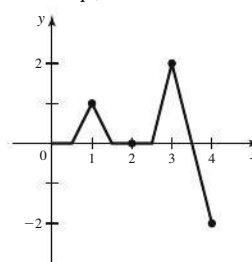


9. Evaluate the function at the critical points and at the endpoints of the interval. 11. Abs. min at $x = c_2$; abs. max at $x = b$ 13. Abs. min at $x = a$; no abs. max 15. Local min at $x = q$; s; local max at $x = p$; r; abs. min at $x = a$; abs. max at $x = b$ 17. Local max at $x = p$; r; local min at $x = q$; abs. max at $x = p$; abs. min at $x = b$

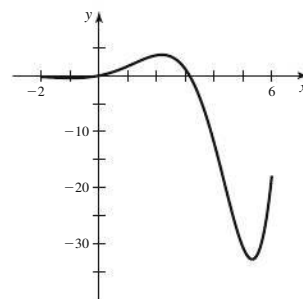
19.



21.

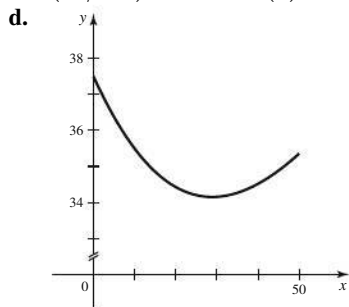


23. $x = \frac{2}{3}$ 25. $x = \pm 3$ 27. $x = -\frac{2}{3}, \frac{1}{3}$ 29. $x = \pm \frac{2a}{\sqrt{3}}$
 31. $t = \pm 1$ 33. $x = 0$ 35. $x = 1$ 37. $x = -4, 0$
 39. If $a \geq 0$, there is no critical point. If $a < 0$, $x = 2a/3$ is the only critical point. 41. $t = \pm a$ 43. Abs. max: -1 at $x = 3$; abs. min: -10 at $x = 0$ 45. Abs. max: 0 at $x = 0, 3$; abs. min: -4 at $x = -1, 2$ 47. Abs. max: 234 at $x = 3$; abs. min: -38 at $x = -1$ 49. Abs. max: 1 at $x = 0, \pi$; abs. min: 0 at $x = \pi/2$ 51. Abs. max: 1 at $x = \pi/6$; abs. min: -1 at $x = -\pi/6$
 53. Abs. min: $(\sqrt{1/e})^{1/e}$ at $x = 1/(2e)$; abs. max: 2 at $x = 1$
 55. Abs. max: $1 + \pi$ at $x = -1$; abs. min: 1 at $x = 1$
 57. Abs. max: 11 at $x = 1$; abs. min: -16 at $x = 4$
 59. Abs. max: 27 at $x = -3$; abs. min: $-\frac{19}{12}$ at $x = \frac{1}{2}$
 61. Abs. max: $\frac{1}{100,000}$ at $x = 1$; abs. min: $-\frac{1}{100,000}$ at $x = -1$
 63. Abs. max: $\sqrt{2}$ at $x = \pm \pi/4$; abs. min: 1 at $x = 0$
 65. Abs. max: $27/e^3$ at $x = 3$; abs. min: $-e$ at $x = -1$
 67. Abs. max: 3 at $x = \pm 1$; abs. min: 0 at $x = -2, 0, 2$
 69. a. The velocity of the downstream wind v_2 is less than or equal to the velocity of the upstream wind, so $0 \leq v_2 \leq v_1$, or $0 \leq \frac{v_2}{v_1} \leq 1$.
 b. $R(1) = 0$ c. $R(0) = \frac{1}{2}$ d. 0.593 is the maximum fraction of power that can be extracted from a wind stream by a wind turbine.
 71. $t = 2$ s 73. $t = 2$ s 75. a. 50 b. 45 77. a. False
 b. False c. False d. True 79. a. $x = -0.96, 2.18, 5.32$
 b. Abs. max: 3.72 at $x = 2.18$; abs. min: -32.80 at $x = 5.32$
 c.



81. a. $x = \tan^{-1} 2 + k\pi$, for $k = -2, -1, 0, 1$
 b. $x = \tan^{-1} 2 + k\pi$, for $k = -2, 0$, correspond to local max; $x = \tan^{-1} 2 + k\pi$, for $k = -1, 1$, correspond to local min.
 c. Abs. max: 2.24 ; abs. min: -2.24 83. a. $x = 5 - 4\sqrt{2}$
 b. $x = 5 - 4\sqrt{2}$ corresponds to a local max. c. No abs. max or min
 85. Abs. max: 4 at $x = -1$; abs. min: -8 at $x = 3$
 87. a. $T(x) = \frac{\sqrt{2500 + x^2}}{2} + \frac{50 - x}{4}$ b. $x = 50/\sqrt{3}$

c. $T(50/\sqrt{3}) \approx 34.15$, $T(0) = 37.50$, $T(50) \approx 35.36$



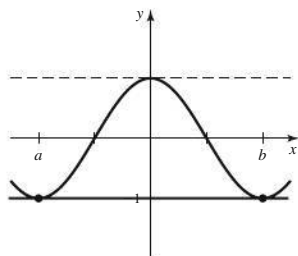
89. a. 1, 3, 0, 1 b. Because $h'(2) \neq 0$, h does not have a local extreme value at $x = 2$. However, g may have a local extremum at $x = 2$ (because $g'(2) = 0$). 91. a. $f(x) - f(c) \leq 0$ for all x near c

b. $\lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c} \leq 0$ c. $\lim_{x \rightarrow c^-} \frac{f(x) - f(c)}{x - c} \geq 0$

d. Because $f'(c)$ exists, $\lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c} = \lim_{x \rightarrow c^-} \frac{f(x) - f(c)}{x - c} = f'(c)$. By parts (b) and (c), we must have that $f'(c) = 0$.

Section 4.2 Exercises, pp. 254–257

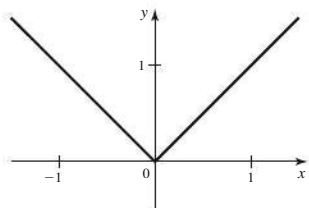
1. If f is a continuous function on the closed interval $[a, b]$ and is differentiable on (a, b) , and the slope of the secant line that joins $(a, f(a))$ to $(b, f(b))$ is zero, then there is at least one value c in (a, b) at which the slope of the line tangent to f at $(c, f(c))$ is also zero.



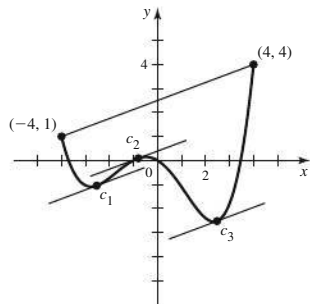
3. $f(x) = |x|$ is not differentiable at 0. 5. b. $c = 1$

7. b. $c = \pm 2/\sqrt[4]{5} \approx \pm 1.34$

9. 11. $x = \frac{1}{3}$



13. $x = \pi/4$ 15. Does not apply 17. $x = \frac{5}{3}$ 19. Average lapse rate $= -6.3^\circ/\text{km}$. You cannot conclude that the lapse rate at a point exceeds the threshold value. 21. a. Yes b. $c = \frac{1}{2}$ 23. a. Does not apply 25. a. Yes b. $c = \ln(e - 1)$ 27. a. Yes b. $c \approx \pm 0.881$ 29. a. Yes b. $c = \sqrt{1 - 9/\pi^2}$ 31. a. Does not apply 33. a. False b. True c. False 37. h and p 39.



41. No such point exists; function is not continuous at 2.

43. The car's average velocity is $(30 - 0)/(28/60) = 64.3$ mi/hr. By the MVT, the car's instantaneous velocity was 64.3 mi/hr at some time.

45. Average speed $= 11.6$ mi/hr. By the MVT, the speed was exactly 11.6 mi/hr at least once. By the Intermediate Value Theorem, all speeds between 0 and 11.6 mi/hr were reached. Because the initial and final speeds were 0 mi/hr, the speed of 11 mi/hr was reached at least twice.

47. $\frac{f(b) - f(a)}{b - a} = A(a + b) + B$ and $f'(x) = 2Ax + B$;

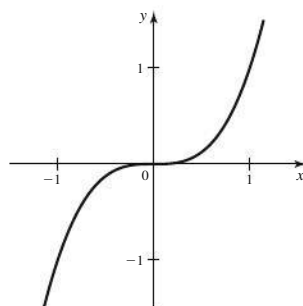
$2Ax + B = A(a + b) + B$ implies that $x = \frac{a + b}{2}$, the midpoint of $[a, b]$. 49. $\tan^2 x$ and $\sec^2 x$ differ by a constant; in fact,

$\tan^2 x - \sec^2 x = -1$. 53. Hint: By the MVT, there is a value of c in $(2, 4)$ for which $\frac{f(4) - f(2)}{4 - 2} = f'(c)$. 57. b. $c = \frac{1}{2}$

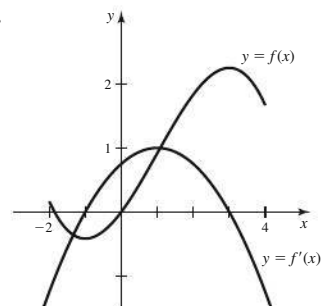
Section 4.3 Exercises, pp. 267–270

1. f is increasing on I if $f'(x) > 0$ for all x in I ; f is decreasing on I if $f'(x) < 0$ for all x in I . 3. a. $x = 3$ b. Increasing on $(3, \infty)$; decreasing on $(-\infty, 3)$

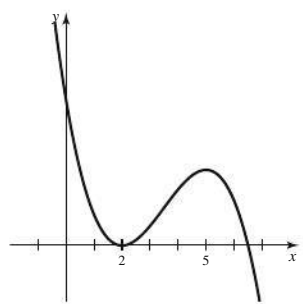
5.



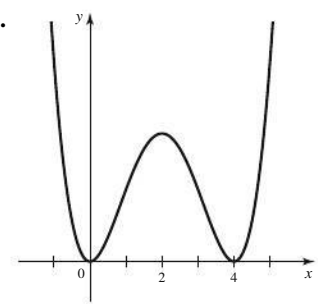
7.



9.



11.



13. a. Concave up on $(-\infty, 2)$; concave down on $(2, \infty)$

b. Inflection point at $x = 2$ 15. Yes; consider the graph of $y = \sqrt{x}$ on $(0, \infty)$. 17. $f(x) = x^4$ 19. Increasing on $(-\infty, 0)$; decreasing on $(0, \infty)$

21. Decreasing on $(-\infty, 1)$; increasing on $(1, \infty)$

23. Increasing on $(-\infty, 1)$ and $(4, \infty)$; decreasing on $(1, 4)$

25. Increasing on $(-\infty, 1/2)$; decreasing on $(1/2, \infty)$

27. Increasing on $(-\infty, 0)$, $(1, 2)$; decreasing on $(0, 1)$, $(2, \infty)$

29. Increasing on $(-\frac{1}{\sqrt{e}}, 0)$, $(\frac{1}{\sqrt{e}}, \infty)$; decreasing on

$(-\infty, -\frac{1}{\sqrt{e}})$, $(0, \frac{1}{\sqrt{e}})$ 31. Increasing on $(\frac{\pi}{6}, \frac{5\pi}{6})$; decreasing

on $(0, \frac{\pi}{6})$, $(\frac{5\pi}{6}, 2\pi)$ 33. Increasing on $(-\pi, -2\pi/3)$,

$(-\pi/3, 0)$, $(\pi/3, 2\pi/3)$; decreasing on $(-2\pi/3, -\pi/3)$,

$(0, \pi/3)$, $(2\pi/3, \pi)$ 35. Increasing on $(-1, 0)$, $(1, \infty)$;

decreasing on $(-\infty, -1)$, $(0, 1)$ 37. Increasing on $(-3, 1)$;

decreasing on $(1, 3)$ 39. Increasing on $(1, 4)$; decreasing on

$(-\infty, 1)$, $(4, \infty)$ 41. Increasing on $(-\infty, -\frac{1}{2})$, $(0, \frac{1}{2})$;

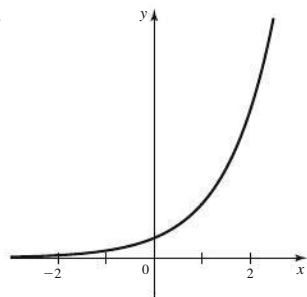
decreasing on $(-\frac{1}{2}, 0)$, $(\frac{1}{2}, \infty)$ 43. Increasing on $(-1, 1)$;

decreasing on $(-\infty, -1)$, $(1, \infty)$ 45. a. $x = 0$ b. Local min

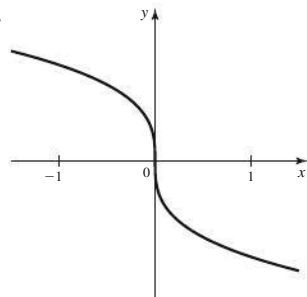
at $x = 0$ c. Abs. min: 3 at $x = 0$; abs. max: 12 at $x = -3$

47. a. $x = \pm\sqrt{2}$ b. Local min at $x = -\sqrt{2}$; local max at $x = \sqrt{2}$
 c. Abs. max: 2 at $x = \sqrt{2}$; abs. min: -2 at $x = -\sqrt{2}$
 49. a. $x = \pm\sqrt{3}$ b. Local min at $x = -\sqrt{3}$; local max at $x = \sqrt{3}$ c. Abs. max: 28 at $x = -4$; abs. min: $-6\sqrt{3}$ at $x = -\sqrt{3}$ 51. a. $x = 2$ and $x = 0$ b. Local max at $x = 0$; local min at $x = 2$ c. Abs. min: $-10\sqrt[3]{25}$ at $x = -5$; abs. max: 0 at $x = 0, 5$ 53. a. $x = e^{-2}$ b. Local min at $x = e^{-2}$ c. Abs. min: $-2/e$ at $x = e^{-2}$; no abs. max 55. Abs. max: $1/e$ at $x = 1$ 57. Abs. min: $36\sqrt[3]{\pi/6}$ at $r = \sqrt[3]{6/\pi}$

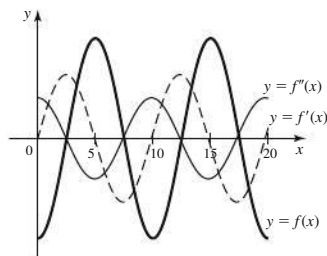
59.



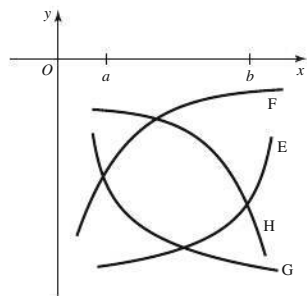
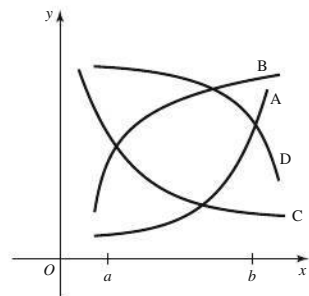
61.



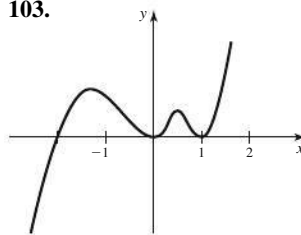
63. Concave up on $(-\infty, 0)$, $(1, \infty)$; concave down on $(0, 1)$; inflection points at $x = 0$ and $x = 1$ 65. Concave up on $(-\infty, 0)$, $(2, \infty)$; concave down on $(0, 2)$; inflection points at $x = 0$ and $x = 2$ 67. Concave down on $(-\infty, 1)$; concave up on $(1, \infty)$; inflection point at $x = 1$ 69. Concave up on $(-1/\sqrt{3}, 1/\sqrt{3})$; concave down on $(-\infty, -1/\sqrt{3})$, $(1/\sqrt{3}, \infty)$; inflection points at $t = \pm 1/\sqrt{3}$ 71. Concave up on $(-\infty, -1)$, $(1, \infty)$; concave down on $(-1, 1)$; inflection points at $x = \pm 1$ 73. Concave up on $(0, 1)$; concave down on $(1, \infty)$; inflection point at $x = 1$ 75. Concave up on $(0, 2)$, $(4, \infty)$; concave down on $(-\infty, 0)$, $(2, 4)$; inflection points at $x = 0, 2, 4$ 77. Critical pts. $x = 0$ and $x = 2$; local max at $x = 0$, local min at $x = 2$ 79. Critical pt. $x = 0$; local max at $x = 0$ 81. Critical pt. $x = 6$; local min at $x = 6$ 83. Critical pts. $x = 0$ and $x = 1$; local max at $x = 0$; local min at $x = 1$ 85. Critical pts. $x = 0$ and $x = 2$; local min at $x = 0$; local max at $x = 2$ 87. Critical pt. $x = e^5$; local min at $x = e^5$ 89. Critical pts. $t = -3$ and $t = 2$; local max at $t = -3$; local min at $t = 2$ 91. Critical pt. $x = -a$; local min at $x = -a$ 93. Critical pts. $x = \pm\sqrt[3]{e}$; local min at $x = \pm\sqrt[3]{e}$ 95. a. True b. False c. True d. False e. False 97. a-C-i, b-B-iii, c-A-ii



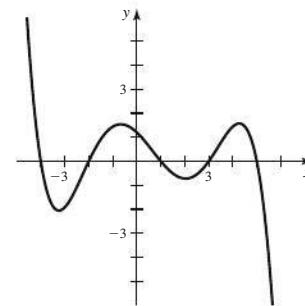
101.



103.

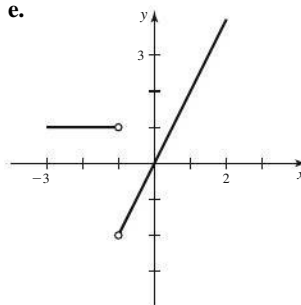


105.

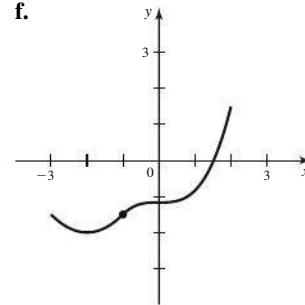


107. a. Increasing on $(-2, 2)$; decreasing on $(-3, -2)$
 b. Critical pts. $x = -2$ and $x = 0$; local min at $x = -2$; neither a local max nor min at $x = 0$ c. Inflection pts. at $x = -1$ and $x = 0$
 d. Concave up on $(-3, -1)$, $(0, 2)$; concave down on $(-1, 0)$

e.



f.



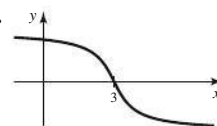
109. a. $E = \frac{p}{p-50}$ b. -1.4% c. $E'(p) = -\frac{ab}{(a-bp)^2} < 0$, for $p \geq 0, p \neq a/b$ d. $E(p) = -b$, for $p \geq 0$

111. a. 300 b. $t = \sqrt{10}$ c. $t = \sqrt{b/3}$

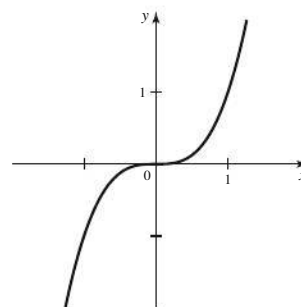
Section 4.4 Exercises, pp. 277-280

1. We need to know on which interval(s) to graph f . 3. No; the domain of any polynomial is $(-\infty, \infty)$; there is no vertical asymptote. Also, $\lim_{x \rightarrow \pm\infty} p(x) = \pm\infty$, where p is any polynomial; there is no horizontal asymptote. 5. Evaluate the function at the critical points and at the endpoints. Then find the largest and smallest values among those candidates.

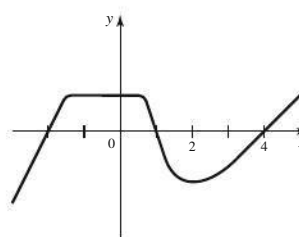
7.



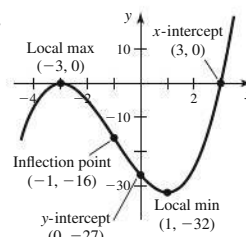
9.

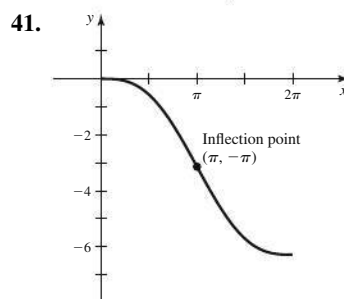
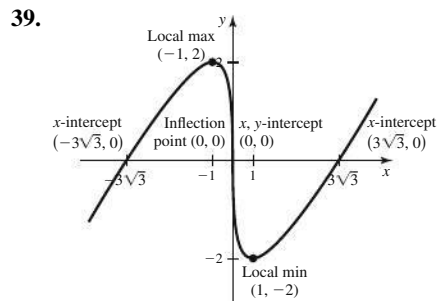
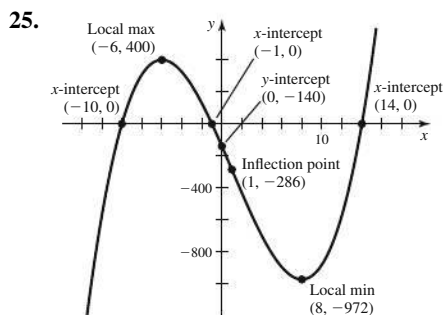
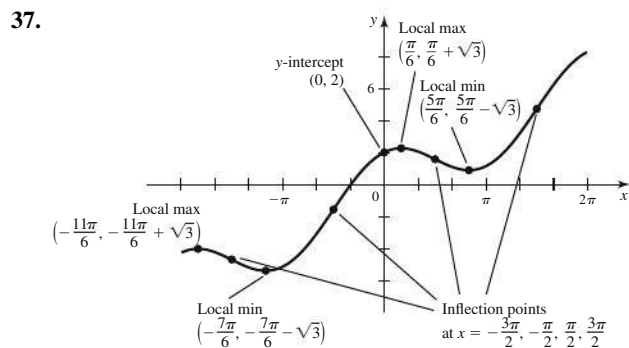
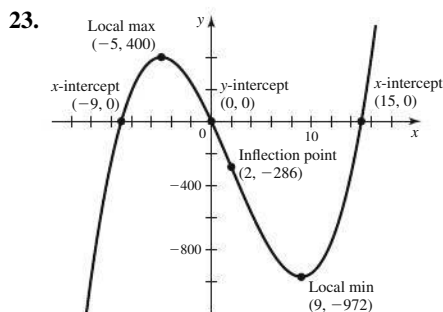
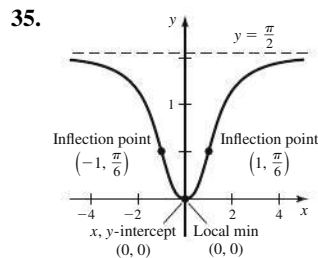
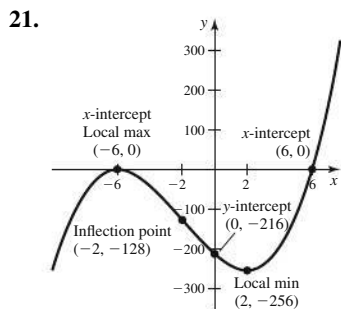
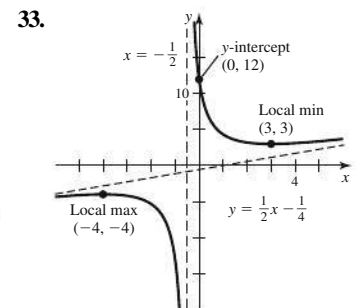
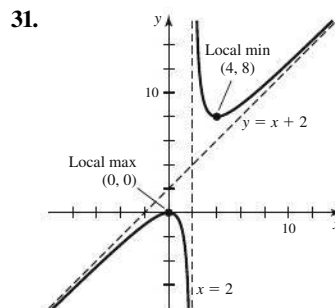
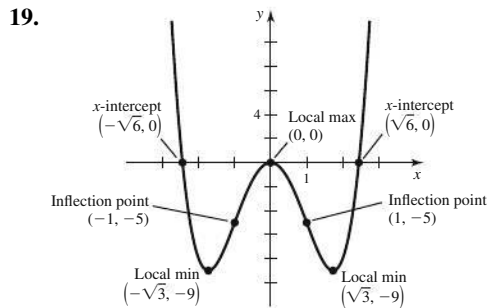
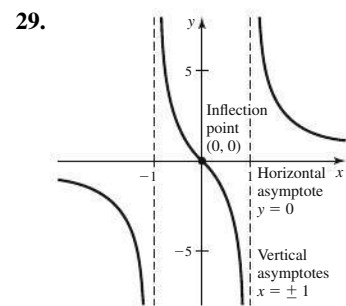
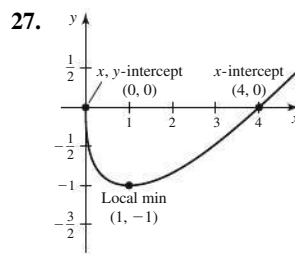
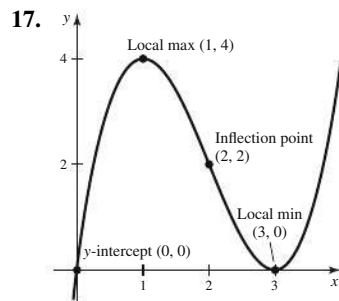
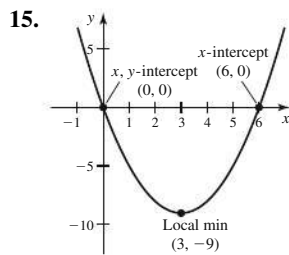


11.

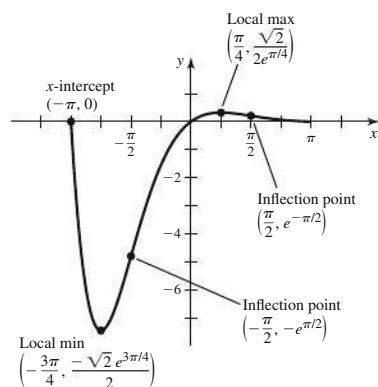


13.

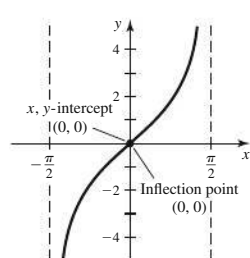




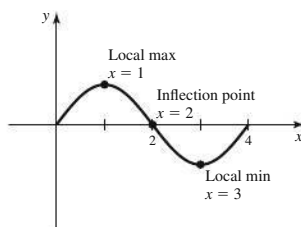
43.



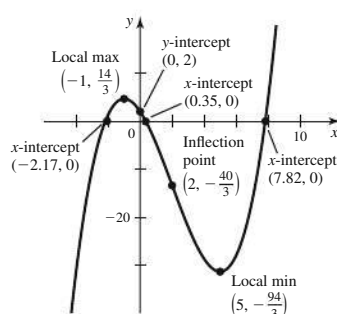
45.



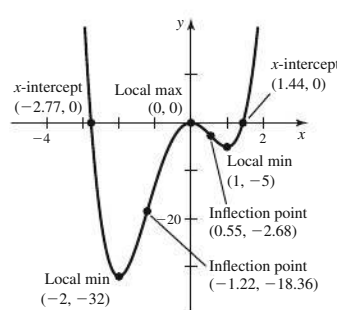
47. Critical pts. at $x = 1, 3$; local max at $x = 1$; local min at $x = 3$; inflection pt. at $x = 2$; increasing on $(0, 1), (3, 4)$; decreasing on $(1, 3)$; concave up on $(2, 4)$; concave down on $(0, 2)$



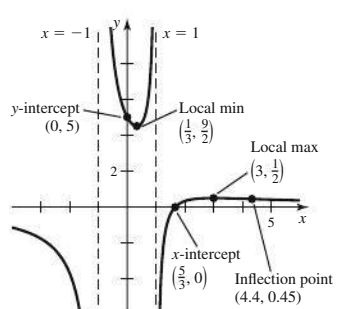
49.



51.

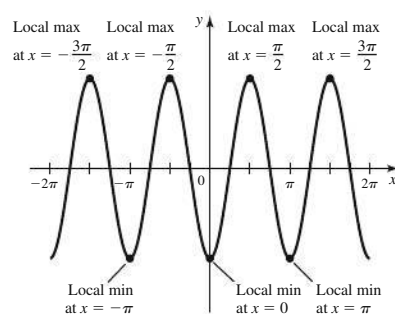


53.

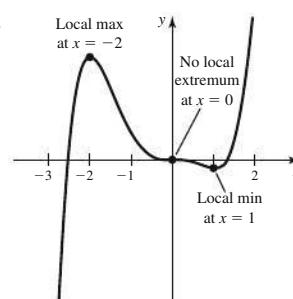


55. a. False b. False
 c. False d. True

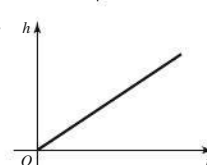
57.



59.

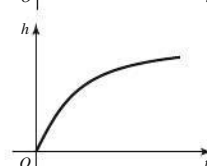


61. (A) a.



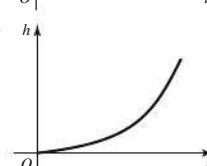
b. Water is being added at all times. c. No concavity
 d. h' has an abs. max at all points of $[0, 10]$.

(B) a.



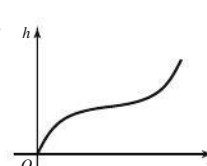
c. Concave down d. h' has abs. max at $t = 0$.

(C) a.



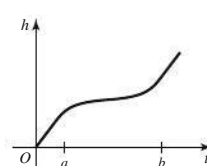
c. Concave up d. h' has abs. max at $t = 10$.

(D) a.



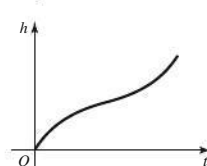
c. Concave down on $(0, 5)$, then concave up on $(5, 10)$; inflection pt. at $t = 5$ d. h' has abs. max at $t = 0$ and $t = 10$.

(E) a.



c. First, no concavity; then concave down, no concavity, concave up, and, finally, no concavity
 d. h' has abs. max at all points of an interval $[0, a]$ and $[b, 10]$.

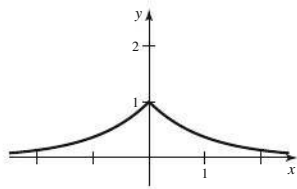
(F) a.



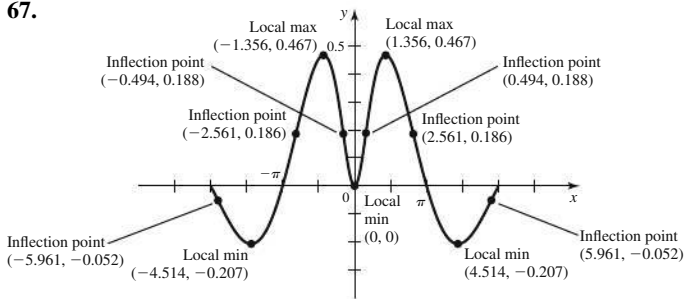
c. Concave down on $(0, 5)$; concave up on $(5, 10)$; inflection pt. at $t = 5$ d. h' has abs. max at $t = 0$ and $t = 10$.

63. Local max of $e^{1/e}$ at $x = e$

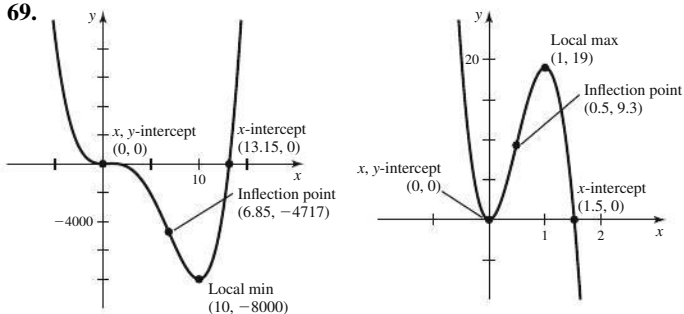
65. $f'(0)$ does not exist.



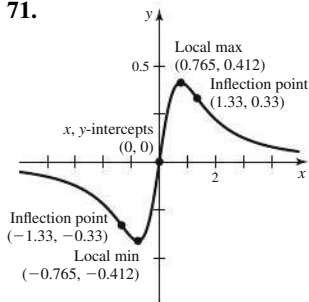
67.



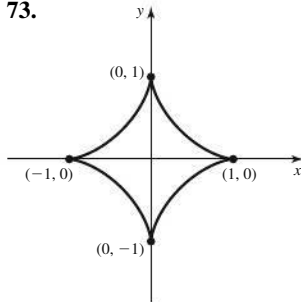
69.



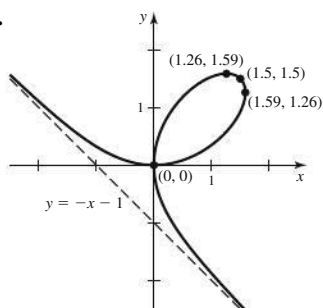
71.



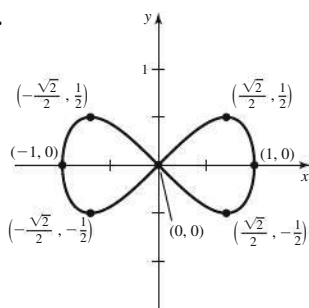
73.



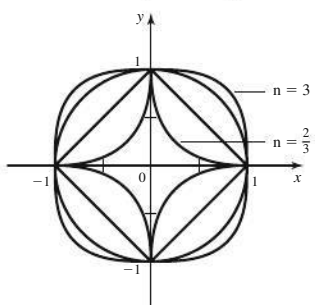
75.



77.



79.



Section 4.5 Exercises, pp. 284–291

1. Objective function, constraint(s) 3. $Q = x^2(10 - x)$; $Q = (10 - y)^2 y$ 5. a. $P(x) = 100x - 10x^2$ b. Abs. max: 250 at $x = 5$ 7. $\frac{23}{2}$ and $\frac{23}{2}$ 9. $5\sqrt{2}$ and $5\sqrt{2}$ 11. Width = length = $\frac{5}{2}$ 13. Width = length = 10 15. $x = \sqrt{6}$, $y = 2\sqrt{6}$

17. $\frac{10}{\sqrt{2}}$ cm by $\frac{5}{\sqrt{2}}$ cm 19. Length = width = height = 2

21. $\frac{4}{\sqrt[3]{5}}$ ft by $\frac{4}{\sqrt[3]{5}}$ ft by $5^{2/3}$ ft 23. Approx. (0.59, 0.65)

25. (5, 15), distance ≈ 47.4 27. a. A point $8/\sqrt{5}$ mi from the point on the shore nearest the woman in the direction of the restaurant b. $9/\sqrt{13}$ mi/hr 29. A point $7\sqrt{3}/6$ mi from the point on shore nearest the island, in the direction of the power station

31. 18.2 ft 33. $h = \frac{20}{\sqrt{3}}$; $r = 20\sqrt{\frac{2}{3}}$

35. a. $r = \sqrt[3]{177/\pi} \approx 3.83$ cm; $h = 2\sqrt[3]{177/\pi} \approx 7.67$ cm

b. $r = \sqrt[3]{177/2\pi} \approx 3.04$ cm; $h = 2\sqrt[3]{708/\pi} \approx 12.17$ cm; part (b) is closer to the real can. 37. $\sqrt{15}$ m by $2\sqrt{15}$ m

39. 12" by 6" by 3"; 216 in³ 41. Lower rectangular pane is approximately 5.6 ft wide by 2.8 ft high. 43. $r/h = \sqrt{2}$

45. $r = \sqrt{2}R/\sqrt{3}$; $h = 2R/\sqrt{3}$ 47. 3:1 49. a. 0, 30, 25

b. 42.5 mi/hr c. The units of $p/g(v)$ are \$/mi and so are the units

of w/v . Therefore, $L\left(\frac{p}{g(v)} + \frac{w}{v}\right)$ gives the total cost of a trip of L

miles. d. Approx. 62.9 mi/hr e. Neither; the zeros of $C'(v)$ are independent of L . f. Decreased slightly, to 62.5 mi/hr

g. Decreased to 60.8 mi/hr 51. $\sqrt{30} \approx 5.5$ ft 53. The point $12/(\sqrt{2} + 1) \approx 5.3$ m from the weaker source

55. b. Because the speed of light is constant, travel time is minimized when distance is minimized. 57. $r = h = \sqrt[3]{450/\pi}$ m 59. a. $\frac{a_1 + a_2}{2}$

b. $\frac{a_1 + a_2 + a_3}{3}$ c. $\frac{a_1 + a_2 + \dots + a_n}{n}$ 61. $\frac{\pi}{3}$

63. When the seat is at its lowest point 65. For $L \leq 4r$, max at $\theta = 0$ and $\theta = 2\pi$; min at $\theta = \cos^{-1}(-L/(4r))$ and $\theta = 2\pi - \cos^{-1}(-L/(4r))$. For $L > 4r$, max at $\theta = 0$ and $\theta = 2\pi$; min at $\theta = \pi$.

67. a. $P = 2/\sqrt{3}$ units from the midpoint of the base 69. You can run 12 mi/hr if you run toward the point

$3/16$ mi ahead of the locomotive (when it passes the point nearest you). 71. a. $r = 2R/3$; $h = \frac{1}{3}H$ b. $r = R/2$; $h = H/2$

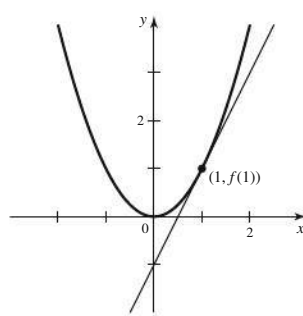
73. (1 + $\sqrt{3}$) mi ≈ 2.7 mi 75. (i) $(p - \frac{1}{2}, \sqrt{p - \frac{1}{2}})$

(ii) (0, 0) 77. Let the angle of the cuts be φ_1 and φ_2 , where $\varphi_1 + \varphi_2 = \theta$. The volume of the notch is proportional to $\tan \varphi_1 + \tan \varphi_2 = \tan \varphi_1 + \tan(\theta - \varphi_1)$, which is minimized

when $\varphi_1 = \varphi_2 = \frac{\theta}{2}$. 79. $x \approx 38.81$, $y \approx 55.03$

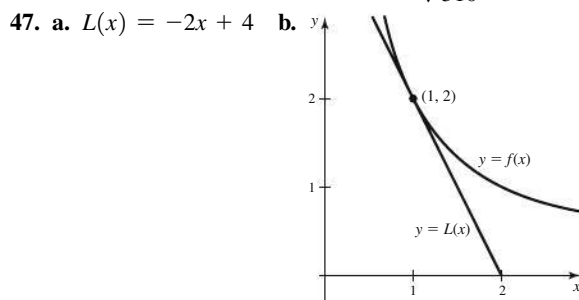
Section 4.6 Exercises, pp. 298–300

1. 3. $f(x) \approx f(a) + f'(a)(x - a)$



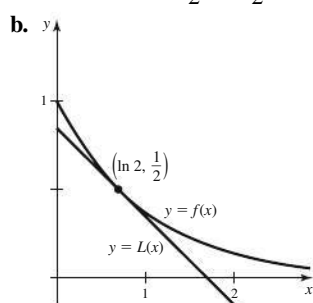
5. $L(x) = 3x - 1$; 2.3 7. 2.7 9. $dy = f'(x) dx$

11. Approx. 25 13. 61 mi/hr; 61.02 mi/hr
 15. $L(x) = T(0) + T'(0)(x - 0) = D - (D/60)x = D(1 - x/60)$
 17. 84 min; 84.21 min 19. $L(x) = 9x - 4$ 21. $L(t) = t + 5$
 23. $L(x) = 3x - 5$ 25. a. $L(x) = -4x + 16$ b. 7.6
 c. 0.13% error 27. a. $L(x) = x$ b. 0.9 c. 40% error
 29. a. $L(x) = 1$ b. 1 c. 0.005% error 31. a. $L(x) = \frac{1}{2} - \frac{x}{48}$
 b. 0.5 c. 0.003% error 33. a. $L(x) = 1 - x$; b. $1/1.1 \approx 0.9$
 c. 1% error 35. a. $L(x) = 1 - x$ b. $e^{-0.03} \approx 0.97$
 c. 0.05% error 37. $a = 200$; $\frac{1}{203} \approx 0.004925$
 39. $a = 144$; $\sqrt{146} \approx \frac{145}{12}$ 41. $a = 1$; $\ln 1.05 \approx 0.05$
 43. $a = 0$; $e^{0.06} \approx 1.06$ 45. $a = 512$; $\frac{1}{\sqrt[3]{510}} \approx 0.125$



c. Underestimates d. $f''(1) = 4 > 0$

49. a. $L(x) = -\frac{1}{2}x + \frac{1}{2}(1 + \ln 2)$



c. Underestimates d. $f''(\ln 2) = \frac{1}{2} > 0$

51. $E(x) \leq 1$ when $-7.26 \leq x \leq 8.26$, which corresponds to driving times for 1 mi from about 53 s to 68 s. Therefore, $L(x)$ gives approximations to $s(x)$ that are within 1 mi/hr of the true value when you drive 1 mi in t seconds, where $53 < t < 68$.

53. a. True b. False c. True d. True

55. $\Delta V \approx 10\pi \text{ ft}^3$ 57. $\Delta V \approx -40\pi \text{ cm}^3$

59. $\Delta S \approx -\frac{59\pi}{5\sqrt{34}} \text{ m}^2$ 61. $dy = 2 dx$ 63. $dy = -\frac{3}{x^4} dx$

65. $dy = a \sin x dx$ 67. $dy = (9x^2 - 4) dx$

69. $dy = \sec^2 x dx$

71. $L(x) = 2 + (x - 8)/12$

x	Linear approx.	Exact value	Percent error
8.1	2.008 $\bar{3}$	2.00829885	1.7×10^{-3}
8.01	2.0008 $\bar{3}$	2.000832986	1.7×10^{-5}
8.001	2.00008 $\bar{3}$	2.00008333	1.7×10^{-7}
8.0001	2.000008 $\bar{3}$	2.000008333	1.7×10^{-9}
7.9999	1.999991 $\bar{6}$	1.999991667	1.7×10^{-9}
7.999	1.99991 $\bar{6}$	1.999916663	1.7×10^{-7}
7.99	1.9991 $\bar{6}$	1.999166319	1.7×10^{-5}
7.9	1.991 $\bar{6}$	1.991631701	1.8×10^{-3}

73. a. f ; the rate at which f' is changing at 1 is smaller than the rate at which g' is changing at 1. The graph of f bends away from the linear function more slowly than the graph of g . b. The larger the value of $|f''(a)|$, the greater the deviation of the curve $y = f(x)$ from the tangent line at points near $x = a$.

Section 4.7 Exercises, pp. 310–312

1. If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x) = 0$, then we say $\lim_{x \rightarrow a} f(x)/g(x)$ is an indeterminate form $0/0$. 3. Take the limit of the quotient of the derivatives of the functions.

5. a. $\lim_{x \rightarrow 0} \left(x^2 \cdot \frac{1}{x^2} \right) = 1$ b. $\lim_{x \rightarrow 0} \left(2x^2 \cdot \frac{1}{x^2} \right) = 2$

7. If $\lim_{x \rightarrow a} f(x)g(x)$ has the indeterminate form $0 \cdot \infty$, then

$\lim_{x \rightarrow a} \left(\frac{f(x)}{1/g(x)} \right)$ has the indeterminate form $0/0$ or ∞/∞ .

9. $\frac{1}{5}$ 11. If $\lim_{x \rightarrow a} f(x) = 1$ and $\lim_{x \rightarrow a} g(x) = \infty$, then

$f(x)^{g(x)}$ has the form 1^∞ as $x \rightarrow a$, which is meaningless; so direct substitution does not work.

13. $\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = 0$ 15. $\ln x, x^3, 2^x, x^x$

17. -1 19. $\frac{3}{4}$ 21. $\frac{1}{2}$ 23. $\frac{1}{2}$ 25. $\frac{1}{e}$ 27. -1 29. $\frac{12}{5}$

31. 4 33. $\frac{9}{16}$ 35. $\frac{1}{2}$ 37. $-\frac{2}{3}$ 39. $\frac{1}{24}$ 41. 1 43. 4

45. 1 47. $-\frac{1}{2}$ 49. $\frac{1}{\pi^2}$ 51. $\frac{1}{3}$ 53. 1 55. $\frac{7}{6}$ 57. 1

59. $-\frac{1}{2}$ 61. 0 63. -8 65. 0 67. $\frac{1}{2}$ 69. $\frac{\ln 3}{\ln 2}$

71. $-\frac{9}{4}$ 73. $\frac{1}{6}$ 75. 1 77. 1 79. e 81. e^{a+1} 83. e

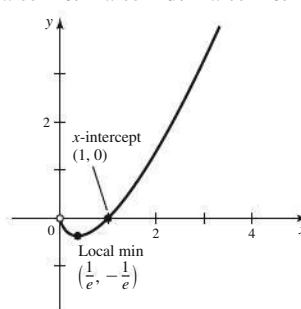
85. b. $\lim_{m \rightarrow \infty} (1 + r/m)^m = \lim_{m \rightarrow \infty} \left(1 + \frac{1}{(m/r)} \right)^{(m/r)r} = e^r$

87. 3 89. $\frac{1}{2}$ 91. e 93. $\ln a - \ln b$ 95. $e^{0.01x}$ 97. Comparable growth rates

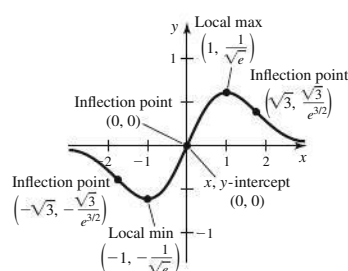
99. x^x 101. 1.00001^x 103. e^{x^2} 105. a. False

b. False c. False d. False e. True f. True

107.



109.



111. $\sqrt{a/c}$ 113. $\lim_{x \rightarrow \infty} \frac{x^p}{b^x} = \lim_{t \rightarrow \infty} \frac{\ln^p t}{t \ln^p b} = 0$, where $t = b^x$

115. Show $\lim_{x \rightarrow \infty} \frac{\log_a x}{\log_b x} = \frac{\ln b}{\ln a} \neq 0$. 117. $1/3$ 121. a. $b > e$

b. e^{ax} grows faster than e^x as $x \rightarrow \infty$, for $a > 1$; e^{ax} grows slower than e^x as $x \rightarrow \infty$, for $0 < a < 1$.

Section 4.8 Exercises, pp. 318–321

1. Newton's method generates a sequence of x -intercepts of lines tangent to the graph of f to approximate the roots of f .

3. $x_1 = 2, x_2 = 1, x_3 = 0$ 5. $x_1 = 0.75$ 7. Generally, if two successive Newton approximations agree in their first p digits, then those approximations have p digits of accuracy. The method is terminated when the desired accuracy is reached.

9. $x_{n+1} = x_n - \frac{x_n^2 - 6}{2x_n} = \frac{x_n^2 + 6}{2x_n}; x_1 = 2.5, x_2 = 2.45$

11. $x_{n+1} = x_n - \frac{e^{-x_n} - x_n}{e^{-x_n} - 1}; x_1 = 0.564382, x_2 = 0.567142$

13.

n	x_n
0	3
1	3.1667
2	3.16228
3	3.16228

$r \approx 3.16228$

15.

n	x_n
0	0.5
1	0.51096
2	0.51097
3	0.51097

$r \approx 0.51097$

17.

n	x_n
0	1.2
1	1.16935
2	1.16561
3	1.16556
4	1.16556

$r \approx 1.16556$

19.

n	x_n
0	0.75
1	0.73915
2	0.73909
3	0.73909

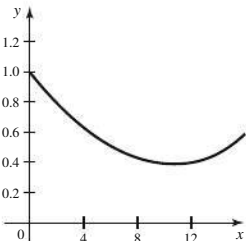
$r \approx 0.73909$

21. $x \approx -0.335408, 1.333057$ 23. $x \approx 0.179295$

25. $x \approx 0.620723, 3.03645$ 27. $x \approx 0, 1.895494, -1.895494$

29. $x \approx -2.114908, 0.254102, 1.860806$

31. $x \approx 0.062997, 2.230120$

33.  The tumor decreases in size and then starts growing again. It decreases to half its size after about 6.4 days.

35. $b. r \approx 7.3\%$ 37. $a. t = \pi/4$ $b. t \approx 1.33897$

$c. t \approx 2.35619$ $d. t \approx 2.90977$

39. $p(x) = x^4 - 7; r \approx 1.62658$

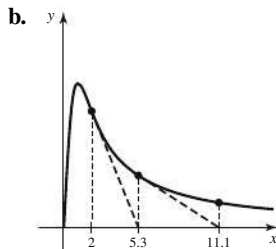
41. $p(x) = x^3 + 9; r \approx -2.08008$ 43. $x \approx 2.798386$

45. $x \approx -0.666667, 1.5, 1.666667$ 47. $a. \text{True}$ $b. \text{False}$

$c. \text{False}$ 49. $x \approx 0.739085$ 51. $x = 0$ and $x \approx 1.047198$

53. a.

n	x_n
0	2
1	5.33333
2	11.0553
3	22.2931



c. The tangent lines intersect the x -axis farther and farther away from the root r . 55. $b. x \approx 0.142857$ is approximately $\frac{1}{7}$.

57. $\lambda = 1.29011, 2.37305, 3.40918$

Section 4.9 Exercises, pp. 331–334

1. The derivative, an antiderivative 3. $x + C$, where C is an

arbitrary constant 5. $\frac{x^{p+1}}{p+1} + C$, where $p \neq -1$ 7. $\ln|x| + C$

9. 0 11. $x^5 + C$ 13. $-2 \cos x + x + C$ 15. $3 \tan x + C$

17. $y^{-2} + C$ 19. $e^x + C$ 21. $\tan^{-1} s + C$ 23. $\frac{1}{2} x^6 - \frac{1}{2} x^{10} + C$

25. $\frac{8}{3} x^{3/2} - 8x^{1/2} + C$ 27. $\frac{25}{3} s^3 + 15s^2 + 9s + C$

29. $\frac{9}{4} x^{4/3} + 6x^{2/3} + 6x + C$ 31. $-x^3 + \frac{11}{2} x^2 + 4x + C$

33. $-x^{-3} + 2x + 3x^{-1} + C$ 35. $x^4 - 3x^2 + C$

37. $\frac{1}{2} x^2 + 6x + C$ 39. $-\cot \theta + 2\theta^3/3 - 3\theta^2/2 + C$

41. $-2 \cot y - 3 \csc y + C$ 43. $\tan x - x + C$

45. $\tan \theta + \sec \theta + C$ 47. $t^3 - 2 \cot t + C$

49. $\sec \theta + \tan \theta + \theta + C$ 51. $\frac{1}{2} \ln |y| + C$ 53. $3 \sin^{-1} x + C$

55. $4 \sec^{-1} |x| + C$ 57. $\frac{1}{6} \sec^{-1} |x| + C$ 59. $t + \ln |t| + C$

61. $e^{x+2} + C$ 63. $e^w - 4w + C$ 65. $\ln |x| + 2\sqrt{x} + C$

67. $\frac{4}{15} x^{15/2} - \frac{24}{11} x^{11/6} + C$ 69. $\frac{1}{6} x^6 - \frac{2}{3} x^3 + x + 1$

71. $2x^4 - \cos x + 3$ 73. $\sec v + 1, -\pi/2 < v < \pi/2$

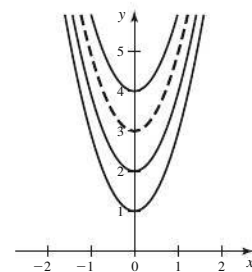
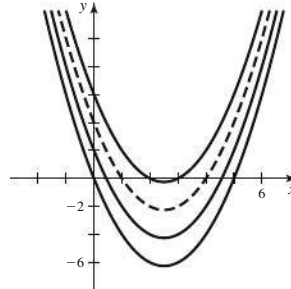
75. $y^3 + 5 \ln y + 2, y > 0$ 77. $f(x) = x^2 - 3x + 4$

79. $g(x) = \frac{7}{8} x^8 - \frac{x^2}{2} + \frac{13}{8}$ 81. $f(u) = 4(\sin u + \cos u) - 4$

83. $y(t) = 3 \ln t + 6t + 2, t > 0$

85. $y(\theta) = \sqrt{2} \sin \theta + \tan \theta + 1, -\pi/2 < \theta < \pi/2$

87. $f(x) = x^2 - 5x + 4$ 89. $f(x) = \frac{3}{2} x^2 - \cos x + 4$



91. $s(t) = t^2 + 4t$ 93. $s(t) = \frac{4}{3} t^{3/2} + 1$

95. $s(t) = 2t^3 + 2t^2 - 10t$ 97. $s(t) = -16t^2 + 20t$

99. $s(t) = \frac{1}{30} t^3 + 1$ 101. $s(t) = t^2 + 4t - 3 \sin t + 10$

103. 200 ft 105. Runner A overtakes runner B at $t = \pi/2$

107. $a. v(t) = -9.8t + 30$ $b. s(t) = -4.9t^2 + 30t$

$c. \text{Approx. } 45.92 \text{ m at time } t \approx 3.06$ $d. t \approx 6.12$

109. $a. v(t) = -9.8t + 10$ $b. s(t) = -4.9t^2 + 10t + 400$

$c. \text{Approx. } 405.10 \text{ m at time } t \approx 1.02$ $d. t \approx 10.11$

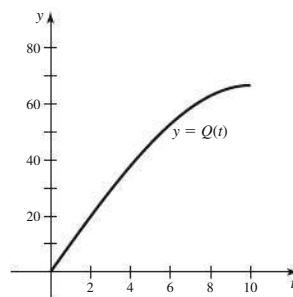
111. $a. \text{True}$ $b. \text{False}$ $c. \text{True}$ $d. \text{False}$

$e. \text{False}$ 113. $F(x) = -\cos x + 3x + 3 - 3\pi$

115. $F(x) = 2x^8 + x^4 + 2x + 1$

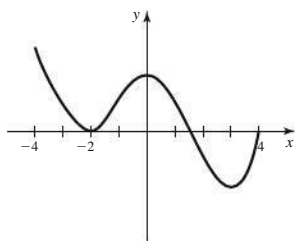
117. $a. Q(t) = 10t - t^3/30 \text{ gal}$

$b. \frac{200}{3} \text{ gal}$

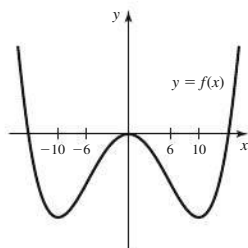


Chapter 4 Review Exercises, pp. 334–337

1. a. False b. False c. True d. True e. True f. False
3.



5. a. $x = 0, \pm 10$; increasing on $(-10, 0)$ and $(10, \infty)$, decreasing on $(-\infty, -10)$ and $(0, 10)$ b. $x = \pm 6$; concave up on $(-\infty, -6)$ and $(6, \infty)$, concave down on $(-6, 6)$ c. Local min at $x = -10, 10$; local max at $x = 0$ d.

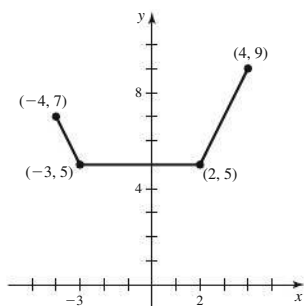


7. Critical pts. $x = 0, \pm 1$; abs. max: 33 at $x = \pm 2$; abs. min: 6 at $x = \pm 1$ 9. $x = 3$ and $x = -2$; no abs. max or min
11. Critical pt. $x = 1$; abs. max: $\ln 2$ at $x = 0, 2$; abs. min: 0 at $x = 1$

13. Critical pts. $x = \frac{2\pi}{3}, \frac{4\pi}{3}$; abs. max: $\frac{3\sqrt{3}}{8}$ at $x = \frac{4\pi}{3}$; abs. min: $-\frac{3\sqrt{3}}{8}$ at $x = \frac{2\pi}{3}$ 15. Critical pt. $x = 1/e$; abs.

min: $10 - 2/e$ at $x = 1/e$

17.



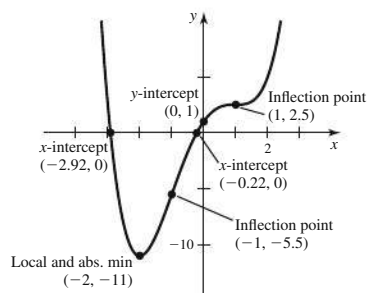
Critical pts.: all x in the interval $[-3, 2]$; abs. max: 9 at $x = 4$; abs. and local min: 5 for x in $[-3, 2]$; local max: 5 for x in $(-3, 2)$

19. a. Increasing on $(-\infty, -1)$ and $(1, \infty)$; decreasing on $(-1, 1)$

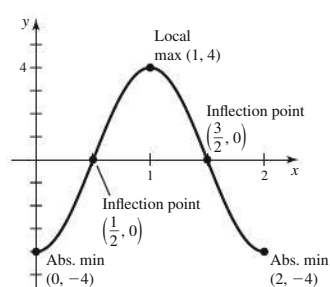
b. Concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$

21. Inflection pt. $x = 0$ 23. Critical pts. $x = -a, a/2$; inflection pts. $x = 0, -a$

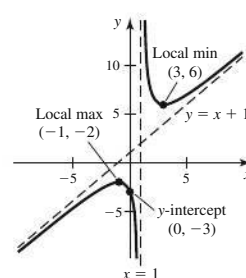
25.



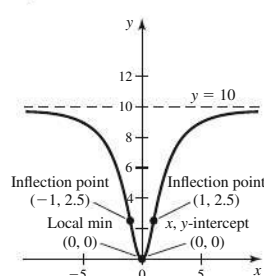
27.



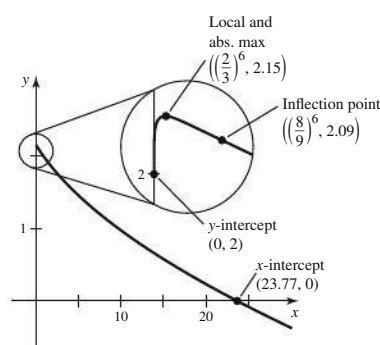
29.



31.



33.



35. Approx. 2.5" by 3.5" by 9.5" 37. Approx. 59 m from the loudest speaker 39. $r = 4\sqrt{6}/3$; $h = 4\sqrt{3}/3$ 41. $x = 7, y = 14$

43. $p = q = 5\sqrt{2}$ 45. a. $L(x) = \frac{2}{9}x + 3$ b. $\frac{85}{9} \approx 9.44$;

overestimate 47. $f(x) = 1/x^2$; $a = 4$; $1/4.2^2 \approx 0.05625$

49. $\Delta h \approx -112$ ft 51. $c = 2.5$ 53. a. $\frac{100}{9}$ cells/week

- b. $t = 2$ weeks 55. $-0.434259, 0.767592, 1$ 57. $0, \pm 0.948683$

59. 0 61. 0 63. 12 65. $\frac{2}{3}$ 67. ∞ 69. 0 71. 1 73. 0

75. 1 77. 1 79. $1/e^3$ 81. 1 83. $x^{1/2}$ 85. $\sqrt{x}$ 87. 3^x

89. Comparable growth rates 91. $\frac{4}{3}x^3 + 2x^2 + x + C$

93. $-\frac{1}{x} + \frac{4}{3}x^{-3/2} + C$ 95. $\theta + 3 \sin \theta + C$ 97. $\tan x + C$

99. $12 \ln |x| + C$ 101. $\tan^{-1} x + C$ 103. $\frac{4}{7}x^{7/4} + \frac{2}{7}x^{7/2} + C$

105. $f(t) = -\cos t + t^2 + 6$ 107. $h(x) = \frac{1}{3}x^3 - x - \tan^{-1} x + \frac{\pi}{4}$

109. $v(t) = -9.8t + 120$; $s(t) = -4.9t^2 + 120t + 125$. The rocket reaches a height of 859.69 m at time $t \approx 12.24$ s and then falls to the ground, hitting at time $t \approx 25.49$ s.

111. a. $v(t) = 64 - 32t$

- b. $s(t) = -16t^2 + 64t + 128$ c. $t = 2$; 192 ft

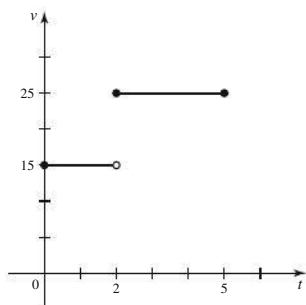
- d. $-64\sqrt{3}$ ft/s ≈ -110.9 ft/s 113. 1; 1

115. $\lim_{x \rightarrow 0^+} f(x) = 1$; $\lim_{x \rightarrow 0^+} g(x) = 0$

CHAPTER 5

Section 5.1 Exercises, pp. 347–352

1. Displacement = 105 m

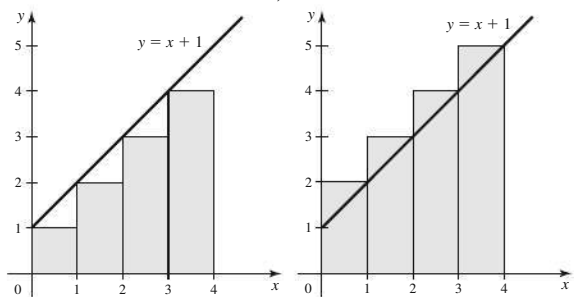


3. a. 440 ft b. 400 ft 5. a. 340 ft b. 330 ft

7. Subdivide the interval $[0, \pi/2]$ into several subintervals, which will be the bases of rectangles that fit under the curve. The heights of the rectangles are computed by taking the value of $\cos x$ at the right-hand endpoint of each base. We calculate the area of each rectangle and add them to get a lower bound on the area. 9. Left sum: 34; right sum: 24 11. 0.5; 1, 1.5, 2, 2.5, 3; 1, 1.5, 2, 2.5; 1.5, 2, 2.5, 3; 1.25, 1.75, 2.25, 2.75 13. Underestimate; the rectangles all fit under the curve. 15. a. 67 ft b. 67.75 ft 17. 40 m

19. 2.78 m 21. 148.96 mi 23. 20; 25

25. a. c.

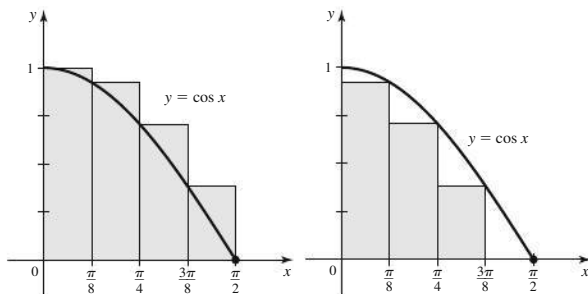


Left Riemann sum underestimates area.

Right Riemann sum overestimates area.

b. $\Delta x = 1$; $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$ d. 10, 14

27. a. c.

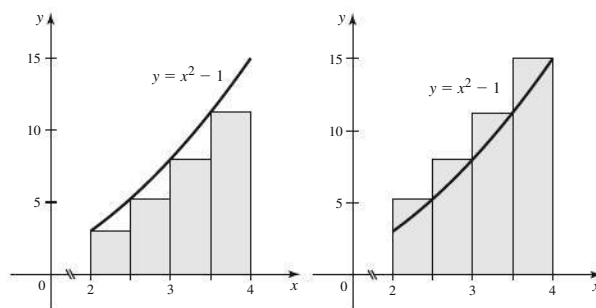


Left Riemann sum overestimates area.

Right Riemann sum underestimates area.

b. $\Delta x = \pi/8$; 0 , $\pi/8$, $\pi/4$, $3\pi/8$, $\pi/2$ d. 1.18; 0.79

29. a. c.

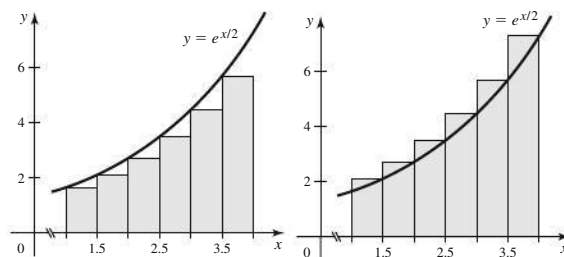


Left Riemann sum underestimates area.

Right Riemann sum overestimates area.

b. $\Delta x = 0.5$; 2, 2.5, 3, 3.5, 4 d. 13.75; 19.75

31. a. c.



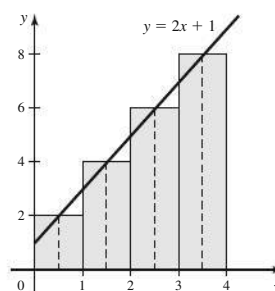
Left Riemann sum underestimates area.

Right Riemann sum overestimates area.

b. $\Delta x = 0.5$; $x_0 = 1$, $x_1 = 1.5$, $x_2 = 2$, $x_3 = 2.5$, $x_4 = 3$, $x_5 = 3.5$, $x_6 = 4$ d. 10.11, 12.98 33. 670 35. a. 10,500 m; 10,350 m

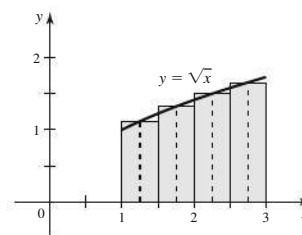
b. Left Riemann sum c. Increase the number of subintervals in the partition.

37. a. c.

b. $\Delta x = 1$; 0, 1, 2, 3, 4

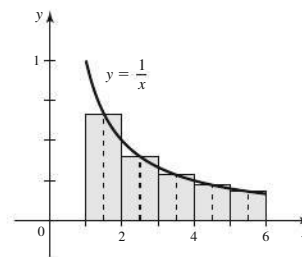
d. 20

39. a. c.

b. $\Delta x = \frac{1}{2}$; 1 , $\frac{3}{2}$, 2 , $\frac{5}{2}$, 3

d. 2.80

41. a. c.

b. $\Delta x = 1$; 1, 2, 3, 4, 5, 6

d. 1.76

43. 5.5, 3.5 45. b. 110, 117.5 47. a. $\sum_{k=1}^5 k$ b. $\sum_{k=1}^6 (k+3)$

c. $\sum_{k=1}^4 k^2$ d. $\sum_{k=1}^4 \frac{1}{k}$ 49. a. 55 b. 48 c. 30 d. 60 e. 6

f. 6 g. 85 h. 0 51. a. Left: $\frac{3}{10} \sum_{k=1}^{40} \sqrt{\frac{k-1}{10}} \approx 15.6809$;

right: $\frac{3}{10} \sum_{k=1}^{40} \sqrt{\frac{k}{10}} \approx 16.2809$; midpoint: $\frac{3}{10} \sum_{k=1}^{40} \sqrt{\frac{k-0.5}{10}} \approx 16.0055$

b. 16 53. a. Left: $\frac{1}{25} \sum_{k=1}^{75} \left(\left(2 + \frac{k-1}{25} \right)^2 - 1 \right) \approx 35.5808$;

right: $\frac{1}{25} \sum_{k=1}^{75} \left(\left(2 + \frac{k}{25} \right)^2 - 1 \right) \approx 36.4208$;

midpoint: $\frac{1}{25} \sum_{k=1}^{75} \left(\left(2 + \frac{k-0.5}{25} \right)^2 - 1 \right) \approx 35.9996$ b. 36

55.

n	Right Riemann sum
10	21.96
30	21.9956
60	21.9989
80	21.9994

The sums approach 22.

57.

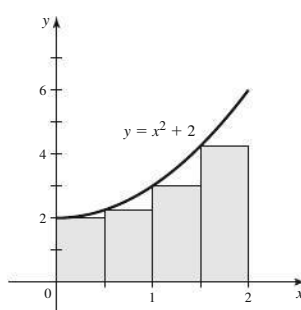
n	Right Riemann sum
10	3.14159
30	3.14159
60	3.14159
80	3.14159

The sums approach π .

59. a. True b. False c. True 61. $\sum_{k=1}^{50} \left(\frac{4k}{50} + 1 \right) \cdot \frac{4}{50} = 12.16$

63. $\sum_{k=1}^{32} \left(3 + \frac{2k-1}{8} \right)^3 \cdot \frac{1}{4} \approx 3639.1$ 65. $[1, 5]; 4$ 67. $[2, 6]; 4$

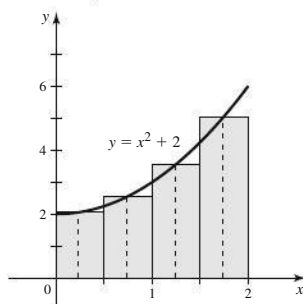
69. a.



Left Riemann sum is

$$\frac{23}{4} = 5.75.$$

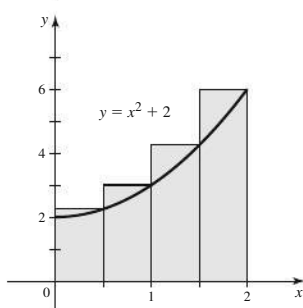
b.



Midpoint Riemann sum is

$$\frac{53}{8} = 6.625.$$

c.



Right Riemann sum is

$$\frac{31}{4} = 7.75.$$

71. a. The object is speeding up on the interval $(0, 1)$, moving at a constant rate on $(1, 3)$, slowing down on $(3, 5)$, and moving at a constant rate on $(5, 6)$. b. 30 m c. 50 m d. $s(t) = 80 + 10t$

73. a. 14.5 g b. 29.5 g c. 44 g d. $x = \frac{19}{3}$ cm

75. $s(t) = \begin{cases} 30t & \text{if } 0 \leq t \leq 2 \\ 50t - 40 & \text{if } 2 < t \leq 2.5 \\ 44t - 25 & \text{if } 2.5 < t \leq 3 \end{cases}$

77.

n	Midpoint Riemann sum
16	4.7257
32	4.7437
64	4.7485

The sums approach 4.75.

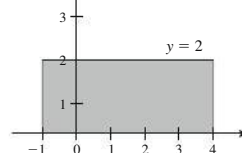
81. Underestimates for decreasing functions, independent of concavity; overestimates for increasing functions, independent of concavity

Section 5.2 Exercises, pp. 364–367

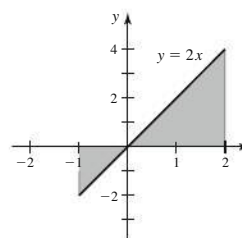
1. The difference between the area bounded by the curve above the x -axis and the area bounded by the curve below the x -axis 3. 60; 0

5. -12 ; -18 ; -16

7. $\int_{-1}^4 2 \, dx = 10$



9.

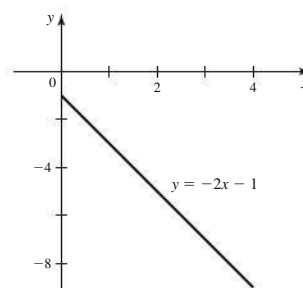


$$\int_{-1}^2 2x \, dx = 3$$

11. Both integrals equal 0. 13. The length of the interval $[a, a]$ is

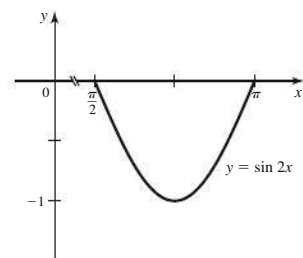
$a - a = 0$, so the net area is 0. 15. $\frac{a^2}{2}$

17. a.

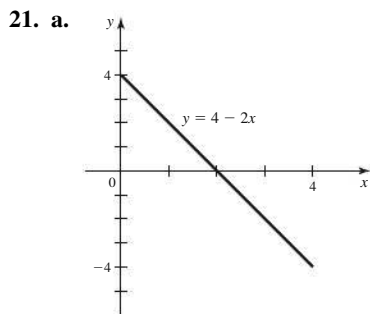


b. $-16, -24, -20$

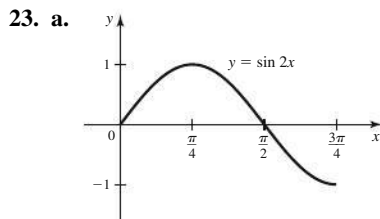
19. a.



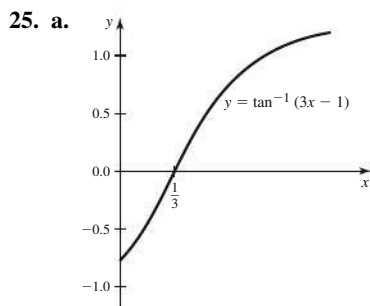
b. $-0.948, -0.948, -1.026$



b. 4, -4, 0 c. Positive contributions on $[0, 2]$; negative contributions on $(2, 4]$

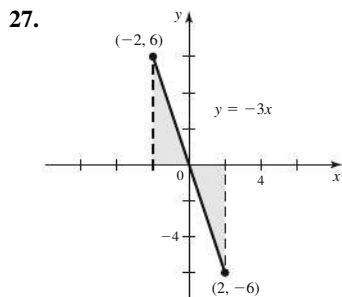


b. 0.735, 0.146, 0.530
c. Positive contributions on $(0, \pi/2)$; negative contributions on $(\pi/2, 3\pi/4]$

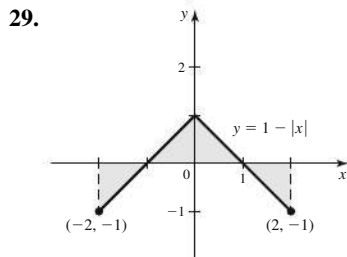


b. 0.082; 0.555; 0.326

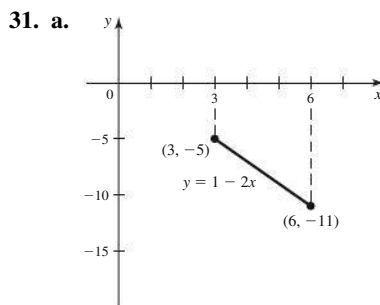
c. Positive contributions on $(\frac{1}{3}, 1]$; negative contributions on $[0, \frac{1}{3})$



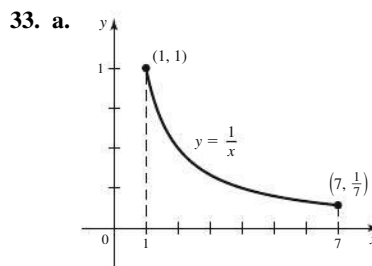
The area is 12; the net area is 0.



The area is 2; the net area is 0.

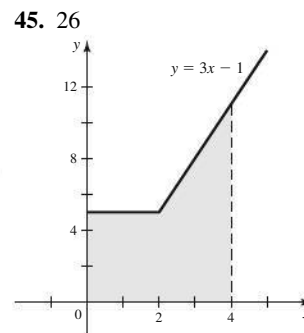
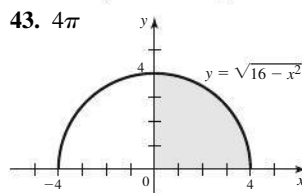
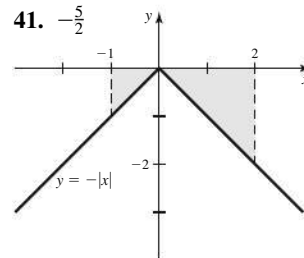
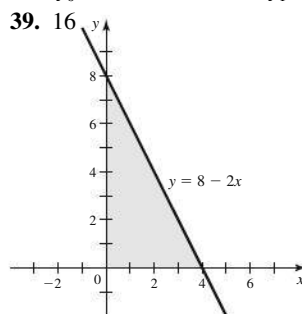


b. $\Delta x = \frac{1}{2}$; 3, 3.5, 4, 4.5, 5, 5.5, 6 c. -22.5; -25.5
d. The left Riemann sum overestimates the integral; the right Riemann sum underestimates the integral.



b. $\Delta x = 1$; 1, 2, 3, 4, 5, 6, 7
c. $\frac{49}{20}$, $\frac{223}{140}$ d. The left Riemann sum overestimates the integral; the right Riemann sum underestimates the integral.

35. $\int_0^2 (x^2 + 1) dx$ 37. $\int_1^2 x \ln x dx$



47. π 49. -2π 51. a. -32 b. $-\frac{32}{3}$ c. -64 d. Not possible

53. a. 10 b. -3 c. -16 d. 3 55. a. 15 b. 5 c. 3

d. -2 e. 24 f. -10 57. a. $\frac{3}{2}$ b. $-\frac{3}{4}$ 59. 16 61. 6

63. 32 65. -16 67. $\frac{\pi}{4} + 2$ 69. a. True b. True c. True

d. False e. False

71. a. Left: $\sum_{k=1}^n \left(\left(\frac{k-1}{n} \right)^2 + 1 \right) \cdot \frac{1}{n}$;

right: $\sum_{k=1}^n \left(\left(\frac{k}{n} \right)^2 + 1 \right) \cdot \frac{1}{n}$

b.

n	Left Riemann sum	Right Riemann sum
20	1.30875	1.35875
50	1.3234	1.3434
100	1.32835	1.33835

Estimate: $\frac{4}{3}$

73. a. Left: $\sum_{k=1}^n \cos^{-1} \left(\frac{k-1}{n} \right) \frac{1}{n}$;

right: $\sum_{k=1}^n \cos^{-1} \left(\frac{k}{n} \right) \frac{1}{n}$

b.

n	Left Riemann sum	Right Riemann sum
20	1.03619	0.95765
50	1.01491	0.983494
100	1.00757	0.99186

Estimate: 1

75. a. $\sum_{k=1}^n 2\sqrt{1 + \left(k - \frac{1}{2}\right)\frac{3}{n} \cdot \frac{3}{n}}$

b.

n	Midpoint Riemann sum
20	9.33380
50	9.33341
100	9.33335

Estimate: $\frac{28}{3}$

77. a. $\frac{4}{n} \sum_{k=1}^n \left(4\left(k - \frac{1}{2}\right)\frac{4}{n} - \left(\left(k - \frac{1}{2}\right)\frac{4}{n}\right)^2 \right)$

b.

n	Midpoint Riemann sum
20	10.6800
50	10.6688
100	10.6672

Estimate: $\frac{32}{3}$

79. 6 81. 104 83. 18 85. 2 87. $25\pi/2$ 89. 25 91. 35

95. For any such partition on $[0, 1]$, the grid points are $x_k = k/n$, for $k = 0, 1, \dots, n$. That is, x_k is rational for each k so that $f(x_k) = 1$, for $k = 0, 1, \dots, n$. Therefore, the left, right, and midpoint Riemann sums are $\sum_{k=1}^n 1 \cdot (1/n) = 1$.

Section 5.3 Exercises, pp. 377–381

1. A is an antiderivative of f ; $A'(x) = f(x)$.

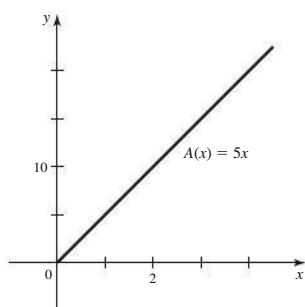
3. $\int_a^b f(x) dx = F(b) - F(a)$, where F is any antiderivative of f .

5. Increasing 7. The derivative of the integral of f is f , or

$\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$. 9. $f(x)$, 0 11. 16 13. a. 0 b. -9

c. 25 d. 0 e. 16

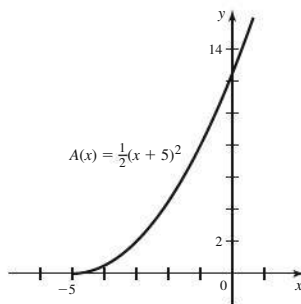
15. a. $A(x) = 5x$



b. $A'(x) = 5$

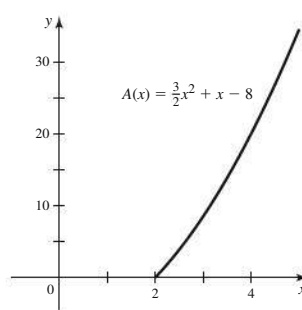
17. a. $A(2) = 2$, $A(4) = 8$; $A(x) = \frac{1}{2}x^2$ b. $F(4) = 6$, $F(6) = 16$; $F(x) = \frac{1}{2}x^2 - 2$ c. $A(x) - F(x) = \frac{1}{2}x^2 - (\frac{1}{2}x^2 - 2) = 2$

19. a.



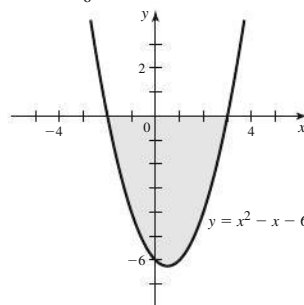
b. $A'(x) = \left(\frac{1}{2}(x+5)^2\right)' = x+5 = f(x)$

21. a.

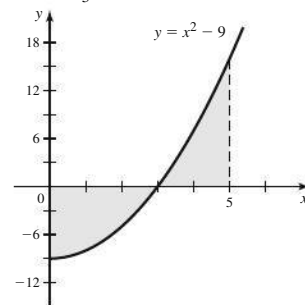


b. $A'(x) = \left(\frac{3}{2}x^2 + x - 8\right)' = 3x + 1 = f(x)$ 23. $\frac{7}{3}$

25. $-\frac{125}{6}$



27. $-\frac{10}{3}$

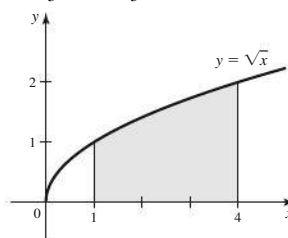


29. 16 31. 90 33. $\frac{7}{6}$ 35. 8 37. $-\frac{32}{3}$ 39. $-\frac{5}{2}$ 41. $\frac{9}{2}$ 43. $-\frac{3}{8}$

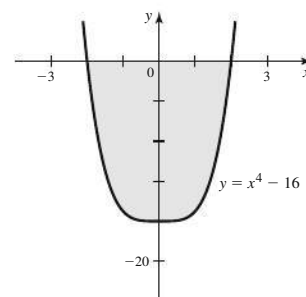
45. 1 47. $3 \ln 2$ 49. $\frac{45}{4}$ 51. $\frac{2}{3}$ 53. 1 55. 2 57. $\frac{\pi}{12}$

59. $\frac{3}{2} + 4 \ln 2$ 61. $\frac{3\pi}{2} - 1$

63. (i) $\frac{14}{3}$ (ii) $\frac{14}{3}$



65. (i) -51.2 (ii) 51.2

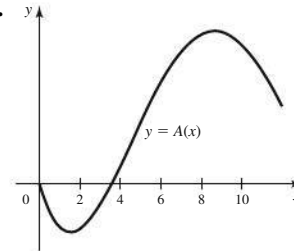


67. $\frac{94}{3}$ 69. $\ln 2$ 71. 2 73. $x^2 + x + 1$ 75. $-\sqrt{x^4 + 1}$

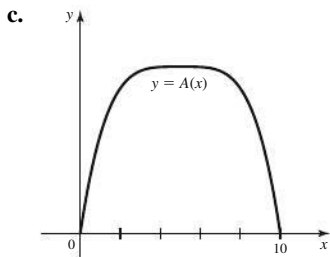
77. $3/x^4$ 79. $-(\cos^4 x + 6) \sin x$ 81. $-\frac{\cos z}{\sin^4 z + 1}$

83. $\frac{9}{t}$ 85. $2\sqrt{1+x^2}$ 87. a-C, b-B, c-D, d-A

89. a. $x = 0$, $x \approx 3.5$ b. Local min at $x \approx 1.5$; local max at $x \approx 8.5$ c.

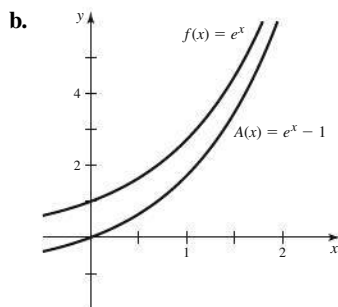


91. a.
- $x = 0, 10$
- b. Local max at
- $x = 5$



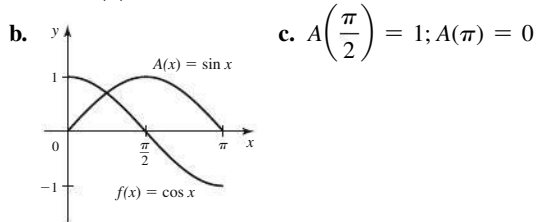
- 93.
- $-\pi, -\pi + \frac{9}{2}, -\pi + 9, 5 - \pi$

95. a.
- $A(x) = e^x - 1$



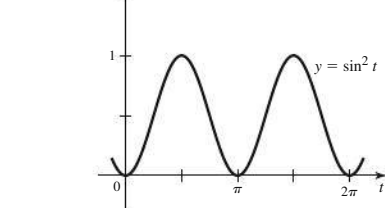
c. $A(\ln 2) = 1; A(\ln 4) = 3$

97. a.
- $A(x) = \sin x$

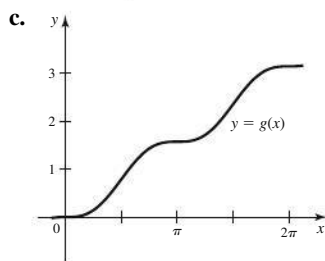


99. Critical pts.
- $x = 0, 3$
- , and
- 4
- ; increasing on
- $(-\infty, 0)$
- ,
- $(0, 3)$
- , and
- $(4, \infty)$
- ; decreasing on
- $(3, 4)$

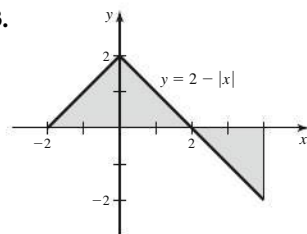
101. a.



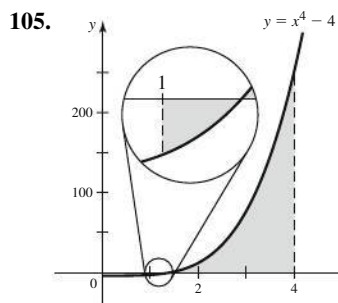
b. $g'(x) = \sin^2 x$



103.



Area = 6

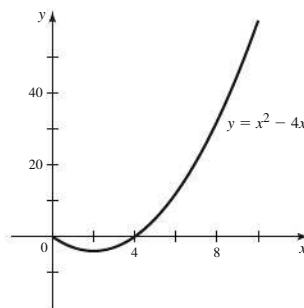
Area ≈ 194.05

107. a. True b. True

- c. False d. True

109. 3

111. a.



- b.
- $b = 6$
- c.
- $b = \frac{3a}{2}$

- 113.
- $f(x) = -2 \sin x + 3$
- 115.
- $\pi/2 \approx 1.57$

$$117. (S'(x))^2 + \left(\frac{S''(x)}{2x}\right)^2 = (\sin x^2)^2 + \left(\frac{2x \cos x^2}{2x}\right)^2$$

$$= \sin^2 x^2 + \cos^2 x^2 = 1$$

119. c. The summation relationship is a discrete analog of the Fundamental Theorem. Summing the difference quotient and integrating the derivative over the relevant interval give the difference of the function values at the endpoints.

Section 5.4 Exercises, pp. 385–387

1. If f is odd, the regions between f and the positive x -axis and between f and the negative x -axis are reflections of each other through the origin. Therefore, on $[-a, a]$, the areas cancel each other.

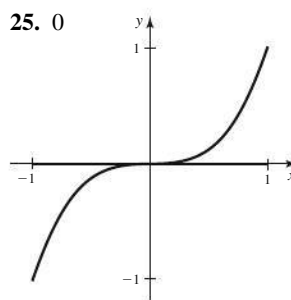
3. a. 9 b. 0 5.
- $3x^3$
- and
- x
- are odd functions. 7. Even; even

9. If f is continuous on $[a, b]$, then there is a c in (a, b) such that

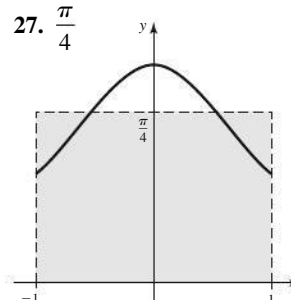
$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx. \quad 11. 0 \quad 13. \frac{1000}{3} \quad 15. \frac{16}{3} \quad 17. -\frac{88}{3}$$

19. 0 21. 2 23. 0

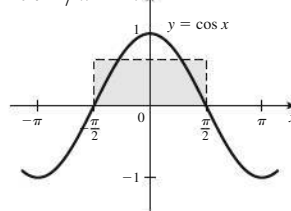
25. 0



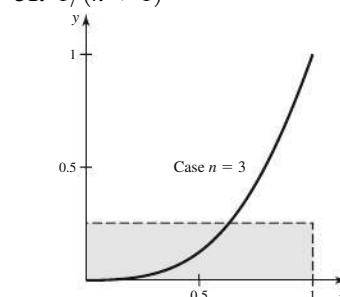
- 27.
- $\frac{\pi}{4}$



- 29.
- $2/\pi$



- 31.
- $1/(n+1)$



33. 2000 35. 21 m/s 37.
- $20/\pi$
39. 2 41.
- $a/\sqrt{3}$

- 43.
- $c = \pm \frac{1}{2}$
45. a. True b. True c. True d. False

47. 420 ft 49.
- $f(g(-x)) = f(g(x)) \Rightarrow$
- the integrand is even;

$$\int_{-a}^a f(g(x)) dx = 2 \int_0^a f(g(x)) dx \quad 51. p(g(-x)) = p(g(x)) \Rightarrow$$

the integrand is even; $\int_{-a}^a p(g(x)) dx = 2 \int_0^a p(g(x)) dx$

53. a.
- $a/6$
- b.
- $(3 \pm \sqrt{3})/6$
- , independent of
- a

- 57.

Even	Even
Even	Odd

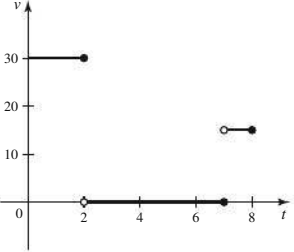
Section 5.5 Exercises, pp. 395–398

1. The Chain Rule 3. $u = g(x)$ 5. The lower bound a becomes $g(a)$ and the upper bound b becomes $g(b)$. 7. $\frac{(x^2 + 1)^5}{5} + C$ 9. $\frac{1}{4}\sin^4 x + C$ 11. $\frac{(x+1)^{13}}{13} + C$ 13. $\frac{(2x+1)^{3/2}}{3} + C$ 15. a. $\frac{1}{10}e^{10x} + C$ b. $\frac{1}{5}\sec 5x + C$ c. $-\frac{1}{7}\cos 7x + C$ d. $7\sin \frac{x}{7} + C$ e. $\frac{1}{27}\tan^{-1} \frac{x}{3} + C$ f. $\sin^{-1} \frac{x}{6} + C$ 17. $\frac{(x^2 - 1)^{100}}{100} + C$ 19. $-\frac{(1 - 4x^3)^{1/2}}{3} + C$ 21. $\frac{(x^2 + x)^{11}}{11} + C$ 23. $\frac{(x^4 + 16)^7}{28} + C$ 25. $\frac{1}{2}\sin^{-1} \frac{x}{3} + C$ 27. $\frac{4x^3}{\ln 2} + C$ 29. $\frac{(x^6 - 3x^2)^5}{30} + C$ 31. $\frac{3}{5}\sin^{-1} 5x + C$ 33. $\frac{1}{6}\tan^{-1} \frac{e^w}{6} + C$ 35. $-\frac{1}{2}\csc x^2 + C$ 37. $\frac{1}{10}\tan(10x + 7) + C$ 39. $\frac{10^{4x+1}}{4\ln 10} + C$ 41. $\frac{1}{2}\tan^2 x + C$ 43. $\frac{1}{7}\sec^7 x + C$ 45. $\frac{\sqrt{2}}{4}$ 47. $\frac{7}{2}$ 49. 1 51. $\frac{1}{3}$ 53. $\frac{2 - \sqrt{2}}{2}$ 55. $(e^9 - 1)/3$ 57. $\sqrt{2} - 1$ 59. $\frac{\pi}{6}$ 61. $\frac{1}{2}\ln 17$ 63. $\frac{\pi}{9}$ 65. $\frac{1}{3}$ 67. $\frac{3}{4}(4 - 3^{2/3})$ 69. $\frac{32}{3}$ 71. $-\ln 3$ 73. $\frac{1}{7}$ 75. 10 m/s 77. a. 160 b. $\frac{4800}{49} \approx 98$ c. $\Delta p = \int_0^T \frac{200}{(t+1)^r} dt$;decreases as r increases d. $r \approx 1.28$ e. As $t \rightarrow \infty$, thepopulation approaches 100. 79. $\frac{2}{3}(x-4)^{1/2}(x+8) + C$ 81. $\frac{3}{5}(x+4)^{2/3}(x-6) + C$ 83. $\frac{3}{112}(2x+1)^{4/3}(8x-3) + C$ 85. $\frac{(x+10)^{10}(x-1)}{11} + C$ 87. π 89. $\frac{\theta}{2} - \frac{1}{4}\sin\left(\frac{6\theta + \pi}{3}\right) + C$ 91. $\frac{\pi}{4}$ 93. $\ln \frac{9}{8}$ 95. a. Trueb. True c. False d. False e. False 97. 1 99. $\frac{2}{3}$; constant101. a. π/p b. 0 103. $2/\pi$ 105. One area is $\int_4^9 \frac{(\sqrt{x}-1)^2}{2\sqrt{x}} dx$.Changing variables by letting $u = \sqrt{x} - 1$ yields $\int_1^2 u^2 du$, which is theother area. 107. $7297/12$ 109. $\frac{2}{15}(3-2a)(1+a)^{3/2} + \frac{4}{15}a^{5/2}$ 111. $\frac{1}{3}\sec^3 \theta + C$ 113. a. $I = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$ b. $I = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$ 117. $\frac{4}{3}(-2 + \sqrt{1+x})\sqrt{1 + \sqrt{1+x}} + C$ 119. $-4 + \sqrt{17}$

Chapter 5 Review Exercises, pp. 398–402

1. a. True b. False c. True d. True e. False

f. True g. True

3. a.  b. 75 c. The area is the distance the diver ascends.

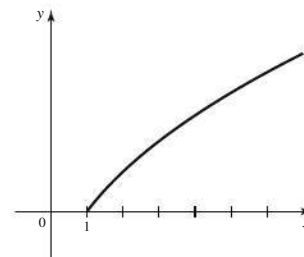
5. 9.34; 10.28; 9.82

n	Midpoint Riemann sum
10	114.167
30	114.022
60	114.006

$$\int_1^{25} \sqrt{2x-1} dx = 114$$

9. a. $1((3 \cdot 2 - 2) + (3 \cdot 3 - 2) + (3 \cdot 4 - 2)) = 21$ b. $\sum_{k=1}^n \frac{3}{n} \left(3 \left(1 + \frac{3k}{n} \right) - 2 \right)$ c. $\frac{33}{2}$ 11. $-\frac{16}{3}$ 13. 56

15. a. 20 b. 0 c. 80 d. 10 e. 0 17. 18 19. 10

21. Not enough information 23. a. 8.5 b. -4.5 c. 0 d. 11.525. 4π 27. A: $\int_0^x f(t) dt$; B: $f(x)$; C: $f'(x)$ 29. $\sqrt{1+x^4+x^6}$ 31. $-\sin x^6$ 33. $\frac{2}{x^{10}+1}$ 35. Increasing on $(3, 6)$; decreasingon $(-\infty, 3)$ and $(6, \infty)$ 39. $\frac{212}{5}$ 41. $x^9 - x^7 + C$ 43. $\frac{7}{6}$ 45. $\frac{4}{\sqrt{3}}$ 47. $\frac{\pi}{12}$ 49. $-\frac{4}{3\sin^{3/4}x} + C$ 51. $\frac{1}{3}\sin x^3 + C$ 53. $\frac{1}{28}\tan^{-1}\left(\frac{\sin 7w}{4}\right) + C$ 55. $\frac{1}{\ln 2}$ 57. 78 59. $\frac{5}{6}e^2(e^3 - 1)$ 61. $e^{e^x} + C$ 63. $\frac{1}{2}\sin^{-1} 2x + C$ 65. $\pi + \frac{3\sqrt{3}}{4}$ 67. $\frac{\pi}{2}$ 69. $\frac{1}{3}\ln \frac{9}{2}$ 71. 0 73. $\cos \frac{1}{x} + C$ 75. $\ln |\tan^{-1} x| + C$ 77. $(x+3)^{11} \left(\frac{11x-3}{132} \right) + C$ 79. 1 81. $\frac{\pi}{12}$ 83. 0 85. 48 87. $\frac{256}{3}$ 89. 8 91. $-\frac{4}{15}; \frac{4}{15}$ 93. Approx. 431.5 ft 95. Displacement = 0; distance = $20/\pi$ 97. $\frac{3}{2\ln 2}$ 99. a. $5/2, c = 3.5$ b. $3, c = 3$ and $c = 5$ 101. 24103. $f(1) = 0$; $f'(x) > 0$ on $[1, \infty)$; $f''(x) < 0$ on $[1, \infty)$ 105. a. $\frac{3}{2}, \frac{5}{6}$ b. x c. $\frac{1}{2}x^2$ d. $-1, \frac{1}{2}$ e. 1, 1 f. $\frac{3}{2}$ 107. e^4 113. a. Increasing on $(-\infty, 1)$ and $(2, \infty)$; decreasing on $(1, 2)$ b. Concave up on $(\frac{13}{8}, \infty)$; concave down on $(-\infty, \frac{13}{8})$ c. Local max at $x = 1$; local min at $x = 2$ d. Inflection pointat $x = \frac{13}{8}$ 115. Differentiating the first equation gives thesecond equation; no. 117. $\sqrt[4]{12}$

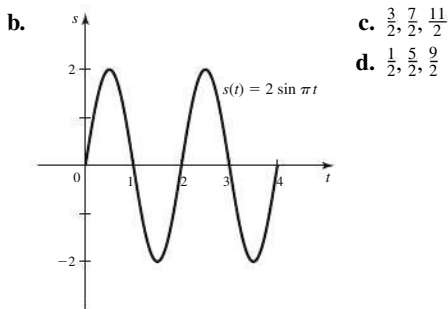
CHAPTER 6

Section 6.1 Exercises, pp. 410–416

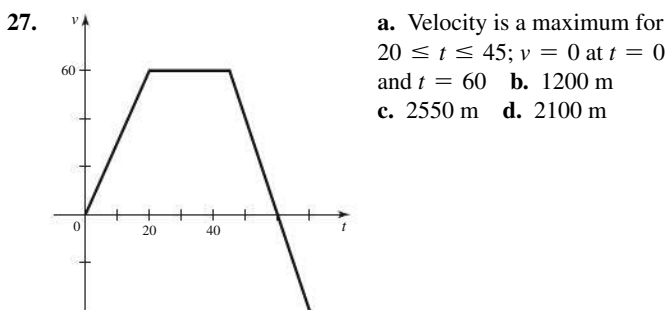
1. The position $s(t)$ is the location of the object relative to the origin. The displacement is the change in position between time $t = a$ and $t = b$. The distance traveled between $t = a$ and $t = b$ is $\int_a^b |v(t)| dt$, where $v(t)$ is the velocity at time t . 3. The displacement between $t = a$ and $t = b$ is $\int_a^b v(t) dt$. 5. $Q(t) = Q(0) + \int_0^t Q'(x) dx$
7. a. $[0, 1)$, $(3, 5)$ b. -4 mi c. 26 mi d. 6 mi e. 6 mi on the positive side of the initial position 9. a. 3 b. $\frac{13}{3}$ c. 3

$$d. s(t) = \begin{cases} -\frac{t^2}{2} + 2t & \text{if } 0 \leq t \leq 3 \\ \frac{3t^2}{2} - 10t + 18 & \text{if } 3 < t \leq 4 \\ -t^2 + 10t - 22 & \text{if } 4 < t \leq 5 \end{cases}$$

11. a. 3 m b. 3 m; 0 m; 3 m; 0 m c. 12 m
13. a. Positive direction for $2 < t \leq 3$; negative direction for $0 < t < 2$ b. 0 m c. 8 m 15. a. Positive direction for $0 \leq t < 2$ and $4 < t \leq 5$; negative direction for $2 < t < 4$ b. 20 m c. 28 m 17. a. $s(t) = 2 - \cos t$ 19. a. $s(t) = 6t - t^2$
21. a. $s(t) = 9t - \frac{t^3}{3} - 2$ 23. a. $s(t) = 2 \sin \pi t$



25. a. $s(t) = 10t(48 - t^2)$ b. 880 mi c. $\frac{2720\sqrt{6}}{9} \approx 740.29$ mi



29. $v(t) = -32t + 70$; $s(t) = -16t^2 + 70t + 10$
31. $v(t) = -9.8t + 20$; $s(t) = -4.9t^2 + 20t$
33. $v(t) = -\frac{1}{200}t^2 + 10$; $s(t) = -\frac{1}{600}t^3 + 10t$
35. $v(t) = \frac{1}{2} \sin 2t + 5$; $s(t) = -\frac{1}{4} \cos 2t + 5t + \frac{29}{4}$
37. a. $s(t) = 44t^2$ b. 704 ft c. $\sqrt{30} \approx 5.477$ s

d. $\frac{5\sqrt{33}}{11} \approx 2.611$ s e. Approx. 180.023 ft

39. 6.154 mi; 1.465 mi 41. a. 2639 people
b. $P(t) = 250 + 20t^{3/2} + 30t$ people 43. a. 1897 cells; 1900 cells b. $N(t) = 1900 - 400e^{-0.25t}$ 45. a. 27,250 barrels

b. 31,000 barrels c. 4000 barrels 47. a. $\frac{10^7(1 - e^{-kt})}{k}$

- b. $\frac{10^7}{k}$ = total number of barrels of oil extracted if the nation extracts the oil indefinitely, and it has at least $\frac{10^7}{k}$ barrels of oil in reserve.

c. $k = \frac{1}{200} = 0.005$ d. Approx. 138.6 yr

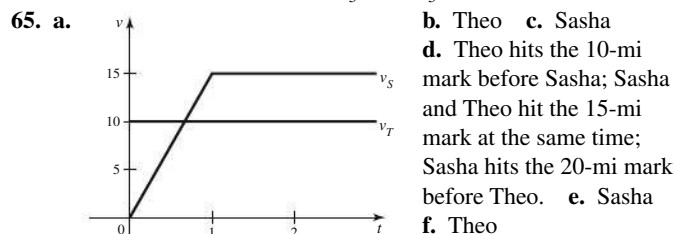
49. a. $\frac{120}{\pi} + 40 \approx 78.20$ m³

b. $Q(t) = 20 \left(t + \frac{12}{\pi} \sin \left(\frac{\pi}{12} t \right) \right)$ c. Approx. 122.6 hr

51. a. $V(t) = 5 + \cos \frac{\pi t}{2}$ b. 15 breaths/min c. 2 L, 6 L

53. a. 7200 MWh or 2.592×10^{13} J b. 16,000 kg; 5,840,000 kg
c. 450 g; 164,250 g d. About 1500 turbines 55. a. \$96,875
b. \$86,875 57. a. \$69,583.33 b. \$139,583.33 59. a. False

b. True c. True d. True 61. $\frac{2}{3}$ 63. $\frac{25}{3}$

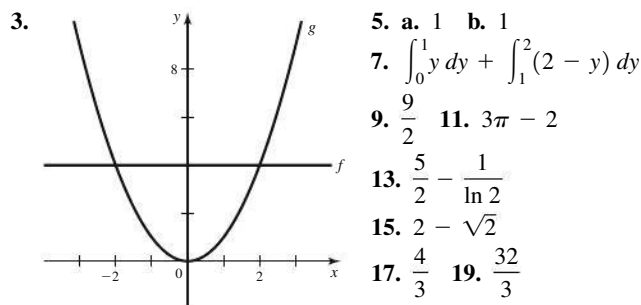


67. Approx. 11:23 A.M.

69. $\int_a^b f'(x) dx = f(b) - f(a) = g(b) - g(a) = \int_a^b g'(x) dx$

Section 6.2 Exercises, pp. 420–425

1. $\int_a^b (f(x) - g(x)) dx + \int_b^c (g(x) - f(x)) dx$



21. 9 23. $\frac{81}{2}$ 25. 2 27. $\frac{8\sqrt{2} - 7}{6}$ 29. $\frac{125}{2}$

31. a. $\int_0^1 (\sqrt{x} - x^3) dx$ b. $\int_0^1 (\sqrt[3]{y} - y^2) dy$

33. $\frac{19}{6} \approx 3.17$ km; the faster runner jogged approximately 3.17 km farther than the slower runner. 35. a. 7 b. 4 37. 25 39. $\frac{81}{32}$

41. $\pi - 2$ 43. $\frac{1}{2} + \ln 2$ 45. $\frac{7}{3}$ 47. 3 49. $\frac{64}{5}$ 51. $\ln 2$

53. $\frac{5}{24}$ 55. $\frac{63}{4}$ 57. $\frac{9}{2}$ 59. $\frac{32}{3}$ 61. $\frac{15}{8} - 2 \ln 2$ 63. $\frac{17}{3}$

65. a. False b. False c. True 67. $\frac{4}{9}$ 69. $\frac{n-1}{2(n+1)}$

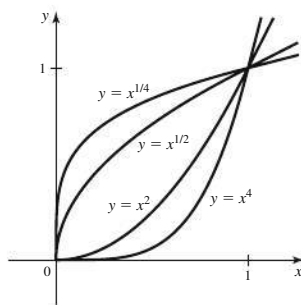
71. $A_n = \frac{n-1}{n+1}$; $\lim_{n \rightarrow \infty} A_n = 1$; the region approximates a square

with side length of 1. 73. $k = 1 - \frac{1}{\sqrt{2}}$ 75. a. The lowest $p\%$ of households own exactly $p\%$ of the wealth for $0 \leq p \leq 100$.

b. The function must be one-to-one and its graph must lie below $y = x$ because the poorest $p\%$ cannot own more than $p\%$ of the wealth. c. $p = 1.1$ is most equitable; $p = 4$ is least equitable.

e. $G(p) = \frac{p-1}{p+1}$ f. $0 \leq G \leq 1$ for $p \geq 1$ g. $\frac{5}{18}$

77. a.



b. $A_n(x)$ is the net area of the region between the graphs of f and g from 0 to x . c. $x = n^{n/(n^2-1)}$; the root decreases with n .

Section 6.3 Exercises, pp. 434–439

1. $A(x)$ is the area of the cross section through the solid at the point x .

3. a. $3 - x$ b. $\int_0^2 (3 - x) dx$ 5. a. $\sqrt{\cos x}$ b. $\pi \cos x$

c. $\int_0^{\pi/2} \pi \cos x dx$ 7. a. $\sqrt{x} + 1$ b. 1 c. $\pi((\sqrt{x} + 1)^2 - 1)$

d. $\int_0^4 \pi((\sqrt{x} + 1)^2 - 1) dx$ 9. a. $\sqrt{x}$ b. πx c. $\int_0^4 \pi x dx$

11. $\frac{4}{3}$ 13. 1 15. $\frac{\pi}{3}$ 17. 36π 19. $\frac{15\pi}{32}$ 21. $\frac{\pi^2}{2}$ 23. $\frac{32\pi}{3}$

25. $\frac{5\pi}{6}$ 27. $\frac{2\pi}{5}$ 29. $\frac{\pi^2}{4}$ 31. $\frac{\pi^2}{2}$ 33. $\frac{\pi(\pi-2)}{8}$

35. $\frac{4\pi - \pi^2}{4}$ 37. $\frac{128\pi}{5}$ 39. $\pi \ln 3$ 41. $\frac{\pi}{2}(e^4 - 1)$

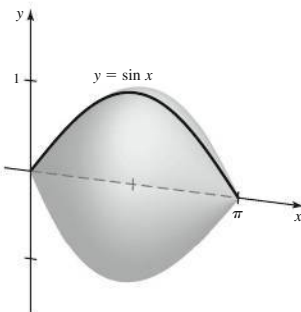
43. $\frac{49\pi}{2}$ 45. Volumes are equal. 47. x -axis 49. $\frac{\pi}{2}$

51. $\pi\sqrt{3}$ 53. $\frac{\pi}{6}$ 55. $2\pi(8 + \pi)$ 57. $(6\sqrt{3} - 2\pi)\pi$

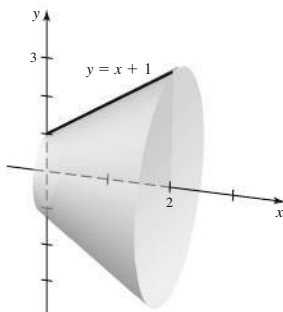
59. 4π 61. a. False b. True c. True

63. Volume (S) = $8\pi a^{5/2}/15$; volume (T) = $\pi a^{5/2}/3$

65. a.



b.



67. Left: 166π ; right: 309π ; midpoint: 219π 69. a. $\frac{1}{3} V_C$ b. $\frac{2}{3} V_C$

71. $24\pi^2$ 73. b. $V = \pi r^2 h$

Section 6.4 Exercises, pp. 447–451

1. $\int_a^b 2\pi x(f(x) - g(x)) dx$ 3. $x; y$ 5. a. x b. $2 - x^2 - x$

c. $\int_0^1 2\pi x(2 - x^2 - x) dx$ 7. a. $2 - y$ b. $4 - (2 - y)^2 = 4y - y^2$

c. $\int_0^2 2\pi(2 - y)(4y - y^2) dy$ 9. $\frac{\pi}{6}$ 11. π 13. 8π 15. $\frac{32\pi}{3}$

17. π 19. $\frac{\pi}{2}$ 21. $\frac{81\pi}{2}$ 23. $\frac{2\pi}{3}$ 25. $\frac{3\pi}{10}$ 27. 90π

29. $2\pi e(e - 1)$ 31. π 33. $\frac{\pi}{5}$ 35. $\frac{4\pi}{15}$ 37. 500π

39. $\frac{11\pi}{6}$ 41. $\frac{5\pi}{6}$ 43. $\frac{23\pi}{15}$ 45. $\frac{52\pi}{15}$ 47. $\frac{36\pi}{5}$

51. a. $4\pi \int_1^5 x\sqrt{4 - (x-3)^2} dx$ b. $12\pi \int_{-2}^2 \sqrt{4 - y^2} dy$

c. $24\pi^2$ 53. $\frac{\pi}{9}$ 55. $\frac{16\pi}{3}$ 57. $\frac{608\pi}{3}$ 59. $\pi(\sqrt{e} - 1)^2$

61. $\frac{5\pi}{6}$ 63. a. True b. False c. True 65. 24π 67. 54π

69. a. $V_1 = \frac{\pi}{15}(3a^2 + 10a + 15)$; $V_2 = \frac{\pi}{2}(a + 2)$

b. $V(S_1) = V(S_2)$, for $a = 0$ and $a = -\frac{5}{6}$ 71. 10π

73. a. $27\sqrt{3}\pi r^3/8$ b. $54\sqrt{2}/(3 + \sqrt{2})^3$ c. $500\pi/3$

Section 6.5 Exercises, pp. 455–457

1. Determine whether f has a continuous derivative on $[a, b]$.

If so, calculate $f'(x)$ and evaluate the integral $\int_a^b \sqrt{1 + f'(x)^2} dx$.

3. $\int_{-2}^5 \sqrt{1 + 9x^4} dx$ 5. $\int_0^2 \sqrt{1 + 4e^{-4x}} dx$ 7. $4\sqrt{5}$

9. $8\sqrt{65}$ 11. 168 13. $\frac{4}{3}$ 15. $\frac{123}{32}$ 17. $\frac{123}{32}$ 19. $7\sqrt{5}$

21. a. $\int_{-1}^1 \sqrt{1 + 4x^2} dx$ b. 2.96 23. a. $\int_1^4 \sqrt{1 + \frac{1}{x^2}} dx$ b. 3.34

25. a. $\int_3^4 \sqrt{\frac{4y-7}{4y-8}} dy$ b. 1.08 27. a. $\int_0^\pi \sqrt{1 + 4\sin^2 2x} dx$

b. 5.27 29. a. $\int_1^{10} \sqrt{1 + 1/x^4} dx$ b. 9.15

31. Approx. 1326 m 33. a. False b. True c. False

35. a. $f(x) = \pm 4x^3/3 + C$ b. $f(x) = \pm 3 \sin 2x + C$

37. $y = 1 - x^2$ 39. a. $L/2$ b. L/c

Section 6.6 Exercises, pp. 463–465

1. 15π 3. Evaluate $\int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2} dx$ 5. a. $4\sqrt{2}\pi$

7. $156\sqrt{10}\pi$ 9. $\frac{2912\pi}{3}$ 11. $\frac{\pi}{9}(17^{3/2} - 1)$ 13. 2π

15. $15\sqrt{17}\pi$ 17. $\frac{\pi}{8}(16 + e^8 - e^{-8})$ 19. 96π

21. $\frac{9\pi}{125} m^3$ 23. a. False b. False c. True d. False

25. a. $\int_0^{\pi/2} 2\pi(\cos x)\sqrt{1 + \sin^2 x} dx$ b. Approx. 7.21

27. a. $\int_0^{\pi/4} 2\pi(\tan x)\sqrt{1 + \sec^4 x} dx$ b. Approx. 3.84

29. $\frac{12\pi a^2}{5}$ 31. $\frac{53\pi}{9}$ 33. $\frac{275\pi}{32}$ 35. $\frac{48,143\pi}{48}$ 39. a. $\frac{6}{a}$ b. $\frac{3}{a}$

c. $\frac{3}{2a} + \frac{3a}{2\sqrt{a^2 - 1}} \sin^{-1}\left(\frac{\sqrt{a^2 - 1}}{a}\right)$ d. The sphere e. A sphere

41. a. $c^2 A$ b. A

Section 6.7 Exercises, pp. 473–477

1. 150 g 3. 25 J 5. Horizontal cross sections of water at various locations in the tank are lifted different distances. 7. 39,200 N/m²

9. $\int_5^{10} 25\pi g(15 - y) dy$ 11. $\int_0^{10} 25\pi g(10 - y) dy$ 13. $\pi + 2$

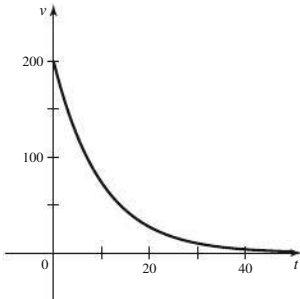
15. 3 17. $(2\sqrt{2} - 1)/3$ 19. 10 21. 9 J 23. a. $k = 150$

b. 12 J c. 6.75 J d. 9 J 25. a. 112.5 J b. 12.5 J

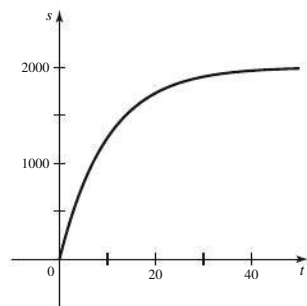
27. a. 31.25 J b. 312.5 J 29. a. 625 J b. 391 J
 31. a. 22,050 J b. 36,750 J 33. 3675 J 35. 1.15×10^7 J
 37. 3.94×10^6 J 39. a. $66,150\pi$ J b. No 41. a. 2.10×10^8 J
 b. 3.78×10^8 J 43. a. 32,667 J b. Yes 45. 7.70×10^3 J
 47. 1.47×10^7 N 49. 2.94×10^7 N 51. 6533 N 53. 6737.5 N
 55. 8×10^5 N 57. a. True b. True c. True d. False
 59. a. Compared to a linear spring, $F(x) = 16x$, the restoring force is less for large displacements. b. 17.87 J c. 31.6 J 61. 1,381,800 J
 63. 0.28 J 65. a. Yes b. 4.296 m 67. Left: 16,730 N; right: 14,700 N 69. a. 8.87×10^9 J
 b. $500 GMx/(R(x+R)) = (2 \times 10^{17})x/(R(x+R))$ J
 c. GMm/R d. $v = \sqrt{2GM/R}$

Chapter 6 Review Exercises, pp. 478–482

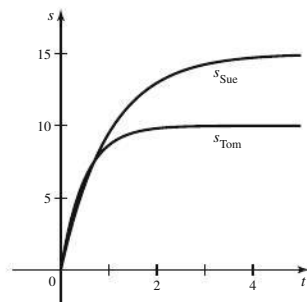
1. a. True b. True c. True 3. a. Positive direction for $0 \leq t < \frac{1}{2}$ and $2 < t \leq 3$; negative direction for $\frac{1}{2} < t < 2$
 b. 9 m c. 22.5 m d. $s(t) = 4t^3 - 15t^2 + 12t + 1$
 5. $s(t) = 20t - 5t^2$; displacement = $20t - 5t^2$;
 $D(t) = \begin{cases} 20t - 5t^2 & \text{if } 0 \leq t < 2 \\ 5t^2 - 20t + 40 & \text{if } 2 \leq t \leq 4 \end{cases}$
 7. a. $v(t) = -\frac{8}{\pi} \cos \frac{\pi t}{4}$; $s(t) = -\frac{32}{\pi^2} \sin \frac{\pi t}{4}$ b. Min value = $-\frac{32}{\pi^2}$;
 max value = $\frac{32}{\pi^2}$ c. 0; 0 9. a. $R(t) = 3t^{4/3}$
 b. $R(t) = \begin{cases} 3t^{4/3} & \text{if } 0 \leq t \leq 8 \\ 2t + 32 & \text{if } t > 8 \end{cases}$ c. $t = 59$ min
 11. a. b. $10 \ln 4 \approx 13.86$ s



- c. $s(t) = 2000(1 - e^{-t/10})$ d. No



13. a. $s_{\text{Tom}}(t) = -10e^{-2t} + 10$
 $s_{\text{Sue}}(t) = -15e^{-t} + 15$



- b. $t = 0$ and $t = \ln 2$ c. Sue 15. $1 - \frac{\pi}{4}$ 17. $e - 2$ 19. $\frac{7}{3}$

21. 8 23. 1 25. $\frac{1}{3}$ 27. $R_1: \frac{7}{6}; R_2: \frac{10}{3}; R_3: 4\sqrt{3} - \frac{10}{3}$ 29. $\frac{11\pi}{15}$
 31. $\frac{14\pi}{3}$ 33. $\int_1^3 2\pi(3-x)(2\sqrt{x}-3+x) dx$ 35. $\frac{7}{3}$ 37. $\frac{31\pi}{5}$
 39. $R_1: \sqrt{3}; R_2: \frac{4\pi}{3} - \sqrt{3}$ 41. $\frac{1}{3}$ 43. $\frac{5}{6}$ 45. $\frac{8}{15}$ 47. $\frac{8\pi}{5}$
 49. $\pi(e-1)^2$ 51. π 53. $\frac{512\pi}{15}$ 55. About $y = -2$; 80π ;
 about $x = -2$; 112π 57. $c = 5$ 59. 1 61. $2\sqrt{3} - \frac{4}{3}$
 63. $\int_2^4 \sqrt{4x^2 + 8x + 5} dx \approx 16.127$

65. $\sqrt{b^2 + 1} - \sqrt{2} + \ln \left(\frac{(\sqrt{b^2 + 1} - 1)(1 + \sqrt{2})}{b} \right)$; $b \approx 2.715$

67. a. 9π b. $\frac{9\pi}{2}$ 69. a. $\frac{263,439\pi}{4096}$ b. $\frac{483}{64}$ c. $\frac{\pi}{8}(84 + \ln 2)$

d. $\frac{264,341\pi}{18,432}$ 71. $\left(450 - \frac{450}{e}\right)$ g 73. a. 562.5 J b. 56.25 J

75. a. 980 J b. 627.2 J 77. a. 1,411,200 J b. 940,800 J
 79. a. 1,477,805 J b. The work required to pump out the top 3 m of water is 1,015,991 J, and the work required to pump out the bottom 3 m of water is 461,814 J. More work is required to pump out the top 3 m of water. 81. 4,987,592 J 83. 5716.7 N 85. 5.2×10^7 N

CHAPTER 7

Section 7.1 Exercises, pp. 490–492

1. $D = (0, \infty)$, $R = (-\infty, \infty)$ 3. $\frac{4^x}{\ln 4} + C$
 5. $e^{x \ln 3}$, $e^{\pi \ln x}$, $e^{(\sin x)(\ln x)}$ 7. $3(\ln x + 1)$ 9. $\frac{\cos(\ln x)}{x}$, $x > 0$
 11. $-\frac{5}{x(\ln 2x)^6}$ 13. $4^{2x+1} x^{4x}(1 + \ln 2x)$ 15. $(\ln 2) 2^{x^2+1} x$
 17. $2(x+1)^{2x} \left(\frac{x}{x+1} + \ln(x+1) \right)$
 19. $y^{\sin y} \left(\cos y \ln y + \frac{\sin y}{y} \right)$ 21. $-20xe^{-10x^2}$ 23. $x^{2x}(2 \ln x + 2)$
 25. $-(1/x)^x(1 + \ln x)$ 27. $\left(-\frac{4}{x+4} + \ln \left(\frac{x+4}{x} \right) \right) \left(1 + \frac{4}{x} \right)^x$
 29. $6(1 - \ln 2)$ 31. $\frac{3}{8}$ 33. $\frac{1}{2} \ln(4 + e^{2x}) + C$ 35. $\frac{1}{\ln 2} - \frac{1}{\ln 3}$
 37. $4 - \frac{4}{e^2}$ 39. $2e^{\sqrt{x}} + C$ 41. $\ln|e^x - e^{-x}| + C$ 43. $\frac{99}{10 \ln 10}$
 45. 3 47. $\frac{6^{x^3+8}}{3 \ln 6} + C$ 49. $\frac{1}{6} e^{3x^2+1} + C$ 51. $-\frac{1}{9^x \ln 9} + C$
 53. $\frac{10^{x^3}}{3 \ln 10} + C$ 55. $\frac{3 \cdot 3^{\ln 2} - 1}{\ln 3}$ 57. $\frac{32}{3}$ 59. $\frac{1}{3} \ln \frac{65}{16}$

61. $2e^{5+\sqrt{x}} + C$ 63.

h	$(1+2h)^{1/h}$	h	$(1+2h)^{1/h}$
10^{-1}	6.1917	-10^{-1}	9.3132
10^{-2}	7.2446	-10^{-2}	7.5404
10^{-3}	7.3743	-10^{-3}	7.4039
10^{-4}	7.3876	-10^{-4}	7.3905
10^{-5}	7.3889	-10^{-5}	7.3892
10^{-6}	7.3890	-10^{-6}	7.3891

$\lim_{h \rightarrow 0} (1+2h)^{1/h} = e^2$

65.

x	$\frac{2^x - 1}{x}$	x	$\frac{2^x - 1}{x}$
10^{-1}	0.71773	-10^{-1}	0.66967
10^{-2}	0.69556	-10^{-2}	0.69075
10^{-3}	0.69339	-10^{-3}	0.69291
10^{-4}	0.69317	-10^{-4}	0.69312
10^{-5}	0.69315	-10^{-5}	0.69314
10^{-6}	0.69315	-10^{-6}	0.69315

$$\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \ln 2$$

67. a. True b. False c. False d. False e. True

69. $\frac{\ln p}{p-1}, 0$ 71. a. No b. No

75. $\ln 2 = \int_1^2 \frac{dt}{t} < L_2 = \frac{5}{6} < 1$

$$\ln 3 = \int_1^3 \frac{dt}{t} > R_7$$

$$= 2 \left(\frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \frac{1}{15} + \frac{1}{17} + \frac{1}{19} + \frac{1}{21} \right) > 1$$

Section 7.2 Exercises, pp. 499–501

1. The relative growth is constant. 3. The time it takes a function to double in value 5. $T_2 = (\ln 2)/k$ 7. $\frac{\ln 2}{20} \approx 0.03466$

9. Compound interest, world population 11. $\ln 1.11 \approx 0.1044$.

13. $\frac{df}{dt} = 10.5$; $\frac{dg}{dt} \cdot \frac{1}{g} = \frac{1}{10}$

15. a. $\ln 1.024 \approx 0.02372$; $y(t) = 90,000 e^{t \ln 1.024}$ b. 2028

17. a. $\frac{\ln 1.1}{10} \approx 0.009531$; $y(t) = 50,000 e^{t \ln 1.1/10}$ b. 60,500

19. a. $\ln 1.016 \approx 0.01587$; $y(t) = 100 e^{t \ln 1.016}$ b. \$126.88

21. 3.71% 23. a. 88.1 years; 423.4 million

b. 99.4 years; 412.2 million 25. 28.7 million 27. 2026

29. $a(t) = 20e^{(t/36) \ln 0.5}$ mg with $t = 0$ at midnight; 15.87 mg;

119.6 hr $\approx$ 5 days 31. 1.798 million; the downward turn in the

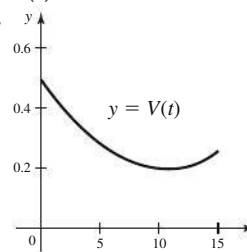
population size may be temporary. 33. 18,928 ft; 125,754 ft

35. 1.055 billion yr 37. 6.2 hours 39. 2 dollars 41. 1044 days

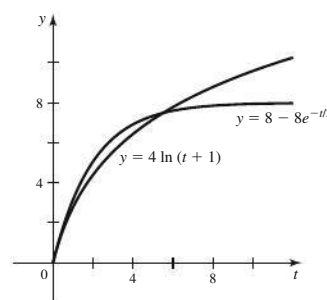
43. a. False b. False c. True d. True e. True

45. a. $V_1(t) = 0.495e^{-0.1216t}$ b. $V_2(t) = 0.005e^{0.239t}$

c. $V(t) = 0.495e^{-0.1216t} + 0.005e^{0.239t}$

d.  The tumor initially shrinks significantly in size but eventually starts growing again. e. 10.9 days; give a second treatment just before the end of the 10th day after the first treatment.

47. a. Bob; Abe b. $y = 4 \ln(t+1)$ and $y = 8 - 8e^{-t/2}$; Bob



49. 10.034%; no 51. 1.3 s

53. $k = \ln(1+r)$; $r = 2^{1/T_2} - 1$; $T_2 = (\ln 2)/k$

Section 7.3 Exercises, pp. 513–517

1. $\cosh x = \frac{e^x + e^{-x}}{2}$; $\sinh x = \frac{e^x - e^{-x}}{2}$ 3. $\cosh^2 x - \sinh^2 x = 1$

5. $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ 7. Evaluate $\sinh^{-1} \frac{1}{5}$.

9. $\int \frac{dx}{16 - x^2} = \frac{1}{4} \coth^{-1} \frac{x}{4} + C$ when $|x| > 4$; the values in the interval of integration $6 \leq x \leq 8$ satisfy $|x| > 4$.

23. $2 \cosh x \sinh x$ 25. $2 \tanh x \operatorname{sech}^2 x$ 27. $-2 \tanh 2x$

29. $2x(3x \sinh 3x + \cosh 3x) \cosh 3x$ 31. $4/\sqrt{16x^2 - 1}$

33. $2v/\sqrt{v^4 + 1}$ 35. $\sinh^{-1} x$ 37. $(\sinh 2x)/2 + C$

39. $\ln(1 + \cosh x) + C$ 41. $x - \tanh x + C$

43. $(\cosh^4 3 - 1)/12 \approx 856$ 45. $\ln(5/4)$

47. $\frac{1}{2\sqrt{2}} \coth^{-1} \left(\frac{x}{2\sqrt{2}} \right) + C$ 49. $\tanh^{-1}(e^x/6)/6 + C$

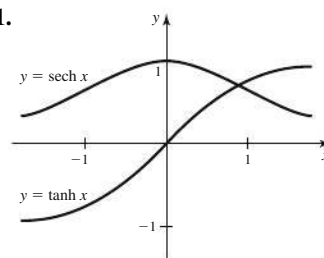
51. $-\operatorname{sech}^{-1}(x^4/2)/8 + C$ 53. $-\operatorname{csch} z + C$

55. $\ln \sqrt{3} \cdot \ln(4/3) \approx 0.158$ 57. $\frac{x^2 + 1}{2x} + C$

59. a. The values of $y = \coth x$ are close to 1 on $[5, 10]$.

b. $\ln(\sinh 10) - \ln(\sinh 5) \approx 5.0000454$; $|\text{error}| \approx 0.0000454$

61. a. $x = \sinh^{-1} 1 = \ln(1 + \sqrt{2})$ b. $\pi/4 - \ln \sqrt{2} \approx 0.44$



63. $\sinh^{-1} 2 = \ln(2 + \sqrt{5})$ 65. $-(\ln 5)/3 \approx -0.54$

67. $3 \ln \left(\frac{\sqrt{5} + 2}{\sqrt{2} + 1} \right) = 3(\sinh^{-1} 2 - \sinh^{-1} 1)$

69. $\frac{1}{15} \left(17 - \frac{8}{\ln(5/3)} \right) \approx 0.09$

71. a. $\text{Sag} = f(50) - f(0) = a(\cosh(50/a) - 1) = 10$; now divide by a . b. $t \approx 0.08$ c. $a = 10/t \approx 125$;

$L = 250 \sinh(2/5) \approx 102.7$ ft 73. $\lambda \approx 32.81$ m

75. b. When $d/\lambda < 0.05$, $2\pi d/\lambda$ is small. Because $\tanh x \approx x$ for small values of x , $\tanh(2\pi d/\lambda) \approx 2\pi d/\lambda$; therefore,

$$v = \sqrt{\frac{g\lambda}{2\pi} \tanh \left(\frac{2\pi d}{\lambda} \right)} \approx \sqrt{\frac{g\lambda}{2\pi} \cdot \frac{2\pi d}{\lambda}} = \sqrt{gd}.$$

c. $v = \sqrt{gd}$ is a function of depth alone; when depth d decreases, v also decreases. 77. a. False b. False c. True d. False

79. a. 1 b. 0 c. Undefined d. 1 e. 13/12 f. 40/9

g. $\left(\frac{e^2 + 1}{2e}\right)^2$ h. Undefined i. $\ln 4$ j. 1 81. $x = 0$

83. $x = \pm \tanh^{-1}(1/\sqrt{3}) = \pm \ln(2 + \sqrt{3})/2 \approx \pm 0.658$

85. $\tan^{-1}(\sinh 1) - \pi/4 \approx 0.08$ 87. Applying l'Hôpital's Rule twice brings you back to the initial limit; $\lim_{x \rightarrow \infty} \tanh x = 1$.

89. $2/\pi$ 91. 1 93. $12(3 \ln(3 + \sqrt{8}) - \sqrt{8}) \approx 29.5$

95. a. Approx. 360.8 m b. First 100 m: $t \approx 4.72$ s, $v_{av} \approx 21.2$ m/s; second 100 m: $t \approx 2.25$ s, $v_{av} \approx 44.5$ m/s 97. a. $\sqrt{mg/k}$

b. $35\sqrt{3} \approx 60.6$ m/s c. $t = \sqrt{\frac{m}{kg}} \tanh^{-1} 0.95 = \frac{\ln 39}{2} \sqrt{\frac{m}{kg}}$

d. Approx. 736.5 m 109. $\ln(21/4) \approx 1.66$

Chapter 7 Review Exercises, pp. 518–519

1. a. False b. False c. False d. True 3. $\ln 4$

5. $\frac{1}{2} \ln(x^2 + 8x + 25) + C$

7. $\cosh^{-1}(x/3) + C = \ln(x + \sqrt{x^2 - 9}) + C$

9. $\tanh^{-1}(1/3)/9 = (\ln 2)/18 \approx 0.0385$

11. $x^{3x^2+1} \left(6x \ln x + 3x + \frac{1}{x}\right)$ 13. $\sinh^2 t + \cosh^2 t$

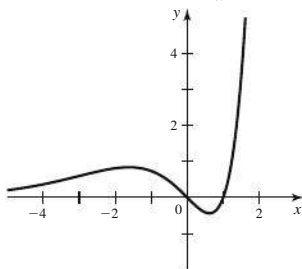
15. $3 \sinh(6x - 2)$ 17. $-\csc x$ 19. $\frac{2x}{\sqrt{x^4 - 1}}$

21. Approx. 7.3 hours 23. a. $y(t) = 29,000e^{(t \ln 2)/2}$

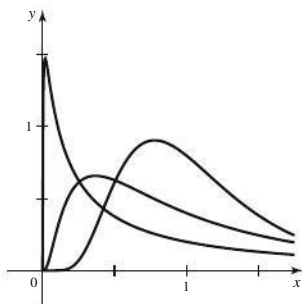
b. Approx. 41,996,486 transistors (which closely approximates the actual number of transistors) 25. 48.37 yr

27. Local max at $x = -\frac{1}{2}(\sqrt{5} + 1)$; local min at $x = \frac{1}{2}(\sqrt{5} - 1)$; inflection points at $x = -3$ and $x = 0$; $\lim_{x \rightarrow -\infty} f(x) = 0$;

$\lim_{x \rightarrow \infty} f(x) = \infty$



29. a.



b. $\lim_{x \rightarrow 0} f(x) = 0$

d. $f(x^*) = \frac{1}{\sqrt{2\pi}} \frac{e^{\sigma^2/2}}{\sigma}$

e. $\sigma = 1$

31. $L(x) = \frac{5}{3} + \frac{4}{3}(x - \ln 3)$; $\cosh 1 \approx 1.535$

33. a. $\cosh x$ b. $(1 - x \tanh x) \operatorname{sech} x$

CHAPTER 8

Section 8.1 Exercises, pp. 523–525

1. $u = 4 - 7x$ 3. $\sin^2 x = \frac{1 - \cos 2x}{2}$ 5. Complete the square in

$x^2 - 4x - 9$. 7. $\frac{1}{15(3 - 5x)^3} + C$ 9. $\frac{\sqrt{2}}{4}$ 11. $\frac{1}{2} \ln^2 2x + C$

13. $\ln(e^x + 1) + C$ 15. $\frac{32}{3}$ 17. $\frac{21}{110}$

19. $\frac{(\ln w - 1)^9}{9} + \frac{(\ln w - 1)^8}{8} + C$

21. $\frac{1}{2} \ln(x^2 + 4) + \tan^{-1} \frac{x}{2} + C$

23. $-\frac{1}{3} \ln |\csc(3e^x + 4) + \cot(3e^x + 4)| + C$ 25. 1

27. $3\sqrt{1 - x^2} + 2 \sin^{-1} x + C$ 29. $\ln(\sqrt{2} + 1)$

31. $\frac{1}{3} \tan^{-1}\left(\frac{x-1}{3}\right) + C$ 33. $\frac{x^2}{2} + x + \ln(x^2 + x + 2) + C$

35. $\frac{3\pi + 10}{12}$ 37. $\sin^{-1}\left(\frac{\theta + 3}{6}\right) + C$ 39. $\tan \theta - \sec \theta + C$

41. $-x - \cot x - \csc x + C$ 43. $\frac{1}{3} \ln(1 + \sinh 3x) + C$

45. $\frac{1}{2} \ln |e^{2x} - 2| + C$ 47. $x - \ln |x + 1| + C$

49. $\frac{4}{5} (9 + \sqrt{t+1})^{3/2} (\sqrt{t+1} - 6) + C$ 51. $\frac{\ln 4 - \pi}{4}$

53. $\ln |\sec(e^x + 1) + \tan(e^x + 1)| + C$

55. $\frac{2 \sin^3 x}{3} + C$ 57. $2 \tan^{-1} \sqrt{x} + C$

59. $\frac{1}{2} \ln(x^2 + 6x + 13) - \frac{5}{2} \tan^{-1}\left(\frac{x+3}{2}\right) + C$

61. $-\frac{1}{e^x + 1} + C$ 63. $\frac{1}{2}$ 65. a. False b. False c. False

d. False 69. a. $\frac{\tan^2 x}{2} + C$ b. $\frac{\sec^2 x}{2} + C$ c. The antiderivatives differ by a constant. 71. a. $\frac{1}{2}(x+1)^2 - 2(x+1) + \ln|x+1| + C$

b. $\frac{x^2}{2} - x + \ln|x+1| + C$ c. The antiderivatives differ by a

constant. 73. $\frac{\ln 26}{3}$ 75. $\frac{2}{3}(5\sqrt{5} - 1)\pi$

77. $\pi\left(\frac{9}{2} - \frac{5\sqrt{5}}{6}\right)$ 79. $\frac{2048 + 1763\sqrt{41}}{9375}$

Section 8.2 Exercises, pp. 529–532

1. Product Rule 3. $\frac{x^2(2 \ln x - 1)}{4} + C$ 5. Products for which the

choice for dv is easily integrated and when the resulting new integral is no more difficult than the original integral

7. $(\tan x + 2) \ln(\tan x + 2) - \tan x + C$

9. $\frac{1}{5} x \sin 5x + \frac{1}{25} \cos 5x + C$ 11. $\frac{e^{6t}}{36} (6t - 1) + C$

13. $\frac{x^2}{4} (2 \ln 10x - 1) + C$ 15. $(w + 2) \sin 2w + \frac{1}{2} \cos 2w + C$

17. $\frac{3^x}{\ln 3} \left(x - \frac{1}{\ln 3}\right) + C$ 19. $-\frac{1}{9x^9} \left(\ln x + \frac{1}{9}\right) + C$

21. $\frac{1}{8} \sin 2x - \frac{x}{4} \cos 2x + C$ 23. $\frac{1}{4} (1 - 2x^2) \cos 2x + \frac{x}{2} \sin 2x + C$

25. $-e^{-t}(t^2 + 2t + 2) + C$ 27. $\frac{e^x}{2} (\sin x + \cos x) + C$

29. $-\frac{e^{-x}}{17} (\sin 4x + 4 \cos 4x) + C$

31. $-e^{2x} \cos e^x + 2e^x \sin e^x + 2 \cos e^x + C$ 33. π 35. $-\frac{1}{2}$

37. $\frac{1}{9} (5e^6 + 1)$ 39. $\frac{\pi - 2}{2}$ 41. a. $x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2) + C$

b. $\frac{1}{2}x^2 \tan^{-1} x^2 - \frac{1}{4} \ln(1 + x^4) + C$ 43. $\pi(1 - \ln 2)$ 45. π

47. $\frac{2\pi}{27}(13e^6 - 1)$ 49. a. False b. True c. True

51. Let $u = x^n$ and $dv = \cos ax \, dx$. 53. Let $u = \ln^n x$ and $dv = dx$.

55. $\frac{x^2 \sin 5x}{5} + \frac{2x \cos 5x}{25} - \frac{2 \sin 5x}{125} + C$

57. $6 - 2e$ 59. a. $\frac{2}{3}(x - 2)\sqrt{x + 1} + C$

61. $\int \log_b x \, dx = \int \frac{\ln x}{\ln b} \, dx = \frac{1}{\ln b}(x \ln x - x) + C$

63. $2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$

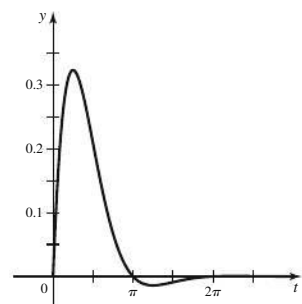
65. Let $u = x$ and $dv = f''(x) \, dx$.

67. $2e^3$ 69. x-axis: $\pi^2/2$; y-axis: $2\pi^2$ 71. $\pi(\pi - 2)$

75. a. $t = k\pi$, for $k = 0, 1, 2, \dots$ b. $\frac{e^{-\pi} + 1}{2\pi}$

c. $(-1)^n \left(\frac{e^\pi + 1}{2\pi e^{(n+1)\pi}} \right)$

d. $a_n = a_{n-1} \cdot \frac{1}{e^\pi}$



77. c. $\int f(x)g(x) \, dx = f(x)G_1(x) - f'(x)G_2(x) + f''(x)G_3(x) - \int f'''(x)G_3(x) \, dx$

f and its derivatives	g and its integrals
$f(x) \xrightarrow{+}$	$g(x)$
$f'(x) \xrightarrow{-}$	$G_1(x)$
$f''(x) \xrightarrow{+}$	$G_2(x)$
$f'''(x) \xrightarrow{-}$	$G_3(x)$

d. $\int x^2 e^{x/2} \, dx = 2x^2 e^{x/2} - 8xe^{x/2} + 16e^{x/2} + C$

f and its derivatives	g and its integrals
$x^2 \xrightarrow{+}$	$e^{x/2}$
$2x \xrightarrow{-}$	$2e^{x/2}$
$2 \xrightarrow{+}$	$4e^{x/2}$
$0 \xrightarrow{-}$	$8e^{x/2}$

$\frac{d^n}{dx^n}(x^2) = 0$, for $n \geq 3$, so all entries in the left column of the table beyond row three are 0, which results in no additional contribution to the antiderivative. e. $x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + C$;

five rows are needed because $\frac{d^n}{dx^n}(x^3) = 0$, for $n \geq 4$.

f. $\frac{d^k}{dx^k}(p_n(x)) = 0$, for $k \geq n + 1$

79. a. $\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

b. $\frac{1}{2}(e^x \sin x + e^x \cos x) + C$ c. $-\frac{3}{13}e^{-2x} \cos 3x - \frac{2}{13}e^{-2x} \sin 3x + C$

81. a. $I_1 = -\frac{1}{2}e^{-x^2} + C$ b. $I_3 = -\frac{1}{2}e^{-x^2}(x^2 + 1) + C$

c. $I_5 = -\frac{1}{2}e^{-x^2}(x^4 + 2x^2 + 2) + C$

Section 8.3 Exercises, pp. 536–538

1. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$; $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ 3. Rewrite $\sin^3 x$ as $(1 - \cos^2 x) \sin x$. 5. A reduction formula expresses an integral with a power in the integrand in terms of another integral with a smaller power in the integrand. 7. Let $u = \tan x$.

9. $\sin x - \frac{1}{3} \sin^3 x + C$ 11. $\frac{x}{2} - \frac{1}{12} \sin 6x + C$

13. $-\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C$ 15. $\frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$

17. $\frac{2}{3} \sin^{3/2} x - \frac{2}{7} \sin^{7/2} x + C$ 19. $\frac{7}{24}$ 21. $\frac{8}{45}$

23. $\frac{1}{8}x - \frac{1}{32} \sin 4x + C$ 25. $\frac{1}{48} \sin^3 2x + \frac{1}{16}x - \frac{1}{64} \sin 4x + C$

27. $\tan x - x + C$ 29. $-\frac{1}{3} \cot^3 x + \cot x + x + C$

31. $4 \tan^5 x - \frac{20}{3} \tan^3 x + 20 \tan x - 20x + C$ 33. $\tan^{10} x + C$

35. $\frac{1}{3} \sec^3 x + C$ 37. $\frac{1}{3} \tan^3(\ln \theta) + \tan(\ln \theta) + C$ 39. $\ln 4$

41. $\frac{7}{6}$ 43. $\frac{1}{8} \tan^2 4x + \frac{1}{4} \ln |\cos 4x| + C$ 45. $\frac{2}{3} \tan^{3/2} x + C$

47. $\tan x - \cot x + C$ 49. $\frac{1}{25}$ 51. $-2 \cot x - \frac{\cot^3 x}{3} + C$

53. $\frac{4}{3}$ 55. $\frac{4}{3} - \ln \sqrt{3}$ 57. $8\sqrt{2}/3$ 59. $\sqrt{2}$ 61. $2\sqrt{2}/3$

63. a. True b. False 65. $\frac{2\pi}{35}$ 67. $\frac{1}{8} \cos 4x - \frac{1}{20} \cos 10x + C$

69. $\frac{1}{2} \sin x - \frac{1}{10} \sin 5x + C$ 73. $\frac{1}{2} - \ln \sqrt{2}$ 75. a. $\frac{\pi}{2}; \frac{\pi}{2}$

b. $\frac{\pi}{2}$, for all n d. Yes e. $\frac{3\pi}{8}$, for all n

Section 8.4 Exercises, pp. 543–546

1. $x = 3 \sec \theta$ 3. $x = 10 \sin \theta$ 5. $\sqrt{4 - x^2}/x$ 7. $\pi/6$

9. $\frac{25\pi}{3}$ 11. $\frac{\pi}{12}$ 13. $\sin^{-1} \frac{x}{4} + C$ 15. $-\frac{\sqrt{x^2 + 9}}{9x} + C$

17. $2 - \frac{\pi}{2}$ 19. $\ln(\sqrt{x^2 - 81} + x) + C$

21. $\frac{x}{2} \sqrt{64 - x^2} + 32 \sin^{-1} \frac{x}{8} + C$ 23. $\frac{x}{25\sqrt{25 - x^2}} + C$

25. $-3 \ln \left| \frac{\sqrt{9 - x^2} + 3}{x} \right| + \sqrt{9 - x^2} + C$ 27. $\sqrt{2}/6$

29. $\frac{1}{16} \left(\tan^{-1} \frac{x}{2} + \frac{2x}{x^2 + 4} \right) + C$

31. $8 \sin^{-1}(x/4) - x\sqrt{16 - x^2}/2 + C$

33. $\sqrt{x^2 - 9} - 3 \sec^{-1}(x/3) + C$

35. $-1/\sqrt{x^2 - 1} - \sec^{-1} x + C$ 37. $2 - \sqrt{2}$

39. $x/\sqrt{100 - x^2} - \sin^{-1}(x/10) + C$ 41. $x/\sqrt{1 + 4x^2} + C$

43. $\frac{\ln 3}{2}$ 45. $81/(2(81 - x^2)) + \ln \sqrt{81 - x^2} + C$

47. $\frac{1}{16}(1 - \sqrt{3} - \ln(21 - 12\sqrt{3}))$ 49. $\frac{1}{3} + \frac{\ln 3}{4}$

51. $\frac{x}{2} \sqrt{4 + x^2} - 2 \ln(x + \sqrt{4 + x^2}) + C$

53. $\frac{9}{10} \cos^{-1} \frac{5}{3x} - \frac{\sqrt{9x^2 - 25}}{2x^2} + C$

55. $\frac{\sec^{-1} \frac{x}{10}}{2000} + \frac{\sqrt{x^2 - 100}}{200x^2} + C$

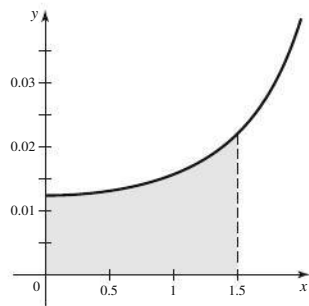
57. a. False b. True c. False d. False

61. $\sin^{-1} \left(\frac{x+1}{2} \right) + C$ 63. $\frac{1}{3} \tan^{-1} \left(\frac{x+3}{3} \right) + C$

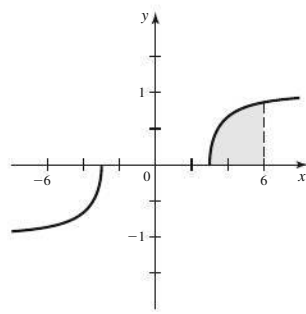
65. $\frac{\pi\sqrt{2}}{48}$ 67. $\frac{x-4}{\sqrt{9+8x-x^2}} - \sin^{-1} \left(\frac{x-4}{5} \right) + C$

69. $\ln((2 + \sqrt{3})(\sqrt{2} - 1))$

71. $\frac{1}{81} + \frac{\ln 3}{108}$



73. $3\sqrt{3} - \pi$



75. $\frac{3}{80}$ 77. $\frac{1}{4a} (20a\sqrt{1+400a^2} + \ln(20a + \sqrt{1+400a^2}))$

81. b. $\lim_{L \rightarrow \infty} \frac{kQ}{a\sqrt{a^2 + L^2}} = \lim_{L \rightarrow \infty} 2\rho k \frac{1}{a\sqrt{\left(\frac{a}{L}\right)^2 + 1}} = \frac{2\rho k}{a}$

85. a. $\frac{1}{\sqrt{g}} \left(\frac{\pi}{2} - \sin^{-1} \left(\frac{2 \cos b - \cos a + 1}{\cos a + 1} \right) \right)$

b. For $b = \pi$, the descent time is $\frac{\pi}{\sqrt{g}}$, a constant.

Section 8.5 Exercises, pp. 554–556

1. Rational functions 3. a. $\frac{A}{x-3}$ b. $\frac{A}{x-4}, \frac{B}{(x-4)^2}, \frac{C}{(x-4)^3}$

c. $\frac{Ax+B}{x^2+2x+6}$ 5. $\frac{A}{x-4} + \frac{B}{x-5}$ 7. $\frac{A}{x-5} + \frac{B}{(x-5)^2}$

9. $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} + \frac{D}{x+2} + \frac{E}{x-2}$ 11. $\frac{A}{x} + \frac{Bx+C}{x^2+1}$

13. $\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+x+2} + \frac{Ex+F}{(x^2+x+2)^2}$

15. $\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2} + \frac{D}{(x+2)^2} + \frac{Ex+F}{x^2+4} + \frac{Gx+H}{(x^2+4)^2}$

17. $\frac{2}{x-1} + \frac{3}{x-2}$ 19. $\frac{1}{x-4} - \frac{1}{x+2}$ 21. $2 + \frac{3}{x+1} - \frac{4}{x+2}$

23. $\ln \left| \frac{x-1}{x+2} \right| + C$ 25. $3 \ln \left| \frac{x-1}{x+1} \right| + C$

27. $3 \ln|x-1| - \frac{1}{3} \ln|3x-2| + C$ 29. $-\ln 4$

31. $\ln \left| \frac{(x-2)^2(x+1)}{(x+2)^2(x-1)} \right| + C$ 33. $3x + \ln \frac{(x-2)^{14}}{|x-1|} + C$

35. $\ln \left| \frac{x(x-2)^3}{(x+2)^3} \right| + C$ 37. $\ln \left| \frac{(x-3)^{1/3}(x+1)}{(x+3)^{1/3}(x-1)} \right|^{1/16} + C$

39. $\frac{9}{x} + \ln \left| \frac{x-9}{x} \right| + C$ 41. $\ln 2 - \frac{3}{4}$ 43. $-\frac{2}{x} + \ln \left| \frac{x+1}{x} \right|^2 + C$

45. $\frac{5}{x} + \ln \left| \frac{x}{x+1} \right|^6 + C$ 47. $\frac{x^2}{2} + 2 \ln|x-5| - \frac{10}{x-5} + C$

49. $\frac{3}{x-1} + \ln \left| \frac{(x-1)^5}{x^4} \right| + C$ 51. $\ln|x+1| + \tan^{-1}x + C$

53. $\ln(x+1)^2 + \tan^{-1}(x+1) + C$

55. $\ln \left| \frac{(x-1)^2}{x^2+4x+5} \right| + 14 \tan^{-1}(x+2) + C$

57. $\frac{1}{2} \ln|x^2+3| - \frac{1}{x^2+3} + C$

59. $\frac{1}{2} \ln(x^2+6x+10) - 3 \tan^{-1}(x+3) - \frac{1}{x^2+6x+10} + C$

61. $\ln \left(\frac{x^2}{x^2+1} \right) + \frac{1}{x^2+1} + C$

63. $\sqrt{\frac{3}{7}} \tan^{-1} \left(\sqrt{\frac{3}{7}} x \right) - \frac{1}{6(3x^2+7)} + C$

65. a. False b. False c. False d. True 67. $\ln 6$

69. $\left(\frac{24}{5} - 2 \ln 5 \right) \pi$ 71. $\frac{2}{3} \pi \ln 2$ 73. $\ln \sqrt{\left| \frac{x-1}{x+1} \right|} + C$

75. $\frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} + \frac{Fx+G}{x^2+x+4} + \frac{Hx+I}{(x^2+x+4)^2};$

$\frac{1}{16x} - \frac{x+10}{100(x^2+1)} + \frac{4x+3}{50(x^2+1)^2} - \frac{21x-19}{400(x^2+x+4)} -$

$\frac{4x+1}{20(x^2+x+4)^2}$ 77. $\ln \left| \frac{e^x-1}{e^x+2} \right|^{1/3} + C$

79. $\frac{1}{4} \ln \left(\frac{1+\sin t}{1-\sin t} - \frac{2}{1+\sin t} \right) + C$

81. $\tan^{-1}e^x - \frac{1}{2(e^{2x}+1)} + C$ 83. $x - \ln(1+e^x) + C$

89. $-\cot x - \csc x + C = -\cot(x/2) + C$

91. $\frac{1}{\sqrt{2}} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1}$ 93. a. Car A b. Car C

c. $S_A(t) = 88t - 88 \ln|t+1|;$

$S_B(t) = 88 \left(t - \ln(t+1)^2 - \frac{1}{t+1} + 1 \right);$

$S_C(t) = 88(t - \tan^{-1}t)$

d. Car C 95. Because $\frac{x^4(1-x)^4}{1+x^2} > 0$ on $(0, 1)$,

$\int_0^1 \frac{x^4(1-x^4)}{1+x^2} dx > 0$; therefore, $\frac{22}{7} > \pi$.

Section 8.6 Exercises, pp. 560–562

1. Integrate by parts. 3. Let $x = 8 \sin \theta$. 5. Use the method of

partial fractions. 7. $\frac{\pi}{4}$ 9. $\frac{\pi}{6}$ 11. $\frac{5}{4} - \frac{3\pi}{8}$ 13. $-\frac{\sqrt{1-e^{2x}}}{e^x} + C$

15. $\frac{4}{\ln 2}$ 17. $\frac{3e^4}{2}$ 19. $\frac{16}{35}$ 21. $\frac{x^{10}}{10} \ln 3x - \frac{x^{10}}{100} + C$

23. $\ln|\cos x + 1| - \ln|\cos x| + C$

25. $\ln \left| \frac{x}{1 + \sqrt{1-x^2}} \right| + C$ 27. $\frac{3x}{8} - \frac{1}{2} \sin x + \frac{1}{16} \sin 2x + C$

29. $\ln|\sin x + \sin^2 x| + C$ 31. $6 \sin^{-1} \frac{x}{2} + \frac{3}{2} x \sqrt{4-x^2} + C$

33. $\frac{1}{a} \tan^{-1} \frac{e^x}{a} + C$ 35. $\frac{11}{6}$ 37. $\frac{\sqrt{3+1}}{2}$

39. $-\cos x \ln(\sin x) - \ln|\csc x + \cot x| + \cos x + C$

41. $-\frac{2}{5} \cot^{5/2} x - \frac{2}{9} \cot^{9/2} x + C$ 43. $\frac{\sin^{-1} x^{10}}{10} + C$ 45. $\ln \frac{4}{3} - \frac{1}{6}$

47. $x^2 + 3x + 4 \ln|x-2| + \ln|x+1| + C$

49. $\frac{\sec^{11} x}{11} - \frac{\sec^9 x}{9} + C$ 51. $\frac{4}{7} (2^{7/4} - 1)$

53. $-\frac{\cot^2 e^x}{2} - \ln|\sin e^x| + C$ 55. $\ln|x^3+x| + 3 \tan^{-1}x + C$

57. $-2\sqrt{x} \cos \sqrt{x} + 2 \sin \sqrt{x} + C$ 59. $-\frac{1}{x} - \tan^{-1}x + C$

61. $\frac{\sqrt{2}e^{\pi/4} - 1}{2}$ 63. $\frac{x^{a+1}}{a+1} \left(\ln x - \frac{1}{a+1} \right) + C$
 65. $\frac{\pi}{18}$ 67. $\frac{1}{54} (\sin^{-1} 3x - 3x \sqrt{1-9x^2}) + C$
 69. $\frac{1 - \sqrt{1-x^2}}{x} + C$ 71. $-2 \cot x + 2 \csc x - x + C$
 73. $\frac{40\sqrt{5}}{3} - \frac{224}{15}$ 75. $\frac{7\pi^2}{144}$ 77. $x \cos^{-1} x - \sqrt{1-x^2} + C$
 79. $-\frac{\sin^{-1} x}{x} + \ln \left| \frac{x}{1 + \sqrt{1-x^2}} \right| + C$
 81. $\ln |x| + 2 \tan^{-1} x - \frac{3}{2(x^2 + 1)} + C$
 83. $\frac{\sin^{999} e^x}{999} - \frac{\sin^{1001} e^x}{1001} + C$ 85. a. True b. True c. False
 d. False 87. $\pi(\sqrt{2} + \ln(1 + \sqrt{2})) \approx 7.212$
 89. $\frac{\pi(4\sqrt{2} + 3)}{3} \approx 9.065$ 91. $9800\pi \ln 2 \approx 21,340.3$
 93. $4x - 2 \ln(e^{2x} + 2e^x + 17) - \tan^{-1} \left(\frac{e^x + 1}{4} \right) + C$
 95. $\frac{1}{4} \ln |\tan x + 1| - \frac{1}{4} \ln |\tan x - 1| + \frac{x}{2} + C$
 97. $x \tan^{-1} \sqrt[3]{x} - \frac{x^{2/3}}{2} + \frac{1}{2} \ln(1 + x^{2/3}) + C$
 99. $\pi \left(\sqrt{5} - \sqrt{2} + \frac{1}{2} \ln \left(\frac{\sqrt{5} - 1}{\sqrt{5} + 1} \right) - \frac{1}{2} \ln \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) \right) \approx 3.839$

Section 8.7 Exercises, pp. 565–567

1. Substitutions, integration by parts, partial fractions 3. The CAS may not include the constant of integration, and it may use a trigonometric identity or other algebraic simplification.

5. $-\frac{1}{3} \sin^3 e^x + \sin e^x + C$ 7. $x \cos^{-1} x - \sqrt{1-x^2} + C$
 9. $\ln(x + \sqrt{16+x^2}) + C$ 11. $\frac{3}{4}(2u - 7 \ln |7 + 2u|) + C$
 13. $-\frac{1}{4} \cot 2x + C$ 15. $\frac{1}{12}(2x - 1)\sqrt{4x + 1} + C$
 17. $\frac{1}{3} \ln \left| x + \sqrt{x^2 - \left(\frac{10}{3}\right)^2} \right| + C$ 19. $\ln(e^x + \sqrt{4 + e^{2x}}) + C$
 21. $-\frac{1}{2} \ln \left| \frac{2 + \sin x}{\sin x} \right| + C$
 23. $\frac{2 \ln^2 x - 1}{4} \sin^{-1}(\ln x) + \frac{\ln x \sqrt{1 - \ln^2 x}}{4} + C$
 25. $\frac{x}{16\sqrt{16 + 9x^2}} + C$ 27. $-\frac{1}{12} \ln \left| \frac{12 + \sqrt{144 - x^2}}{x} \right| + C$
 29. $2x + x \ln^2 x - 2x \ln x + C$
 31. $\frac{x + 5}{2} \sqrt{x^2 + 10x} - \frac{25}{2} \ln |x + 5 + \sqrt{x^2 + 10x}| + C$
 33. $\frac{1}{3} \tan^{-1} \left(\frac{x + 1}{3} \right) + C$ 35. $\ln x - \frac{1}{10} \ln(x^{10} + 1) + C$
 37. $2 \ln(\sqrt{x-6} + \sqrt{x}) + C$
 39. $-\frac{\tan^{-1} x^3}{3x^3} + \ln \left| \frac{x}{(x^6 + 1)^{1/6}} \right| + C$ 41. $4\sqrt{17} + \ln(4 + \sqrt{17})$
 43. $\sqrt{5} - \sqrt{2} + \ln \left(\frac{2 + 2\sqrt{2}}{1 + \sqrt{5}} \right)$ 45. $\frac{128\pi}{3}$ 47. $\frac{\pi^2}{4}$

49. $\frac{(x-3)\sqrt{3+2x}}{3} + C$ 51. $\frac{1}{3} \tan 3x - x + C$
 53. $\frac{1540 + 243 \ln 3}{8}$
 55. $\frac{(x^2 - a^2)^{3/2}}{3} - a^2 \sqrt{x^2 - a^2} + a^3 \cos^{-1} \frac{a}{x} + C$ 57. $\frac{\pi}{4}$
 59. $-\frac{x}{8} (2x^2 - 5a^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} \frac{x}{a} + C$
 61. $2 - \frac{\pi^2}{12} - \ln 4$ 63. $\frac{27,456\sqrt{15}}{7} \approx 15,190.9$
 65. $\frac{1}{8} e^{2x} (4x^3 - 6x^2 + 6x - 3) + C$ 67. $\frac{\tan^3 3y}{9} - \frac{\tan 3y}{3} + y + C$
 69. $\frac{1}{24} (128 - 78\sqrt{2} - 3 \ln(3 + 2\sqrt{2}))$
 71. $\frac{1}{a^2} (ax - b \ln |b + ax|) + C$
 73. $\frac{1}{a^2} \left(\frac{(ax + b)^{n+2}}{n+2} - \frac{b(ax + b)^{n+1}}{n+1} \right) + C$
 75. a. True b. True
 79. $\frac{1}{16} ((8x^2 - 1) \sin^{-1} 2x + 2x \sqrt{1 - 4x^2}) + C$
 81. $-\frac{\tan^{-1} x}{x} + \ln \left(\frac{|x|}{\sqrt{x^2 + 1}} \right) + C$ 83. b. $\frac{\pi}{8} \ln 2$
 85. a.

θ_0	T
0.10	6.27927
0.20	6.26762
0.30	6.24854
0.40	6.22253
0.50	6.19021
0.60	6.15236
0.70	6.10979
0.80	6.06338
0.90	6.01399
1.00	5.96247

b. All are within 10%.

87. b. $\frac{63\pi}{512}$ c. Decrease

Section 8.8 Exercises, pp. 578–582

1. $\frac{1}{2}$ 3. The Trapezoid Rule approximates areas under curves using trapezoids. 5. 42 7. $\frac{112}{3}$ 9. $-1, 1, 3, 5, 7, 9$
 11. $1.59 \times 10^{-3}; 5.04 \times 10^{-4}$ 13. $1.72 \times 10^{-3}; 6.32 \times 10^{-4}$
 15. 576; 640; 656 17. 0.643950551 19. 704; 672; 664
 21. 0.622 23. 2.28476811; 2.33512377 25. 1.76798499
 27. $M(25) \approx 0.63703884, T(25) \approx 0.63578179; 6.58 \times 10^{-4}, 1.32 \times 10^{-3}$

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	99	102	1.00	2.00
8	99.75	100.5	0.250	0.500
16	99.9375	100.125	6.3×10^{-2}	0.125
32	99.984375	100.03125	1.6×10^{-2}	3.1×10^{-2}

31.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	1.50968181	1.48067370	9.7×10^{-3}	1.9×10^{-2}
8	1.50241228	1.49517776	2.4×10^{-3}	4.8×10^{-3}
16	1.50060256	1.49879502	6.0×10^{-4}	1.2×10^{-3}
32	1.50015061	1.49969879	1.5×10^{-4}	3.0×10^{-4}

33.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	-1.96×10^{-16}	0	2.0×10^{-16}	0
8	7.63×10^{-17}	-1.41×10^{-16}	7.6×10^{-17}	1.4×10^{-16}
16	1.61×10^{-16}	1.09×10^{-17}	1.6×10^{-16}	1.1×10^{-17}
32	6.27×10^{-17}	-4.77×10^{-17}	6.3×10^{-17}	4.8×10^{-17}

35. $T(4) \approx 690.3$ million ft^3 ; $S(4) \approx 692.2$ million ft^3 (answers may vary) 37. 54.5°F , Trapezoid Rule 39. 35.0°F , Trapezoid Rule

41. a. Left sum: 204.917; right sum: 261.375; Trapezoid Rule: 233.146; the approximations measure the average temperature of the curling iron on $[0, 120]$. b. Left sum: underestimate; right sum: overestimate; Trapezoid Rule: underestimate c. 305°F is the change in temperature over $[0, 120]$. 43. a. 5907.5 b. 5965 c. 5917

45. a. $T(25) \approx 3.19623162$ $T(50) \approx 3.19495398$ b. $S(50) \approx 3.19452809$ c. $e_T(50) \approx 4.3 \times 10^{-4}$ $e_S(50) \approx 4.5 \times 10^{-8}$ 47. a. $T(50) \approx 1.00008509$ $T(100) \approx 1.00002127$ b. $S(100) \approx 1.00000000$ c. $e_T(100) \approx 2.1 \times 10^{-5}$ $e_S(100) \approx 4.6 \times 10^{-9}$

49.

n	$T(n)$	$S(n)$	Error in $T(n)$	Error in $S(n)$
4	1820.0000	—	284	—
8	1607.7500	1537.0000	71.8	1
16	1553.9844	1536.0625	18.0	6.3×10^{-2}
32	1540.4990	1536.0039	4.50	3.9×10^{-3}

51.

n	$T(n)$	$S(n)$	Error in $T(n)$	Error in $S(n)$
4	0.46911538	—	5.3×10^{-2}	—
8	0.50826998	0.52132152	1.3×10^{-2}	2.9×10^{-4}
16	0.51825968	0.52158957	3.4×10^{-3}	1.7×10^{-5}
32	0.52076933	0.52160588	8.4×10^{-4}	1.1×10^{-6}

53. a. True b. False c. True

55.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	0.40635058	0.40634782	1.4×10^{-6}	1.4×10^{-6}
8	0.40634920	0.40634920	7.6×10^{-10}	7.6×10^{-10}
16	0.40634920	0.40634920	6.6×10^{-13}	6.6×10^{-13}
32	0.40634920	0.40634920	8.9×10^{-16}	7.8×10^{-16}

57.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	4.72531819	4.72507878	1.2×10^{-4}	1.2×10^{-4}
8	4.72519850	4.72519849	9.1×10^{-9}	9.1×10^{-9}
16	4.72519850	4.72519850	0	8.9×10^{-16}
32	4.72519850	4.72519850	0	8.9×10^{-16}

63. Approximations will vary; exact value is 68.26894921

65. a. Approx. 1.6×10^{11} barrels b. Approx. 6.8×10^{10} barrels67. a. $M(50) \approx 34.4345566$ b. $f''(x) = \frac{3(x^4 + 4x)}{4(x^3 + 1)^{3/2}}$ d. $E_M \leq 0.0028$ 69. a. $T(40) = 0.874799972 \dots$ b. $f''(x) = e^x \cos e^x - e^{2x} \sin e^x$ d. $E_T \leq \frac{1}{3200}$ 71. a. $S(20) \approx 0.97774576$ b. $E_S \leq 3.5 \times 10^{-8}$ 73. Approximations will vary; exact value is 38.753792 77. Overestimate 79. $S(20) \approx 1.00000175$

Section 8.9 Exercises, pp. 590–593

1. The interval of integration is infinite or the integrand is unbounded

on the interval of integration. 3. $\lim_{b \rightarrow \infty} \int_2^b \frac{dx}{x^{1/5}}$ 5. $\int_{-\infty}^{\infty} f(x) dx$ 7. $\frac{1}{3}$ 9. Diverges 11. $\frac{1}{a}$ 13. Diverges 15. $\frac{\pi}{10}$ 17. Diverges 19. Diverges 21. $\frac{1}{\pi}$ 23. $\frac{\pi}{4}$ 25. $\frac{\pi}{6}$ 27. 029. $\frac{\pi^3}{12}$ 31. $\ln 2$ 33. Diverges 35. Diverges 37. 639. Diverges 41. Diverges 43. $2(e - 1)$ 45. Diverges47. $4 \cdot 10^{3/4}/3$ 49. Diverges 51. π 53. -1 55. $\ln(2 + \sqrt{3})$ 57. 2 59. \$41,666.67 61. 0.76 63. 20,000 hr65. $\frac{\pi}{3}$ 67. $3\pi/2$ 69. $\pi/\ln 2$ 71. 2π 73. Does not exist75. $\frac{72 \cdot 2^{1/3} \pi}{5}$ 77. Converges 79. Diverges 81. Converges

83. Diverges 85. Converges 87. a. True b. False c. False

d. True e. True 89. $1/b - 1/a$ 91. a. $A(a, b) = \frac{e^{-ab}}{a}$, for $a > 0$ b. $b = g(a) = -\frac{1}{a} \ln 2a$ c. $b^* = -2/e$ 93. π 107. a. π b. $\pi/(4e^2)$ 109. a. $6.28 \times 10^7 \text{ m J}$ b. 11.2 km/s c. $\leq 9 \text{ mm}$

Chapter 8 Review Exercises, pp. 593–596

1. a. True b. False c. False d. True e. False

3. $2(x - 8)\sqrt{x + 4} + C$ 5. $\frac{1}{3}\sqrt{x + 2}(x - 4) + C$ 7. $\frac{\pi}{4}$ 9. $\frac{4}{105}$ 11. $\sqrt{t - 1} - \tan^{-1}\sqrt{t - 1} + C$ 13. $\frac{2}{15}(1 - e^{3\pi})$ 15. $7 + \ln 40 - \ln 17$ 17. $2 \ln |x| + 3 \tan^{-1}(x + 1) + C$ 19. $\frac{2}{x + 3} - \frac{2}{(x + 3)^2} + \ln |x + 3| + C$ 21. $\sqrt{3} - 1 - \frac{\pi}{12}$ 23. $\frac{1}{5} \tan^5 t + C$ 25. $\frac{\pi}{8}$ 27. $\frac{\sqrt{w^2 + 2w - 8}}{9(w + 1)} + C$ 29. $-\frac{\cot^5 x}{5} + C$ 31. $\frac{x \cosh 2x}{2} - \frac{\sinh 2x}{4} + C$ 33. $\frac{1}{15} \sec^5 3\theta - \frac{1}{9} \sec^3 3\theta + C$

$$35. \frac{1}{6}(x^2 - 8)\sqrt{x^2 + 4} + C \quad 37. \frac{1}{x+1} + \ln|(x+1)(x^2 + 4)| + C$$

$$39. \frac{t - \ln(2 + e^t)}{2} + C \quad 41. \frac{1}{4}(\csc 4\theta - \cot 4\theta) + C$$

$$43. \frac{e^x}{2}(\sin x - \cos x) + C$$

$$45. \ln|x| - \frac{1}{x} + \frac{1}{2}\ln(x^2 + 4x + 9) - \frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + C$$

$$47. \frac{\theta}{2} + \frac{1}{16}\sin 8\theta + C \quad 49. \frac{\sec^{49} 2z}{98} + C \quad 51. \frac{4}{15}$$

$$53. 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln(\sqrt[6]{x} + 1) + C$$

$$55. -\frac{\sqrt{9-y^2}}{9\sqrt{2}y} + C \quad 57. \frac{\pi}{9} \quad 59. -\operatorname{sech} x + C \quad 61. \frac{\pi}{3}$$

$$63. \frac{1}{8}\ln\left|\frac{x-5}{x+3}\right| + C \quad 65. \frac{\ln 2}{4} + \frac{\pi}{8} \quad 67. 3 \quad 69. \frac{1}{3}\ln\left|\frac{x-2}{x+1}\right| + C$$

$$71. 2(x - 2\ln|x+2|) + C \quad 73. e^{2t}/(2\sqrt{1+e^{4t}}) + C$$

$$75. \text{a. } \sec e^x + C \quad \text{b. } e^x \sec e^x - \ln|\sec e^x + \tan e^x| + C$$

$$77. \frac{\sqrt{6}}{3}\tan^{-1}\sqrt{\frac{2x-3}{3}} + C$$

$$79. \frac{1}{4}\sec^3 x \tan x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| + C$$

$$81. 2(\ln^3 2 - 3\ln^2 2 + \ln 64 - 3) \quad 83. 1 \quad 85. \frac{\pi}{2}$$

$$87. \frac{2\pi}{\sqrt{3}} \quad 89. \text{Converges} \quad 91. \text{Diverges} \quad 93. 1.196288$$

$$95. M(4) = 44; T(4) = 42; S(4) = \frac{124}{3}$$

$$97. M(40) \approx 0.398236; T(40) \approx 0.398771; S(40) \approx 0.398416$$

$$99. 0.886227 \quad 101. y\text{-axis} \quad 103. \pi(e-2) \quad 105. \frac{\pi}{2}(e^2 - 3)$$

$$107. \text{a. } 1.603 \quad \text{b. } 1.870 \quad \text{c. } b \ln b - b = a \ln a - a$$

$$\text{d. Decreasing} \quad 109. 20/(3\pi) \quad 111. 1901 \text{ cars}$$

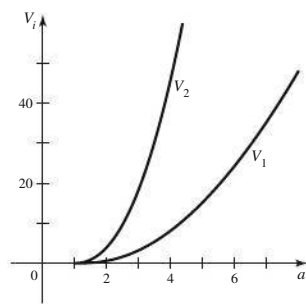
$$113. \text{a. } I(p) = \frac{1}{(p-1)^2}(1 - pe^{1-p}) \text{ if } p \neq 1, I(1) = \frac{1}{2} \quad \text{b. } 0, \infty$$

$$\text{c. } I(0) = 1 \quad 115. 0.4054651 \quad 117. n = 2$$

$$119. \text{a. } V_1(a) = \pi(a \ln^2 a - 2a \ln a + 2(a-1))$$

$$\text{b. } V_2(a) = \frac{\pi}{2}(2a^2 \ln a - a^2 + 1)$$

$$\text{c. } V_2(a) > V_1(a) \text{ for all } a > 1$$



$$121. a = \ln 2/(2b) \quad 123. \ln(1 + \sqrt{2}/2)$$

CHAPTER 9

Section 9.1 Exercises, pp. 604–606

$$1. \text{a. } 1 \quad \text{b. Linear} \quad 3. \text{Yes} \quad 5. \frac{\pi}{2} < t < \frac{3\pi}{2}$$

$$21. y = 3t - \frac{e^{-2t}}{2} + C \quad 23. y = 2 \ln|\sec 2x| - 3 \sin x + C$$

$$25. y = 2t^6 + 6t^{-1} - 2t^2 + C_1 t + C_2$$

$$27. u = \frac{x^{11}}{2} + \frac{x^9}{2} - \frac{x^7}{2} + \frac{5}{x} + C_1 x + C_2$$

$$29. u = \ln(x^2 + 4) - \tan^{-1} \frac{x}{2} + C \quad 31. y = \sin^{-1} x + C_1 x + C_2$$

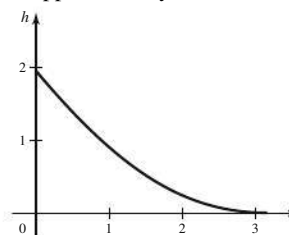
$$33. y = e^t + t + 3 \quad 35. y = x^3 + x^{-3} - 2, x > 0$$

$$37. y = -t^5 + 2t^3 + 1 \quad 39. y = e^t(t-2) + 2(t+1)$$

$$41. u = \frac{1}{4}\tan^{-1} \frac{x}{4} - 4x + 2 \quad 43. \text{a. } v(t) = -9.8t + 29.4;$$

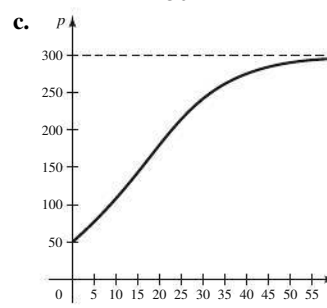
$s(t) = -4.9t^2 + 29.4t + 30$; the object is above the ground for approximately $0 \leq t \leq 6.89$. **b.** The highest point of 74.1 m is reached at $t = 3$ s. **45.** The amount of resource is increasing for $H < 75$ and is constant if $H = 75$. If $H = 100$, the resource vanishes at approximately 28 time units.

47. $h = (\sqrt{1.96} - 0.1t\sqrt{2g})^2 \approx (1.4 - 0.44t)^2, 0 \leq t \leq 3.16$; the tank is empty after approximately 3.16 s.



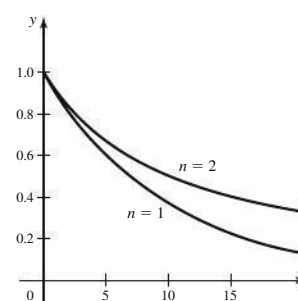
$$49. \text{a. False} \quad \text{b. False} \quad \text{c. True} \quad 51. \text{c. } y = C_1 \sin kt + C_2 \cos kt$$

$$53. \text{b. } C = \frac{K-50}{50}$$



$$\text{d. } 300$$

55. c. The decay rate is greater for the $n = 1$ model.

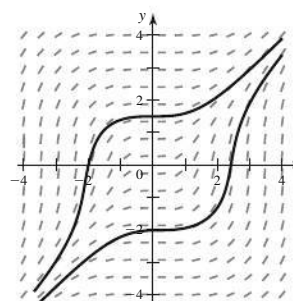


Section 9.2 Exercises, pp. 611–614

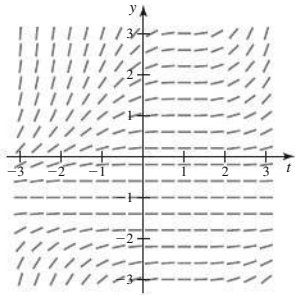
1. At selected points (t_0, y_0) in the region of interest draw a short line segment with slope $f(t_0, y_0)$. **3.** $y(3.1) \approx 1.6$

$$5. \text{a. D} \quad \text{b. B} \quad \text{c. A} \quad \text{d. C}$$

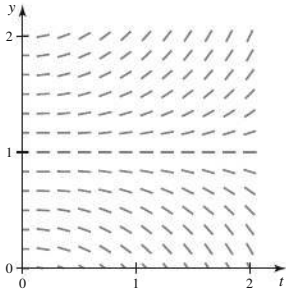
7.



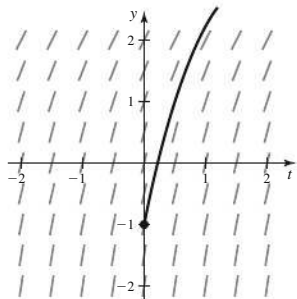
9. An initial condition of $y(0) = -1$ leads to a constant solution. For any other initial condition, the solutions are increasing over time.



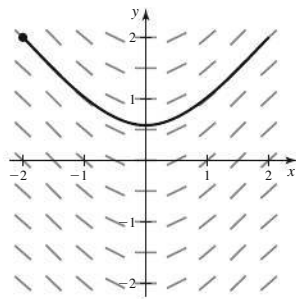
11. An initial condition of $y(0) = 1$ leads to a constant solution. Initial conditions $y(0) = A$ lead to solutions that are increasing over time if $A > 1$.



13.

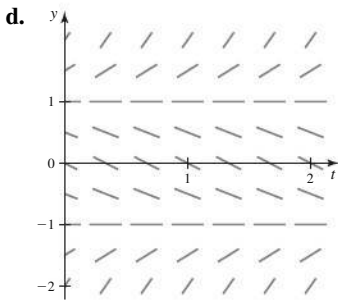


15.



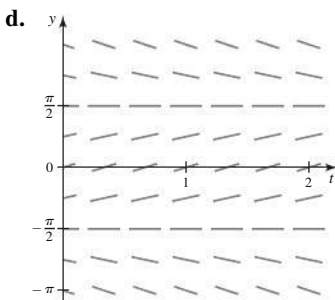
17. a. $y = 1, y = -1$

b. Solutions are increasing for $|y| > 1$ and decreasing for $|y| < 1$. c. Initial conditions $y(0) = A$ lead to increasing solutions if $|A| > 1$ and decreasing solutions if $|A| < 1$.

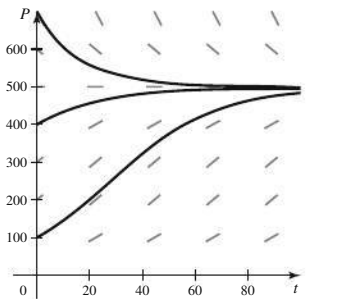


19. a. $y = \pi/2, y = -\pi/2$

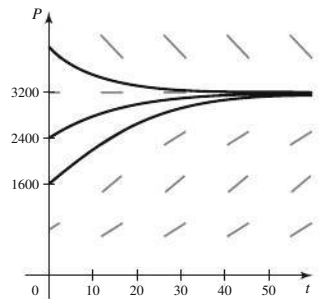
b. Solutions are increasing for $|y| < \pi/2$ and decreasing for $|y| > \pi/2$. c. Initial conditions $y(0) = A$ lead to increasing solutions if $|A| < \pi/2$ and decreasing solutions if $\pi/2 < |A| < \pi$.



21. The equilibrium solutions are $P = 0$ and $P = 500$.



23. The equilibrium solutions are $P = 0$ and $P = 3200$.



25. $y(0.5) \approx u_1 = 4; y(1) \approx u_2 = 8$

27. $y(0.1) \approx u_1 = 1.1; y(0.2) \approx u_2 = 1.19$

29. a.

Δt	approximation to $y(0.2)$	approximation to $y(0.4)$
0.20000	0.80000	0.64000
0.10000	0.81000	0.65610
0.05000	0.81451	0.66342
0.02500	0.81665	0.66692

b.

Δt	errors for $y(0.2)$	errors for $y(0.4)$
0.20000	0.01873	0.03032
0.10000	0.00873	0.01422
0.05000	0.00422	0.00690
0.02500	0.00208	0.00340

c. Time step $\Delta t = 0.025$; smaller time steps generally produce more accurate results. d. Halving the time steps results in approximately halving the error.

31. a.

Δt	approximation to $y(0.2)$	approximation to $y(0.4)$
0.20000	3.20000	3.36000
0.10000	3.19000	3.34390
0.05000	3.18549	3.33658
0.02500	3.18335	3.33308

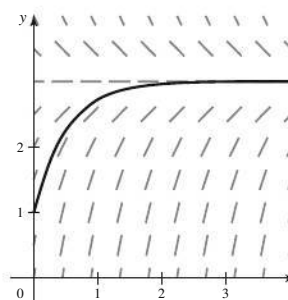
b.

Δt	errors for $y(0.2)$	errors for $y(0.4)$
0.20000	0.01873	0.03032
0.10000	0.00873	0.01422
0.05000	0.00422	0.00690
0.02500	0.00208	0.00340

c. Time step $\Delta t = 0.025$; smaller time steps generally produce more accurate results. d. Halving the time steps results in approximately halving the error.

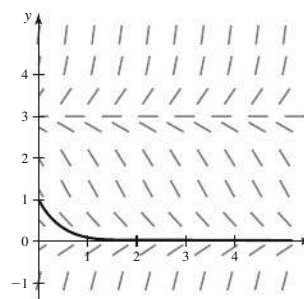
33. a. $y(2) \approx 0.00604662$ b. 0.012269
c. $y(2) \approx 0.0115292$ d. Error in part (c) is approximately half of the error in part (b).
35. a. $y(4) \approx 3.05765$ b. 0.0339321
c. $y(4) \approx 3.0739$ d. Error in part (c) is approximately half of the error in part (b).
37. a. True b. False

39. a. $y = 3$ b, c.



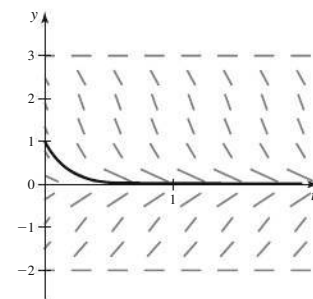
41. a. $y = 0$ and $y = 3$

b, c.



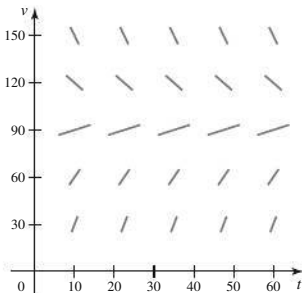
43. a. $y = -2, y = 0$, and $y = 3$

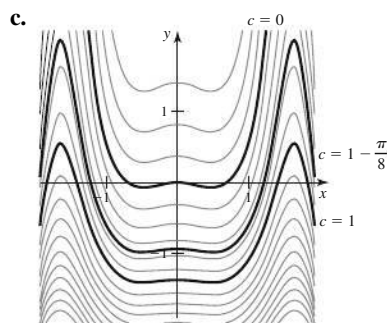
b, c.



45. a. $\Delta t = \frac{b-a}{N}$ b. $u_1 = A + f(a, A) \frac{b-a}{N}$

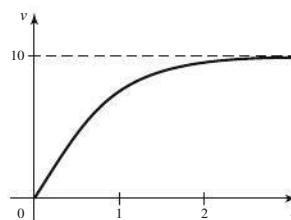
c. $u_{k+1} = u_k + f(t_k, u_k) \frac{b-a}{N}$, where $u_0 = A$ and $t_k = a + k(b-a)/N$, for $k = 0, 1, 2, \dots, N-1$.

47. a.  b. Increasing for $A < 98$ and decreasing for $A > 98$
c. $v(t) = 98$



45. $y = kx$ 47. b. $\sqrt{gm/k}$

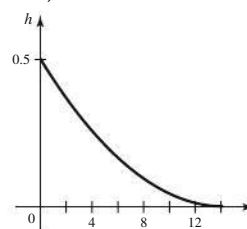
c. $v = \sqrt{\frac{g}{a} \frac{Ce^{2\sqrt{ag}t} - 1}{Ce^{2\sqrt{ag}t} + 1}}$, $t \geq 0$, where $a = \frac{k}{m}$



49. a. $h = \left(\sqrt{H} - \frac{kt}{2}\right)^2$, $0 \leq t \leq \frac{2\sqrt{H}}{k}$

b. $h = (\sqrt{0.5} - 0.05t)^2$, $0 \leq t \leq 14.1$

c. Approx. 14.1 s



Section 9.3 Exercises, pp. 618–620

1. A first-order separable differential equation has the form $g(y)y'(t) = h(t)$, where the factor $g(y)$ is a function of y and

$h(t)$ is a function of t . 3. No 5. $y = \frac{t^4}{4} + C$

7. $y = \pm \sqrt{2t^3 + C}$ 9. $y = -2 \ln\left(\frac{1}{2} \cos t + C\right)$

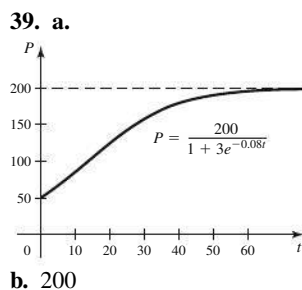
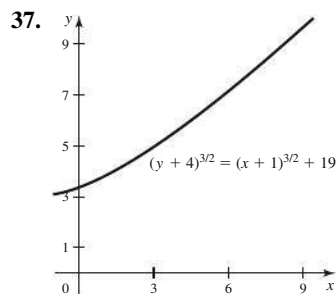
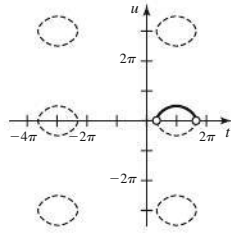
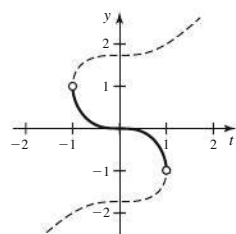
11. $y = \frac{x}{1 + Cx}$ 13. $y = \pm \frac{1}{\sqrt{C - \cos t}}$ 15. $u = \ln\left(\frac{e^{2x}}{2} + C\right)$

17. $y = \sqrt{t^3 + 81}$ 19. Not separable 21. $y(t) = -e^{e^t - 1}$

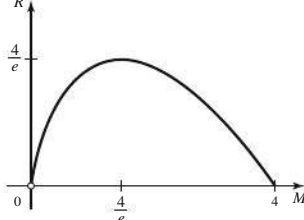
23. $y = \ln(e^x + 2)$ 25. $y = \ln\left(\frac{\ln^4 t}{4} + 1\right)$

27. $y = \sqrt{\tan t}$, $0 < t < \pi/2$ 29. $y = \sqrt{t^2 + 3}$ 31. $y = \ln t + 2$

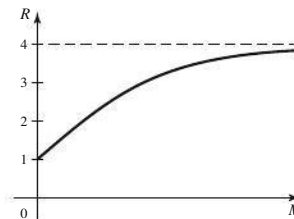
33. $y^3 - 3y = 2t^3$, $-1 < t < 1$ 35. $\cos u = 2 - 2 \sin \frac{x}{2}$, $\frac{\pi}{3} < x < \frac{5\pi}{3}$



b. 200

51. a.  R is positive if $0 < M < 4$; R has a maximum value when $M = \frac{4}{e}$; $\lim_{M \rightarrow 0} R(M) = 0$.

b. $M(t) = 4^{1-e^{-t}}$, $t \geq 0$; the tumor grows quickly at first and then the rate of growth slows down; the limiting size of the tumor is 4.



53. a. $y = \frac{1}{1-t}$, $t < 1$ b. $y = \frac{1}{\sqrt{2}\sqrt{1-t}}$, $t < 1$

c. $y = \frac{1}{(n(1-t))^{1/n}}$, $t < 1$; as $t \rightarrow 1^-$, $y \rightarrow \infty$

41. a. True b. False c. True

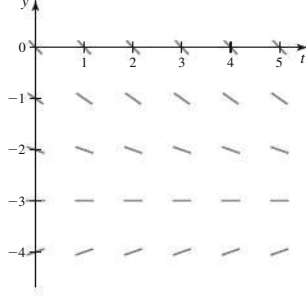
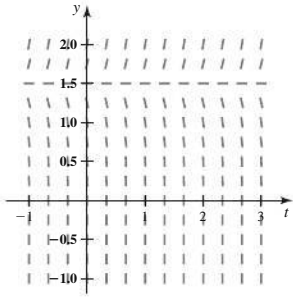
43. a. $y = -2 \ln\left(\frac{x^2}{4} + \cos x^2 + C\right)$ b. $C = 0, 1, 1 - \frac{\pi}{8}$

Section 9.4 Exercises, pp. 625–627

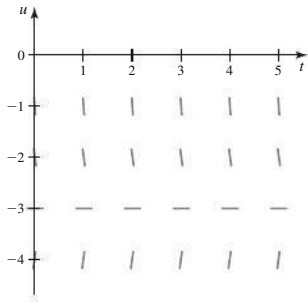
1. $y = 17e^{-10t} - 13$ 3. $y = Ce^{-4t} + \frac{3}{2}$ 5. $y = Ce^{3t} + \frac{4}{3}$
 7. $y = Ce^{-2x} - 2$ 9. $u = Ce^{-12t} + \frac{5}{4}$ 11. $y = 7e^{3t} + 2$
 13. $y = 4(e^{2t} - 1)$ 15. $y = 4(2e^{3t-3} - 1)$

17. $y = \frac{3}{2}$; unstable

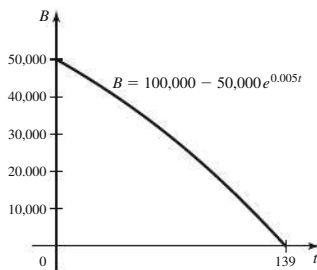
19. $y = -3$; stable



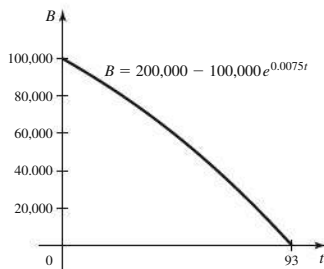
21. $u = -3$; stable



23. $B = 100,000 - 50,000e^{0.005t}$; reaches a balance of zero after approximately 139 months

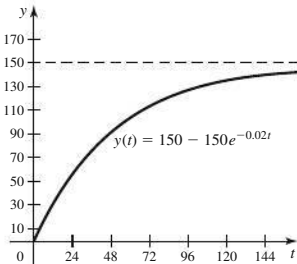


25. $B = 200,000 - 100,000e^{0.0075t}$; reaches a balance of zero after approximately 93 months



27. Approx. 32 min 29. Approx. 14 min

31. a. $y(t) = 150 - 150e^{-0.02t}$ b. 150 c. Approx. 115.1 hr



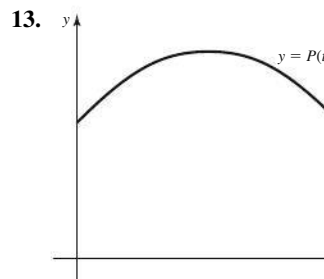
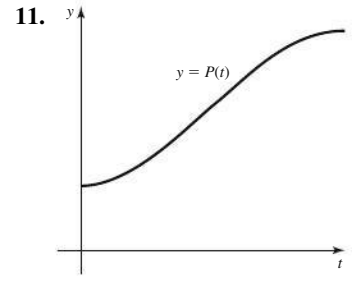
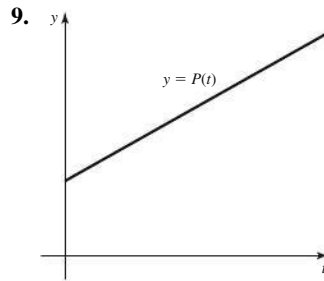
33. a. $h = 16 \text{ yr}^{-1}$ b. 25,000 35. a. False b. True c. False
 d. False 37. a. $B = 20,000 + 20,000e^{0.03t}$, the unpaid balance is growing because the monthly payment of \$600 is less than the interest on the unpaid balance. b. \$20,000 c. $\frac{m}{r}$

39. $y = 1 + \frac{t}{2} + \frac{5}{2t}, t > 0$ 41. $y = \frac{1}{2}e^{3t} + \frac{7}{2}e^t$

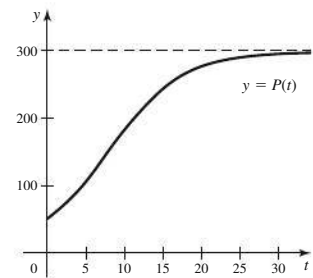
45. $y(t) = \frac{6}{t}, t > 0$ 47. $y = \frac{9t^5 + 20t^3 + 15t + 76}{15(t^2 + 1)}$

Section 9.5 Exercises, pp. 634–636

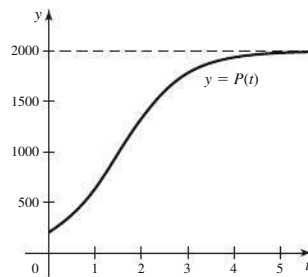
1. The growth rate function specifies the rate of growth of the population. The population is increasing when the growth rate function is positive, and the population is decreasing when the growth rate function is negative. 3. If the growth rate function is positive (it does not matter whether it is increasing or decreasing), then the population is increasing. 5. It is a linear, first-order differential equation. 7. The solution curves in the FH -plane are closed curves that circulate around the equilibrium point.



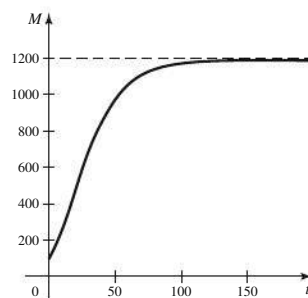
15. $P' = 0.2P\left(1 - \frac{P}{300}\right)$;
 $P = \frac{300}{5e^{-0.2t} + 1}, t \geq 0$



17. $P = \frac{2000}{9e^{-\ln(27/7)t} + 1}, t \geq 0$ 19. $M = K\left(\frac{M_0}{K}\right)e^{-rt}, t \geq 0$



21. $M = 1200 \cdot 0.075^{\exp(-0.05t)}, t \geq 0$



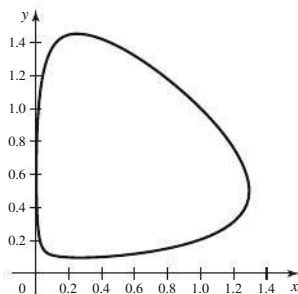
23. a. $m'(t) = -0.008t + 80, m(0) = 0$

b. $m = 10,000 - 10,000e^{-0.008t}, t \geq 0$

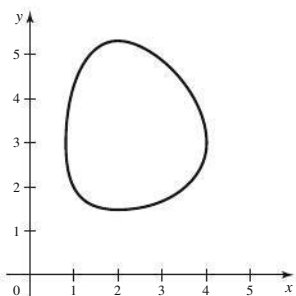
25. a. $m'(t) = -0.005t + 100, m(0) = 80,000$

b. $m = 60,000e^{-0.005t} + 20,000, t \geq 0$

27. **a.** x is the predator population; y is the prey population.
b. $x' = 0$ on the lines $x = 0$ and $y = \frac{1}{2}$; $y' = 0$ on the lines $y = 0$ and $x = \frac{1}{4}$. **c.** $(0, 0)$, $(\frac{1}{4}, \frac{1}{2})$
d. $x' > 0$ and $y' > 0$ for $0 < x < \frac{1}{4}$, $y > \frac{1}{2}$
 $x' > 0$ and $y' < 0$ for $x > \frac{1}{4}$, $y > \frac{1}{2}$
 $x' < 0$ and $y' < 0$ for $x > \frac{1}{4}$, $0 < y < \frac{1}{2}$
 $x' < 0$ and $y' > 0$ for $0 < x < \frac{1}{4}$, $0 < y < \frac{1}{2}$
e. The solution evolves in the clockwise direction.



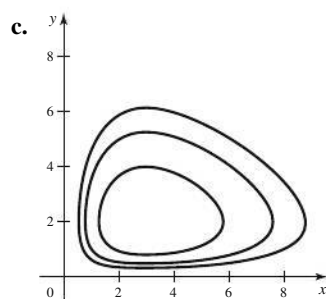
29. **a.** x is the predator population; y is the prey population.
b. $x' = 0$ on the lines $x = 0$ and $y = 3$; $y' = 0$ on the lines $y = 0$ and $x = 2$. **c.** $(0, 0)$, $(2, 3)$
d. $x' > 0$ and $y' > 0$ for $0 < x < 2$, $y > 3$
 $x' > 0$ and $y' < 0$ for $x > 2$, $y > 3$
 $x' < 0$ and $y' < 0$ for $x > 2$, $0 < y < 3$
 $x' < 0$ and $y' > 0$ for $0 < x < 2$, $0 < y < 3$
e. The solution evolves in the clockwise direction.



31. **a.** True **b.** True **c.** True **35. c.** $\lim_{t \rightarrow \infty} m(t) = C_i V$, which is the amount of substance in the tank when the tank is filled with the inflow solution. **d.** Increasing R increases the rate at which the solution in the tank reaches the steady-state concentration.

37. **a.** $I = \frac{V}{R} e^{-t/(RC)}$ **b.** $Q = VC(1 - e^{-t/(RC)})$

39. **a.** $y'(x) = \frac{y(c - dx)}{x(-a + by)}$ **c.**

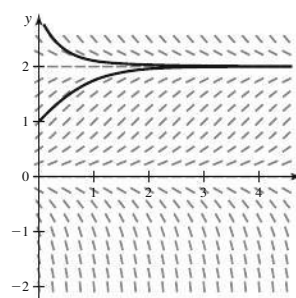


Chapter 9 Review Exercises, pp. 636–638

1. **a.** False **b.** False **c.** True **d.** True **e.** False
 3. $y = Ce^{-2t} + 3$ **5.** $y = Ce^{t^2}$ **7.** $y = Ce^{\tan^{-1}t}$
 9. $y = \tan(t^2 + t + C)$ **11.** $y = \sin t + t^2 + 1$
 13. $Q = 8(1 - e^{t-1})$ **15.** $u = (3 + t^{2/3})^{3/2}$, $t > 0$

17. $s = \frac{t\sqrt{2}}{\sqrt{t^2 + 1}}$

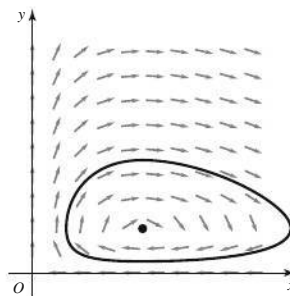
19. **a, b.**



- c.** $0 < A < 2$
d. $A > 2$ or $A < 0$
e. $y = 0$ and $y = 2$

21. **a.** 1.05, 1.09762 **b.** 1.04939, 1.09651 **c.** 0.00217, 0.00106; the error in part (b) is smaller. **23.** $y = -3$ (unstable), $y = 0$ (stable), $y = 5$ (unstable) **25.** $y = -1$ (unstable), $y = 0$ (stable), $y = 2$ (unstable) **27. a.** 0.0713 **b.** $P = \frac{1600}{79e^{-0.0713t} + 1}$, $t \geq 0$

- c.** Approx. 61 hours **29. a.** $m = 2000(1 - e^{-0.005t})$
b. 2000 g **c.** Approx. 599 minutes **31. a.** x represents the predator. **b.** $x'(t) = 0$ when $x = 0$ and $y = 2$. $y'(t) = 0$ when $y = 0$ and $x = 5$. **c.** $(0, 0)$ and $(5, 2)$ **d.** $x' > 0$, $y' > 0$ when $0 < x < 5$ and $y > 2$; $x' > 0$, $y' < 0$ when $x > 5$ and $y > 2$; $x' < 0$, $y' < 0$ when $x > 5$ and $0 < y < 2$; $x' < 0$, $y' > 0$ when $0 < x < 5$ and $0 < y < 2$
e. Clockwise direction



33. **a.** $p_1 = 3$, $p_2 = -4$ **b.** $y(t) = t^3 - t^{-4}$, $t > 0$

CHAPTER 10

Section 10.1 Exercises, pp. 647–649

1. A sequence is an ordered list of numbers. Example: $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$
 3. 1, 1, 2, 6, 24 **5.** $a_n = (-1)^{n+1}n$, for $n = 1, 2, 3, \dots$;
 $a_n = (-1)^n(n + 1)$, for $n = 0, 1, 2, \dots$ (Answers may vary.)
 7. e **9.** 1, 5, 14, 30 **11.** $\sum_{k=1}^{\infty} 10$ (Answer is not unique.)
 13. $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10,000}$ **15.** $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}$ **17.** $\frac{4}{3}, \frac{8}{5}, \frac{16}{9}, \frac{32}{17}$
 19. 2, 1, 0, 1 **21.** 2, 4, 8, 16 **23.** 10, 18, 42, 114 **25.** $1, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}$
 27. **a.** $\frac{1}{32}, \frac{1}{64}$ **b.** $a_1 = 1$, $a_{n+1} = \frac{1}{2}a_n$, for $n \geq 1$ **c.** $a_n = \frac{1}{2^{n-1}}$, for $n \geq 1$ **29. a.** 32, 64 **b.** $a_1 = 1$, $a_{n+1} = 2a_n$, for $n \geq 1$
c. $a_n = 2^{n-1}$, for $n \geq 1$ **31. a.** 243, 729 **b.** $a_1 = 1$, $a_{n+1} = 3a_n$, for $n \geq 1$ **c.** $a_n = 3^{n-1}$, for $n \geq 1$ **33. a.** -5, 5 **b.** $a_1 = -5$, $a_{n+1} = -a_n$, for $n \geq 1$ **c.** $a_n = (-1)^n \cdot 5$, for $n \geq 1$
 35. 9, 99, 999, 9999; diverges **37.** $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10,000}$; converges to 0 **39.** 2, 4, 2, 4; diverges **41.** 2, 2, 2, 2; converges to 2
 43. 54.545, 54.959, 54.996, 55.000; converges to 55

45.

n	a_n
1	0.83333333
2	0.96153846
3	0.99206349
4	0.99840256
5	0.99968010
6	0.99993600
7	0.99998720
8	0.99999744
9	0.99999949
10	0.99999990

The limit appears to be 1.

49. a. $\frac{5}{2}, \frac{9}{4}, \frac{17}{8}, \frac{33}{16}$ b. 2

51.

n	a_n
1	3.00000000
2	3.50000000
3	3.75000000
4	3.87500000
5	3.93750000
6	3.96875000
7	3.98437500
8	3.99218750
9	3.99609375
10	3.99804688

The limit appears to be 4.

55.

n	a_n
1	8.00000000
2	4.41421356
3	4.05050150
4	4.00629289
5	4.00078630
6	4.00009828
7	4.00001229
8	4.00000154
9	4.00000019
10	4.00000002

The limit appears to be 4.

69. a. 9, 9.9, 9.99, 9.999 b. $S_n = 10 - (0.1)^{n-1}$; 9.9999, 9.99999, 9.999999, 9.9999999 c. 10 71. a. True b. False c. True
 73. a. 20, 10, 5, $\frac{5}{2}, \frac{5}{4}$ b. $M_n = 20(\frac{1}{2})^n$, for $n \geq 0$ c. $M_0 = 20$, $M_{n+1} = \frac{1}{2}M_n$, for $n \geq 0$ d. $\lim_{n \rightarrow \infty} M_n = 0$ 75. a. 200, 190, 180.5, 171.475, 162.90125 b. $d_n = 200(0.95)^n$, for $n \geq 0$
 c. $d_0 = 200$, $d_{n+1} = (0.95)d_n$, for $n \geq 0$ d. $\lim_{n \rightarrow \infty} d_n = 0$
 77. a. 40, 70, 92.5, 109.375 b. 160 79. 0.739

Section 10.2 Exercises, pp. 659–662

1. $a_n = \frac{1}{n}$, $n \geq 1$ 3. $a_n = \frac{n}{n+1}$, $n \geq 1$ 5. Converges for $-1 < r \leq 1$, diverges otherwise 7. Diverges monotonically
 9. Converges, oscillates; 0 11. $\{e^{n/100}\}$ grows faster than $\{n^{100}\}$.

47.

n	a_n
1	2
2	6
3	12
4	20
5	30
6	42
7	56
8	72
9	90
10	110

The sequence appears to diverge.

53.

n	a_n
0	1
1	5
2	21
3	85
4	341
5	1365
6	5461
7	21,845
8	87,381
9	349,525

The sequence appears to diverge.

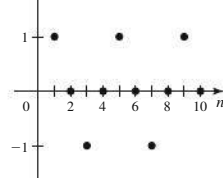
57. a. 20, 10, 5, $\frac{5}{2}$
 b. $h_n = 20(\frac{1}{2})^n$, for $n \geq 0$
 59. a. 30, $\frac{15}{2}, \frac{15}{8}, \frac{15}{32}$
 b. $h_n = 30(\frac{1}{4})^n$, for $n \geq 0$
 61. $S_1 = 0.3$, $S_2 = 0.33$, $S_3 = 0.333$, $S_4 = 0.3333$; $\frac{1}{3}$
 63. $S_1 = 4$, $S_2 = 4.9$, $S_3 = 4.99$, $S_4 = 4.999$; 5
 65. $S_1 = 0.6$, $S_2 = 0.66$, $S_3 = 0.666$, $S_4 = 0.6666$; $\frac{2}{3}$ 67. a. $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}$
 b. $S_n = \frac{2n}{2n+1}$; $\frac{10}{11}, \frac{12}{13}, \frac{14}{15}, \frac{16}{17}$ c. 1

13. 0 15. $\frac{3}{2}$ 17. $\frac{\pi}{4}$ 19. 2 21. 0 23. $\frac{1}{4}$ 25. 2 27. 0

29. 0 31. 3 33. Diverges 35. $\frac{\pi}{2}$ 37. 0 39. e^2 41. e^3

43. $e^{1/4}$ 45. 0 47. 1 49. 0 51. 6

53. a_n 55. 0 57. Diverges



Diverges

59. Diverges 61. 0 63. 0

65. 0 67. 0 69. 0

71. a. $d_{n+1} = \frac{1}{2}d_n + 80$, for $n \geq 1$

b. 160 mg

73. a. \$0, \$100, \$200.75, \$302.26, \$404.53

b. $B_{n+1} = 1.0075B_n + 100$, for $n \geq 0$ c. Approx. 43 months

75. 0 77. Diverges 79. 0 81. 1 83. a. True b. False

c. True d. True e. False f. True 85. a. Nondecreasing

b. $\frac{1}{2}$ 87. a. Nonincreasing b. 2 89. a. $d_{n+1} = 0.4d_n + 75$;

$d_1 = 75$ c. 125; in the long run there will be approximately 125 mg of medication in the blood. 91. 0.607 93. b. 9

95. a. $\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}$,

$\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}$, or 1.41421, 1.84776, 1.96157, 1.99037

c. 2 97. a. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 b. No 99. b. 1, 2,

1.5, 1.6667, 1.6 c. Approx. 1.618 e. $\frac{a + \sqrt{a^2 + 4b}}{2}$

101. Given a tolerance $\varepsilon > 0$, look beyond a_N , where $N > 1/\varepsilon$.

103. Given a tolerance $\varepsilon > 0$, look beyond a_N , where $N > \frac{1}{4}\sqrt{3/\varepsilon}$,

provided $\varepsilon < \frac{3}{4}$. 105. Given a tolerance $\varepsilon > 0$, look beyond a_N ,

where $N > c/(\varepsilon b^2)$. 107. $a < 1$ 109. $\{n^2 + 2n - 17\}_{n=3}^\infty$

111. $n = 4, n = 6, n = 25$

Section 10.3 Exercises, pp. 668–671

1. The constant r in the series $\sum_{k=0}^\infty ar^k$ 3. No 5. a. $a = \frac{2}{3}$; $r = \frac{1}{5}$

b. $a = \frac{1}{27}$; $r = -\frac{1}{3}$ 7. $S_n = \frac{1}{4} - \frac{1}{n+4}$; $S_{36} = \frac{9}{40}$ 9. 9841

11. Approx. 1.1905 13. Approx. 0.5392 15. $\frac{1093}{2916}$

17. \$15,920.22 19. a. $\frac{7}{9}$ 21. $\frac{4}{3}$ 23. $\frac{10}{19}$ 25. 10 27. Diverges

29. $\frac{1}{e^2 - 1}$ 31. $\frac{1}{7}$ 33. $\frac{1}{500}$ 35. $\frac{3\pi}{\pi + 1}$ 37. $\frac{\pi}{\pi - e}$ 39. $\frac{9}{460}$

41. $\frac{4}{11}$ 43. $A_5 = 266.406$; $A_{10} = 266.666$; $A_{30} = 266.667$;

$\lim_{n \rightarrow \infty} A_n = 266\frac{2}{3}$ mg, which is the steady-state level. 45. 400 mg

47. $0.\overline{3} = \sum_{k=1}^\infty 3(0.1)^k = \frac{1}{3}$ 49. $0.\overline{037} = \sum_{k=1}^\infty 37(0.001)^k = \frac{1}{27}$

51. $0.\overline{456} = \sum_{k=0}^\infty 0.456(0.001)^k = \frac{152}{333}$

53. $0.00\overline{952} = \sum_{k=0}^\infty 0.00952(0.001)^k = \frac{238}{24,975}$

55. $S_n = \frac{n}{2n+4}$; $\frac{1}{2}$ 57. $S_n = \frac{1}{7} - \frac{1}{n+7}$; $\frac{1}{7}$

59. $S_n = \frac{1}{9} - \frac{1}{4n+9}$; $\frac{1}{9}$ 61. $S_n = \ln(n+1)$; diverges

63. $S_n = \frac{1}{p+1} - \frac{1}{n+p+1}; \frac{1}{p+1}$
 65. $S_n = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \right) - \left(\frac{1}{\sqrt{n+2}} + \frac{1}{\sqrt{n+3}} \right); \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}$
 67. $S_n = \frac{13}{12} - \frac{1}{n+2} - \frac{3}{n+3} - \frac{1}{n+4}; \frac{13}{12}$
 69. $S_n = \tan^{-1}(n+1) - \tan^{-1}1; \frac{\pi}{4}$ 71. a, b. $\frac{4}{3}$ 73. $-\frac{1}{4}$
 75. $\frac{2500}{19}$ 77. $-\frac{2}{15}$ 79. $\frac{1}{\ln 2}$ 81. -2 83. $\frac{113}{30}$ 85. $\frac{17}{10}$
 87. a. True b. True c. False d. False e. True f. False
 g. True 89. a. $\frac{1}{5}$ b. Approx. 0.19999695 91. Approx. 0.96
 95. 462 months 99. $\sum_{k=0}^{\infty} \left(\frac{1}{4} \right)^k A_1 = \frac{A_1}{1 - 1/4} = \frac{4}{3} A_1$
 101. a. $L_n = 3 \left(\frac{4}{3} \right)^n$, so $\lim_{n \rightarrow \infty} L_n = \infty$ b. $\lim_{n \rightarrow \infty} A_n = \frac{2\sqrt{3}}{5}$
 103. $R_n = S - S_n = \frac{1}{1-r} - \left(\frac{1-r^n}{1-r} \right) = \frac{r^n}{1-r}$ 105. a. 60
 b. 9 107. a. 13 b. 15 109. a. $1, \frac{5}{6}, \frac{2}{3}$, undefined, undefined
 b. $(-1, 1)$ 111. Converges for x in $(-\infty, -2)$ or $(0, \infty)$; $x = \frac{1}{2}$

Section 10.4 Exercises, pp. 680–683

1. The series diverges. 3. $\lim_{k \rightarrow \infty} a_k = 0$ 5. Converges for $p > 1$ and diverges for $p \leq 1$ 7. $R_n = S - S_n$ 9. Diverges
 11. Inconclusive 13. Diverges 15. Diverges 17. Diverges
 19. Diverges 21. Converges 23. Diverges 25. Converges
 27. Diverges 29. Converges 31. Converges 33. Converges
 35. Diverges 37. Diverges 39. a. $S \approx S_2 = 1.0078125$
 b. $R_2 < 0.0026042$ c. $L_2 = 1.0080411$; $U_2 = 1.0104167$
 41. a. $\frac{1}{5n^5}$ b. 3 c. $L_n = S_n + \frac{1}{5(n+1)^5}$; $U_n = S_n + \frac{1}{5n^5}$
 d. $(1.017342754, 1.017343512)$ 43. a. $\frac{3^{-n}}{\ln 3}$ b. 7
 c. $L_n = S_n + \frac{3^{-n-1}}{\ln 3}$; $U_n = S_n + \frac{3^{-n}}{\ln 3}$
 d. $(0.499996671, 0.500006947)$ 45. 1.0083 47. a. False
 b. True c. False d. True e. False f. False 49. Converges
 51. Converges 53. Diverges 55. Diverges 57. Diverges
 59. Converges 61. Converges 63. Converges 65. a. $p > 1$
 b. $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ converges faster. 67. $\zeta(3) \approx 1.202$, $\zeta(5) \approx 1.037$
 69. $\frac{\pi^2}{8}$ 73. a. $\frac{1}{2}, \frac{7}{12}, \frac{37}{60}$

Section 10.5 Exercises, pp. 687–688

1. Find an appropriate comparison series. Then take the limit of the ratio of the terms of the given series and the comparison series as $n \rightarrow \infty$. The value of the limit determines whether the series converges.
 3. $\sum_{k=1}^{\infty} \frac{1}{k^2}$ 5. $\sum_{k=1}^{\infty} \left(\frac{2}{3} \right)^k$ 7. $\sum_{k=1}^{\infty} \frac{1}{k}$ 9. Converges
 11. Diverges 13. Converges 15. Converges 17. Converges
 19. Diverges 21. Diverges 23. Converges 25. Diverges
 27. Converges 29. Diverges 31. Diverges 33. Diverges
 35. Converges 37. a. False b. True c. True d. True

39. Converges 41. Diverges 43. Diverges 45. Diverges
 47. Converges 49. Diverges 51. Converges 53. Converges
 55. Diverges 57. Converges 59. Diverges 61. Converges

Section 10.6 Exercises, pp. 694–696

1. Because $S_{n+1} - S_n = (-1)^n a_{n+1}$ alternates sign
 3. Because the remainder $R_n = S - S_n$ alternates sign
 5. $|R_n| = |S - S_n| \leq |S_{n+1} - S_n| = a_{n+1}$ 7. No; if a series of positive terms converges, it does so absolutely and not conditionally.
 9. Yes, $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$ has this property. 11. Converges 13. Diverges
 15. Converges 17. Converges 19. Diverges 21. Diverges
 23. Diverges 25. Diverges 27. Converges 29. $S_4 = -0.945939$;
 $|S - S_4| \leq 0.0016$ 31. $S_5 = 0.70696$; $|S - S_5| \leq 0.001536$
 33. 10,000 35. 5000 37. 10 39. -0.973 41. -0.269
 43. -0.783 45. Converges conditionally 47. Converges absolutely
 49. Converges absolutely 51. Converges absolutely 53. Diverges
 55. Converges conditionally 57. Diverges 59. Converges absolutely
 61. Converges conditionally 63. Converges absolutely
 65. a. False b. True c. True d. True e. False f. True
 g. True 69. x and y are divergent series.
 71. b. $S_{2n} = \sum_{k=1}^n \left(\frac{1}{k^2} - \frac{1}{k} \right)$

Section 10.7 Exercises, pp. 699–700

1. Take the limit of the magnitude of the ratio of consecutive terms of the series as $k \rightarrow \infty$. The value of the limit determines whether the series converges absolutely or diverges. 3. 999,000
 5. $\frac{1}{(k+2)(k+1)}$ 7. Ratio Test 9. Converges absolutely
 11. Diverges 13. Converges absolutely 15. Converges absolutely
 17. Diverges 19. Diverges 21. Converges absolutely
 23. Converges absolutely 25. Diverges 27. Converges absolutely
 29. Diverges 31. a. False b. True c. True d. True
 33. Converges absolutely 35. Diverges 37. Converges absolutely
 39. Converges conditionally 41. Converges absolutely
 43. Converges absolutely 45. Converges conditionally
 47. Converges absolutely 49. Converges conditionally
 51. $p > 1$ 53. $p > 1$ 55. $p < 1$ 57. Diverges for all p
 59. $-1 < x < 1$ 61. $-1 \leq x \leq 1$ 63. $-2 < x < 2$

Section 10.8 Exercises, pp. 703–704

1. Root Test 3. Divergence Test 5. p -series Test or Limit Comparison Test 7. Comparison Test or Limit Comparison Test
 9. Alternating Series Test 11. Diverges 13. Diverges
 15. Converges 17. Diverges 19. Converges 21. Converges
 23. Converges 25. Converges 27. Converges 29. Diverges
 31. Converges 33. Diverges 35. Converges 37. Diverges
 39. Diverges 41. Converges 43. Diverges 45. Converges
 47. Diverges 49. Converges 51. Diverges 53. Converges
 55. Diverges 57. Converges 59. Converges 61. Diverges
 63. Diverges 65. Converges 67. Converges 69. Diverges
 71. Converges 73. Converges 75. Converges 77. Diverges
 79. Diverges 81. Converges 83. Converges 85. Converges
 87. a. False b. True c. True d. False 89. Diverges
 91. Diverges 93. Diverges

Chapter 10 Review Exercises, pp. 704–707

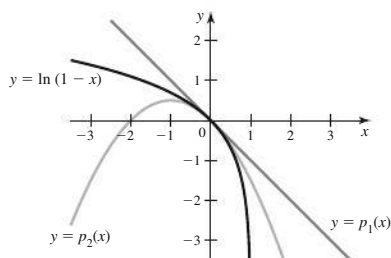
1. a. False b. False c. True d. False e. True f. False
 g. False h. True 3. Approx. 1.25; approx. 0.05 5. $\lim_{k \rightarrow \infty} a_k = 0$,
 $\lim_{n \rightarrow \infty} S_n = 8$ 7. $a_k = \frac{1}{k}$ 9. a. 0 b. $\frac{5}{9}$ 11. a. Yes; $\lim_{k \rightarrow \infty} a_k = 1$
 b. No; $\lim_{k \rightarrow \infty} a_k \neq 0$ 13. Diverges 15. 5 17. 0 19. 0 21. $1/e$
 23. Diverges 25. a. 80, 48, 32, 24, 20 b. 16 27. Diverges
 29. Diverges 31. Diverges 33. $\frac{3\pi}{4}$ 35. 3 37. $2/9$
 39. $\frac{311}{990}$ 41. 200 mg 43. Diverges 45. Diverges 47. Converges
 49. Converges 51. Converges 53. Converges 55. Converges
 57. Diverges 59. Converges 61. Converges 63. Converges
 65. Converges 67. Converges 69. Converges 71. Converges
 73. Diverges 75. Diverges 77. Converges conditionally
 79. Converges absolutely 81. Diverges 83. Converges absolutely
 85. Converges absolutely 87. Diverges 89. a. Approx. 1.03666
 b. 0.0004 c. $L_5 = 1.03685$; $U_5 = 1.03706$ 91. 0.0067
 93. 100 95. a. 803 m, 1283 m, 2000(1 - 0.95^N) m b. 2000 m
 97. a. $\frac{\pi}{2^{n-1}}$ b. 2π 99. a. $T_1 = \frac{\sqrt{3}}{16}$, $T_2 = \frac{7\sqrt{3}}{64}$
 b. $T_n = \frac{\sqrt{3}}{4} \left(1 - \left(\frac{3}{4} \right)^n \right)$ c. $\lim_{n \rightarrow \infty} T_n = \frac{\sqrt{3}}{4}$ d. 0
 101. $\sqrt{\frac{20}{g}} \left(\frac{1 + \sqrt{p}}{1 - \sqrt{p}} \right) s$

CHAPTER 11

Section 11.1 Exercises, pp. 718–721

1. $f(0) = p_2(0)$, $f'(0) = p_2'(0)$, and $f''(0) = p_2''(0)$
 3. 1, 1.05, 1.04875 5. $p_3(x) = 1 + x^2 + x^3$; 1.048
 7. $p_3(x) = 1 + (x - 2) + 2(x - 2)^2$; 0.898
 9. a. $p_1(x) = 8 + 12(x - 1)$
 b. $p_2(x) = 8 + 12(x - 1) + 3(x - 1)^2$ c. 9.2; 9.23
 11. a. $p_1(x) = 1 - 2x$ b. $p_2(x) = 1 - 2x + 2x^2$ c. 0.8, 0.82
 13. a. $p_1(x) = 1 - x$ b. $p_2(x) = 1 - x + x^2$ c. 0.95, 0.9525
 15. a. $p_1(x) = 2 + \frac{1}{12}(x - 8)$
 b. $p_2(x) = 2 + \frac{1}{12}(x - 8) - \frac{1}{288}(x - 8)^2$ c. 1.9583, 1.95747
 17. $p_1(x) = 1$, $p_2(x) = p_3(x) = 1 - 18x^2$, $p_4(x) = 1 - 18x^2 + 54x^4$
 19. $p_3(x) = 1 - 3x + 6x^2 - 10x^3$,
 $p_4(x) = 1 - 3x + 6x^2 - 10x^3 + 15x^4$
 21. $p_1(x) = 1 + 3(x - 1)$, $p_2(x) = 1 + 3(x - 1) + 3(x - 1)^2$,
 $p_3(x) = 1 + 3(x - 1) + 3(x - 1)^2 + (x - 1)^3$
 23. $p_3(x) = 1 + \frac{1}{e}(x - e) - \frac{1}{2e^2}(x - e)^2 + \frac{1}{3e^3}(x - e)^3$
 25. a. $p_1(x) = -x$, $p_2(x) = -x - \frac{x^2}{2}$

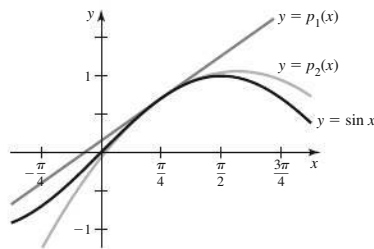
b.



$$27. \text{ a. } p_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \left(x - \frac{\pi}{4} \right),$$

$$p_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \left(x - \frac{\pi}{4} \right) - \frac{\sqrt{2}}{4} \left(x - \frac{\pi}{4} \right)^2$$

b.



29. a. 1.0247 b. 7.6×10^{-6} 31. a. 0.8613 b. 5.4×10^{-4}
 33. a. 1.1274988 b. Approx. 8.85×10^{-6} (Answers may vary if intermediate calculations are rounded.) 35. a. Approx. -0.10033333
 b. Approx. 1.34×10^{-6} (Answers may vary if intermediate calculations are rounded.) 37. a. 1.0295635 b. Approx. 4.86×10^{-7}
 (Answers may vary if intermediate calculations are rounded.) 39. a. Approx. 0.52083333 b. Approx. 2.62×10^{-4} (Answers may vary if intermediate calculations are rounded.)
 41. $R_n(x) = \frac{\sin^{(n+1)}(c)}{(n+1)!} x^{n+1}$, for c between x and 0
 43. $R_n(x) = \frac{(-1)^{n+1} e^{-c}}{(n+1)!} x^{n+1}$, for c between x and 0
 45. $R_n(x) = \frac{\sin^{(n+1)}(c)}{(n+1)!} \left(x - \frac{\pi}{2} \right)^{n+1}$, for c between x and $\frac{\pi}{2}$
 47. 2.0×10^{-5} 49. 1.6×10^{-5} ($e^{0.25} < 2$) 51. 2.6×10^{-4}
 53. With $n = 4$, $|\text{error}| \leq 2.5 \times 10^{-3}$
 55. With $n = 2$, $|\text{error}| \leq 4.2 \times 10^{-2}$ ($e^{0.5} < 2$)
 57. With $n = 2$, $|\text{error}| \leq 5.4 \times 10^{-3}$ 59. 4 61. 3 63. 1
 65. a. False b. True c. True d. True 67. a. C b. E
 c. A d. D e. B f. F 69. a. 0.1; 1.7×10^{-4} b. 0.2;
 1.3×10^{-3} 71. a. 0.995; 4.2×10^{-6} b. 0.98; 6.7×10^{-5}
 73. a. 1.05; 1.3×10^{-3} b. 1.1; 5×10^{-3} 75. a. 1.1; 10^{-2}
 b. 1.2; 4×10^{-2}

77. a.

x	$ \sec x - p_2(x) $	$ \sec x - p_4(x) $
-0.2	3.39×10^{-4}	5.51×10^{-6}
-0.1	2.09×10^{-5}	8.51×10^{-8}
0.0	0	0
0.1	2.09×10^{-5}	8.51×10^{-8}
0.2	3.39×10^{-4}	5.51×10^{-6}

b. The errors decrease as $|x|$ decreases.

79. a.

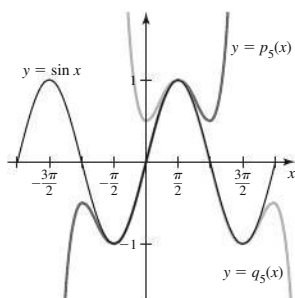
x	$ e^{-x} - p_1(x) $	$ e^{-x} - p_2(x) $
-0.2	2.14×10^{-2}	1.40×10^{-3}
-0.1	5.17×10^{-3}	1.71×10^{-4}
0.0	0	0
0.1	4.84×10^{-3}	1.63×10^{-4}
0.2	1.87×10^{-2}	1.27×10^{-3}

b. The errors decrease as $|x|$ decreases.81. Centered at $x = 0$, for all n

83. a. $y = f(a) + f'(a)(x - a)$ 85. a. $p_5(x) = x - \frac{x^3}{6} + \frac{x^5}{120}$;

$q_5(x) = -(x - \pi) + \frac{1}{6}(x - \pi)^3 - \frac{1}{120}(x - \pi)^5$

b.



p_5 is a better approximation on $[-\pi, \pi/2]$; q_5 is a better approximation on $(\pi/2, 2\pi]$.

c.

x	$ \sin x - p_5(x) $	$ \sin x - q_5(x) $
$\pi/4$	3.6×10^{-5}	7.4×10^{-2}
$\pi/2$	4.5×10^{-3}	4.5×10^{-3}
$3\pi/4$	7.4×10^{-2}	3.6×10^{-5}
$5\pi/4$	2.3	3.6×10^{-5}
$7\pi/4$	20	7.4×10^{-2}

d. p_5 is a better approximation at $x = \pi/4$; at $x = \pi/2$ the errors are equal.

87. a. $p_1(x) = 6 + \frac{1}{12}(x - 36)$; $q_1(x) = 7 + \frac{1}{14}(x - 49)$

b.

x	$ \sqrt{x} - p_1(x) $	$ \sqrt{x} - q_1(x) $
37	5.7×10^{-4}	6.0×10^{-2}
39	5.0×10^{-3}	4.1×10^{-2}
41	1.4×10^{-2}	2.5×10^{-2}
43	2.6×10^{-2}	1.4×10^{-2}
45	4.2×10^{-2}	6.1×10^{-3}
47	6.1×10^{-2}	1.5×10^{-3}

c. p_1 is a better approximation at $x = 37, 39$, and 41 .

Section 11.2 Exercises, pp. 729–730

1. $c_0 + c_1x + c_2x^2 + c_3x^3$ 3. Ratio and Root Tests 5. The radius of convergence does not change. The interval of convergence may change. 7. $R = 10$; $[-8, 12]$ 9. $R = \frac{1}{2}$; $(-\frac{1}{2}, \frac{1}{2})$

11. $R = 0$; $\{x: x = 0\}$ 13. $R = \infty$; $(-\infty, \infty)$ 15. $R = 3$; $(-3, 3)$

17. $R = \infty$; $(-\infty, \infty)$ 19. $R = 2$; $(-2, 2)$ 21. $R = \infty$; $(-\infty, \infty)$

23. $R = 1$; $(0, 2]$ 25. $R = \frac{1}{4}$; $[0, \frac{1}{2}]$ 27. $R = 5$; $(-3, 7)$

29. $R = \infty$; $(-\infty, \infty)$ 31. $R = \sqrt{3}$; $(-\sqrt{3}, \sqrt{3})$ 33. $R = 1$; $(0, 2)$

35. $R = \infty$; $(-\infty, \infty)$ 37. $R = e$ 39. $R = e^4$

41. $\sum_{k=0}^{\infty} (3x)^k$; $(-\frac{1}{3}, \frac{1}{3})$ 43. $2 \sum_{k=0}^{\infty} x^{k+3}$; $(-1, 1)$

45. $4 \sum_{k=0}^{\infty} x^{k+12}$; $(-1, 1)$ 47. $-\sum_{k=1}^{\infty} \frac{(3x)^k}{k}$; $[-\frac{1}{3}, \frac{1}{3})$

49. $-2 \sum_{k=1}^{\infty} \frac{x^{k+6}}{k}$; $[-1, 1)$ 51. $g(x) = 2 \sum_{k=1}^{\infty} k(2x)^{k-1}$; $(-\frac{1}{2}, \frac{1}{2})$

53. $g(x) = \sum_{k=1}^{\infty} (-1)^k kx^{k-1}$; $(-1, 1)$

55. $g(x) = -\sum_{k=1}^{\infty} \frac{3^k x^k}{k}$; $[-\frac{1}{3}, \frac{1}{3})$

57. $\sum_{k=1}^{\infty} (-1)^{k+1} 2kx^{2k-1}$; $(-1, 1)$ 59. $\sum_{k=0}^{\infty} \left(-\frac{x}{3}\right)^k$; $(-3, 3)$

61. $\ln 2 - \frac{1}{2} \sum_{k=1}^{\infty} \frac{x^{2k}}{k 4^k}$; $(-2, 2)$ 63. a. True b. True c. True

d. True 65. $|x - a| < R$ 67. $f(x) = \frac{1}{3 - \sqrt{x}}$; $1 < x < 9$

69. $f(x) = \frac{e^x}{e^x - 1}$; $0 < x < \infty$ 71. $f(x) = \frac{3}{4 - x^2}$; $-2 < x < 2$

73. $\sum_{k=0}^{\infty} \frac{(-3x)^k}{k!}$; $-\infty < x < \infty$

75. $\lim_{k \rightarrow \infty} \left| \frac{c_{k+1} x^{k+1}}{c_k x^k} \right| = \lim_{k \rightarrow \infty} \left| \frac{c_{k+1} x^{k+m+1}}{c_k x^{k+m}} \right|$, so by the Ratio Test,

the two series have the same radius of convergence.

77. a. $f(x)g(x) = c_0d_0 + (c_0d_1 + c_1d_0)x +$

$(c_0d_2 + c_1d_1 + c_2d_0)x^2$ b. $\sum_{k=0}^n c_k d_{n-k}$

Section 11.3 Exercises, pp. 740–742

1. The n th Taylor polynomial is the n th partial sum of the corresponding Taylor series. 3. $\sum_{k=0}^{\infty} \frac{(x-2)^k}{k!}$ 5. Replace x with x^2 in the

Taylor series for $f(x)$; $|x| < 1$. 7. The Taylor series for a function f converges to f on an interval if, for all x in the interval, $\lim_{n \rightarrow \infty} R_n(x) = 0$, where $R_n(x)$ is the remainder at x .

9. a. $1 - 2(x-1) + 3(x-1)^2 - 4(x-1)^2$

b. $\sum_{k=0}^{\infty} (-1)^k (k+1)(x-1)^k$ c. $(0, 2)$ 11. a. $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$

b. $\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k!}$ c. $(-\infty, \infty)$ 13. a. $2 + 6x + 12x^2 + 20x^3$

b. $\sum_{k=0}^{\infty} (k+1)(k+2)x^k$ c. $(-1, 1)$ 15. a. $1 - x^2 + x^4 - x^6$

b. $\sum_{k=0}^n (-1)^k x^{2k}$ c. $(-1, 1)$ 17. a. $1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!}$

b. $\sum_{k=0}^{\infty} \frac{(2x)^k}{k!}$ c. $(-\infty, \infty)$ 19. a. $\frac{x}{2} - \frac{x^3}{3 \cdot 2^3} + \frac{x^5}{5 \cdot 2^5} - \frac{x^7}{7 \cdot 2^7}$

b. $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)2^{2k+1}}$ c. $[-2, 2]$

21. a. $1 + (\ln 3)x + \frac{\ln^2 3}{2}x^2 + \frac{\ln^3 3}{6}x^3$ b. $\sum_{k=0}^{\infty} \frac{\ln^k 3}{k!}x^k$ c. $(-\infty, \infty)$

23. a. $1 + \frac{(3x)^2}{2} + \frac{(3x)^4}{24} + \frac{(3x)^6}{720}$ b. $\sum_{k=0}^{\infty} \frac{(3x)^{2k}}{(2k)!}$ c. $(-\infty, \infty)$

25. a. $(x-3) - \frac{1}{2}(x-3)^2 + \frac{1}{3}(x-3)^3 - \frac{1}{4}(x-3)^4$

b. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-3)^k}{k}$ c. $(2, 4]$

27. a. $1 - \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!} - \frac{(x-\pi/2)^6}{6!}$

b. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} (x-\pi/2)^{2k}$

29. a. $1 - (x-1) + (x-1)^2 - (x-1)^3$ b. $\sum_{k=0}^{\infty} (-1)^k (x-1)^k$

31. a. $\ln 3 + \frac{(x-3)}{3} - \frac{(x-3)^2}{3^2 \cdot 2} + \frac{(x-3)^3}{3^3 \cdot 3}$

b. $\ln 3 + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-3)^k}{k3^k}$

$$33. \mathbf{a.} \ 2 + 2(\ln 2)(x-1) + (\ln^2 2)(x-1)^2 + \frac{\ln^3 2}{3}(x-1)^3$$

$$\mathbf{b.} \ \sum_{k=0}^{\infty} \frac{2(x-1)^k \ln^k 2}{k!} \quad 35. \ x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4}$$

$$37. \ 1 + 2x + 4x^2 + 8x^3 \quad 39. \ 1 + \frac{x}{2} + \frac{x^2}{6} + \frac{x^3}{24}$$

$$41. \ 1 - x^4 + x^8 - x^{12} \quad 43. \ x^2 + \frac{x^6}{6} + \frac{x^{10}}{120} + \frac{x^{14}}{5040}$$

$$45. \mathbf{a.} \ 1 - 2x + 3x^2 - 4x^3 \quad \mathbf{b.} \ 0.826$$

$$47. \mathbf{a.} \ 1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3 \quad \mathbf{b.} \ 1.029$$

$$49. \mathbf{a.} \ 1 - \frac{2}{3}x + \frac{5}{9}x^2 - \frac{40}{81}x^3 \quad \mathbf{b.} \ 0.895 \quad 51. \ 1 + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{16};$$

$$[-1, 1] \quad 53. \ 3 - \frac{3x}{2} - \frac{3x^2}{8} - \frac{3x^3}{16}; [-1, 1]$$

$$55. \ a + \frac{x^2}{2a} - \frac{x^4}{8a^3} + \frac{x^6}{16a^5}; |x| \leq a$$

$$57. \ 1 - 8x + 48x^2 - 256x^3 \quad 59. \ \frac{1}{16} - \frac{x^2}{32} + \frac{3x^4}{256} - \frac{x^6}{256}$$

$$61. \ \frac{1}{9} - \frac{2}{9}\left(\frac{4x}{3}\right) + \frac{3}{9}\left(\frac{4x}{3}\right)^2 - \frac{4}{9}\left(\frac{4x}{3}\right)^3$$

$$63. \ R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}, \text{ where } c \text{ is between } 0 \text{ and } x \text{ and}$$

$$f^{(n+1)}(c) = \pm \sin c \text{ or } \pm \cos c. \text{ Therefore, } |R_n(x)| \leq \frac{|x|^{n+1}}{(n+1)!} \rightarrow 0$$

$$\text{as } n \rightarrow \infty, \text{ for } -\infty < x < \infty. \quad 65. \ R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1},$$

$$\text{where } c \text{ is between } 0 \text{ and } x \text{ and } f^{(n+1)}(c) = (-1)^n e^{-c}. \text{ Therefore,}$$

$$|R_n(x)| \leq \frac{|x|^{n+1}}{e^c(n+1)!} \rightarrow 0 \text{ as } n \rightarrow \infty, \text{ for } -\infty < x < \infty.$$

$$67. \mathbf{a.} \ \text{False} \quad \mathbf{b.} \ \text{True} \quad \mathbf{c.} \ \text{False} \quad \mathbf{d.} \ \text{False} \quad \mathbf{e.} \ \text{True}$$

$$69. \mathbf{a.} \ 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} \quad \mathbf{b.} \ R = \infty$$

$$71. \mathbf{a.} \ 1 - \frac{2}{3}x^2 + \frac{5}{9}x^4 - \frac{40}{81}x^6 \quad \mathbf{b.} \ R = 1$$

$$73. \mathbf{a.} \ 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 \quad \mathbf{b.} \ R = 1$$

$$75. \mathbf{a.} \ 1 - 2x^2 + 3x^4 - 4x^6 \quad \mathbf{b.} \ R = 1 \quad 77. \ \text{Approx. } 3.9149$$

$$79. \ \text{Approx. } 1.8989 \quad 85. \ \sum_{k=0}^{\infty} \left(\frac{x-4}{2}\right)^k \quad 87. \ \text{Use three terms of the}$$

Taylor series for $\cos x$ centered at $a = \pi/4$; 0.766 **89. a.** Use three terms of the Taylor series for $\sqrt[3]{125+x}$ centered at $a = 0$; 5.03968 **b.** Use three terms of the Taylor series for $\sqrt[3]{x}$ centered at $a = 125$; 5.03968 **c.** Yes

Section 11.4 Exercises, pp. 748–750

1. Replace f and g with their Taylor series centered at a and evaluate the limit. **3.** Substitute $x = -0.6$ into the Taylor series for e^x centered at 0. Because the resulting series is an alternating series, the error can be estimated easily. **7.** 1 **9.** $\frac{1}{2}$ **11.** 2 **13.** $\frac{2}{3}$ **15.** $\frac{1}{3}$

$$17. \ \frac{3}{5} \quad 19. \ -\frac{8}{5} \quad 21. \ 1 \quad 23. \ \frac{3}{4} \quad 25. \mathbf{a.} \ 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\mathbf{b.} \ e^x \quad \mathbf{c.} \ -\infty < x < \infty$$

$$27. \mathbf{a.} \ 1 - x + x^2 - \cdots (-1)^{n-1}x^{n-1} + \cdots \quad \mathbf{b.} \ \frac{1}{1+x} \quad \mathbf{c.} \ |x| < 1$$

$$29. \mathbf{a.} \ -2 + 4x - 8 \cdot \frac{x^2}{2!} + \cdots + (-2)^n \frac{x^{n-1}}{(n-1)!} + \cdots$$

$$\mathbf{b.} \ -2e^{-2x} \quad \mathbf{c.} \ -\infty < x < \infty \quad 31. \mathbf{a.} \ 1 - x^2 + x^4 - \cdots$$

$$\mathbf{b.} \ \frac{1}{1+x^2} \quad \mathbf{c.} \ -1 < x < 1$$

$$33. \mathbf{a.} \ 2 + 2t + \frac{2t^2}{2!} + \cdots + \frac{2t^n}{n!} + \cdots \quad \mathbf{b.} \ y(t) = 2e^t$$

$$35. \mathbf{a.} \ 2 + 16t + 24t^2 + 24t^3 + \cdots + \frac{3^{n-1} \cdot 16}{n!}t^n + \cdots$$

$$\mathbf{b.} \ y(t) = \frac{16}{3}e^{3t} - \frac{10}{3} \quad 37. \ 0.2448 \quad 39. \ 0.6958$$

$$41. \ 0.0600 \quad 43. \ 0.4994 \quad 45. \ 1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!}$$

$$47. \ 1 - 2 + \frac{2}{3} - \frac{4}{45} \quad 49. \ \frac{1}{2} - \frac{1}{8} + \frac{1}{24} - \frac{1}{64} \quad 51. \ e - 1$$

$$53. \ \sum_{k=1}^{\infty} \frac{(-1)^{k+1}x^k}{k}, \text{ for } -1 < x \leq 1; \ln 2 \quad 55. \ \frac{2}{2-x} \quad 57. \ \frac{4}{4+x^2}$$

$$59. \ -\ln(1-x) \quad 61. \ -\frac{3x^2}{(3+x)^2} \quad 63. \ \frac{6x^2}{(3-x)^3}$$

$$65. \mathbf{a.} \ \text{False} \quad \mathbf{b.} \ \text{False} \quad \mathbf{c.} \ \text{True} \quad 67. \ \frac{a}{b} \quad 69. \ e^{-1/6}$$

$$71. \ f^{(3)}(0) = 0; f^{(4)}(0) = 4e \quad 73. \ f^{(3)}(0) = 2; f^{(4)}(0) = 0$$

$$75. \ 2 \quad 77. \ 1.575 \text{ using four terms} \quad 79. \mathbf{a.} \ S'(x) = \sin x^2;$$

$$C'(x) = \cos x^2 \quad \mathbf{b.} \ \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} - \frac{x^{15}}{15 \cdot 7!};$$

$$x - \frac{x^5}{5 \cdot 2!} + \frac{x^9}{9 \cdot 4!} - \frac{x^{13}}{13 \cdot 6!} \quad \mathbf{c.} \ S(0.05) \approx 0.00004166664807;$$

$$C(-0.25) \approx -0.2499023614 \quad \mathbf{d.} \ 1 \quad \mathbf{e.} \ 2$$

$$81. \mathbf{a.} \ 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} \quad \mathbf{b.} \ R = \infty, -\infty < x < \infty$$

Chapter 11 Review Exercises, pp. 750–752

$$1. \mathbf{a.} \ \text{True} \quad \mathbf{b.} \ \text{False} \quad \mathbf{c.} \ \text{True} \quad \mathbf{d.} \ \text{True} \quad \mathbf{e.} \ \text{True}$$

$$3. \ p_2(x) = 1 - \frac{3}{2}x^2 \quad 5. \ p_2(x) = 1 - (x-1) + \frac{3}{2}(x-1)^2$$

$$7. \ p_2(x) = 1 - \frac{1}{2}(x-1)^2$$

$$9. \ p_3(x) = \frac{5}{4} + \frac{3}{4}(x - \ln 2) + \frac{5}{8}(x - \ln 2)^2 + \frac{1}{8}(x - \ln 2)^3$$

$$11. \mathbf{a.} \ p_1(x) = 1 + x; p_2(x) = 1 + x + \frac{x^2}{2}$$

$$\mathbf{b.} \begin{array}{|c|c|c|} \hline n & p_n(x) & \text{Error} \\ \hline 1 & 0.92 & 3.1 \times 10^{-3} \\ \hline 2 & 0.9232 & 8.4 \times 10^{-5} \\ \hline \end{array}$$

$$13. \mathbf{a.} \ p_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right);$$

$$p_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$$

$$\mathbf{b.} \begin{array}{|c|c|c|} \hline n & p_n(x) & \text{Error} \\ \hline 1 & 0.5960 & 8.2 \times 10^{-3} \\ \hline 2 & 0.5873 & 4.7 \times 10^{-4} \\ \hline \end{array}$$

$$15. \ |R_3| < \frac{\pi^4}{4!} \quad 17. \ R = \infty, (-\infty, \infty) \quad 19. \ R = \infty, (-\infty, \infty)$$

$$21. \ R = 9, (-9, 9) \quad 23. \ R = 2, [-4, 0] \quad 25. \ R = \frac{3}{2}, [-2, 1]$$

$$27. \ R = \frac{1}{27} \quad 29. \ \sum_{k=0}^{\infty} x^{2k}, (-1, 1) \quad 31. \ \sum_{k=0}^{\infty} (-5x)^k; \left(-\frac{1}{5}, \frac{1}{5}\right)$$

$$33. \sum_{k=1}^{\infty} k(10x)^{k-1}; \left(-\frac{1}{10}, \frac{1}{10}\right) \quad 35. 1 + 3x + \frac{9x^2}{2!}; \sum_{k=0}^{\infty} \frac{(3x)^k}{k!}$$

$$37. -(x - \pi/2) + \frac{(x - \pi/2)^3}{3!} - \frac{(x - \pi/2)^5}{5!};$$

$$\sum_{k=0}^{\infty} (-1)^{k+1} \frac{(x - \pi/2)^{2k+1}}{(2k+1)!}$$

$$39. 4x - \frac{(4x)^3}{3} + \frac{(4x)^5}{5}; \sum_{k=0}^{\infty} \frac{(-1)^k (4x)^{2k+1}}{2k+1}$$

$$41. 1 + 2(x-1)^2 + \frac{2}{3}(x-1)^4; \sum_{k=0}^{\infty} \frac{4^k (x-1)^{2k}}{(2k)!}$$

$$43. 1 + \frac{x}{3} - \frac{x^2}{9} + \cdots \quad 45. 1 - \frac{3}{2}x + \frac{3}{2}x^2 - \cdots$$

$$47. R_n(x) = \frac{(\sinh c + \cosh c) x^{n+1}}{(n+1)!}, \text{ where } c \text{ is between } 0 \text{ and } x;$$

$$\lim_{n \rightarrow \infty} |R_n(x)| = |\sinh c + \cosh c| \lim_{n \rightarrow \infty} \frac{|x|^{n+1}}{(n+1)!} = 0 \text{ because}$$

$$|x|^{n+1} \ll (n+1)! \text{ for any fixed value of } x.$$

$$49. \frac{1}{24} \quad 51. \frac{1}{8} \quad 53. \frac{1}{6} \quad 55. \text{Approx. } 0.4615 \quad 57. \text{Approx. } 0.3819$$

$$59. 11 - \frac{1}{11} - \frac{1}{2 \cdot 11^3} - \frac{1}{2 \cdot 11^5} \quad 61. -\frac{1}{3} + \frac{1}{3 \cdot 3^3} - \frac{1}{5 \cdot 3^5} + \frac{1}{7 \cdot 3^7}$$

$$63. y = 4 + 4x + \frac{4^2}{2!}x^2 + \frac{4^3}{3!}x^3 + \cdots + \frac{4^n}{n!}x^n + \cdots = 3 + e^{4x}$$

$$65. \text{a. } \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \quad \text{b. } \sum_{k=1}^{\infty} \frac{1}{k2^k} \quad \text{c. } 2 \sum_{k=0}^{\infty} \frac{x^{2k+1}}{2k+1}$$

$$\text{d. } x = \frac{1}{3}; 2 \sum_{k=0}^{\infty} \frac{1}{3^{2k+1}(2k+1)} \quad \text{e. Series in part (d)}$$

CHAPTER 12

Section 12.1 Exercises, pp. 763–767

1. Plotting $\{(f(t), g(t)): a \leq t \leq b\}$ generates a curve in the xy -plane.

$$3. x = R \cos(\pi t/5), y = -R \sin(\pi t/5) \quad 5. x = t^2, y = t,$$

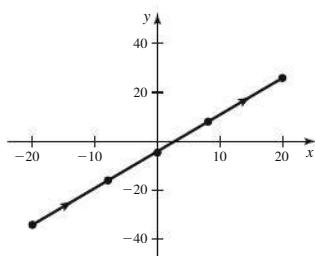
$$-\infty < t < \infty \quad 7. -\frac{1}{2} \quad 9. x = t, y = t, 0 \leq t \leq 6; x = 2t, y = 2t,$$

$$0 \leq t \leq 3; x = 3t, y = 3t, 0 \leq t \leq 2 \text{ (answers will vary)}$$

11. a.

t	-10	-4	0	4	10
x	-20	-8	0	8	20
y	-34	-16	-4	8	26

b.

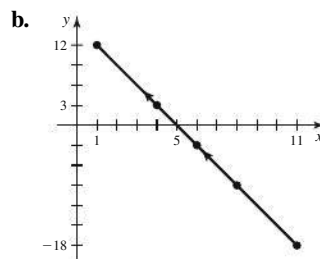


$$\text{c. } y = \frac{3}{2}x - 4$$

d. A line segment rising to the right as t increases

13. a.

t	-5	-2	0	2	5
x	11	8	6	4	1
y	-18	-9	-3	3	12



$$\text{c. } y = -3x + 15$$

d. A line segment rising to the left as t increases

$$15. \text{a. } y = -x + 4 \quad \text{b. A line segment starting at } (3, 1) \text{ and ending at } (4, 0)$$

$$17. \text{a. } y = 3x - 12 \quad \text{b. A line segment starting at } (4, 0) \text{ and ending at } (8, 12)$$

$$19. \text{a. } x^2 + y^2 = 9 \quad \text{b. Lower half of a circle of radius 3 centered at } (0, 0); \text{ starts at } (-3, 0) \text{ and ends at } (3, 0)$$

$$21. \text{a. } y = 1 - x^2, -1 \leq x \leq 1 \quad \text{b. A parabola opening downward with a vertex at } (0, 1) \text{ starting at } (1, 0) \text{ and ending at } (-1, 0)$$

$$23. \text{a. } x^2 + (y - 1)^2 = 1 \quad \text{b. A circle of radius 1 centered at } (0, 1); \text{ generated counterclockwise, starting and ending at } (1, 1)$$

$$25. \text{a. } y = (x + 1)^3 \quad \text{b. A cubic curve rising to the right as } r \text{ increases}$$

$$27. \text{a. } x^2 + y^2 = 49 \quad \text{b. A circle of radius 7 centered at } (0, 0); \text{ generated counterclockwise, starting and ending at } (-7, 0)$$

$$29. \text{a. } y = 1, -\infty < x < \infty \quad \text{b. A horizontal line with } y\text{-intercept } 1,$$

$$\text{generated from left to right} \quad 31. x^2 + y^2 = 4 \quad 33. y = \sqrt{4 - x^2}$$

$$35. y = x^2 \quad 37. x = 4 \cos t, y = 4 \sin t, 0 \leq t \leq 2\pi$$

$$39. x = \cos t + 2, y = \sin t + 3, 0 \leq t \leq 2\pi$$

$$41. x = 2t, y = 8t; 0 \leq t \leq 1$$

$$43. x = t, y = 2t^2 - 4; -1 \leq t \leq 5 \quad 45. x = 2, y = t; 3 \leq t \leq 9$$

$$47. x = 4t - 2, y = -6t + 3; 0 \leq t \leq 1 \text{ and}$$

$$x = t + 1, y = 8t - 11; 1 \leq t \leq 2$$

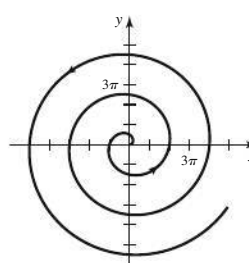
$$49. x = 1 + 2t, y = 1 + 4t; -\infty < t < \infty$$

$$51. x = t^2, y = t; t \geq 0$$

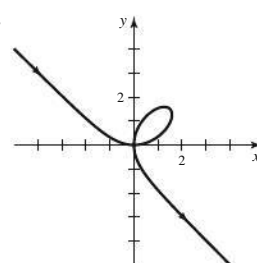
$$53. x = 400 \cos\left(\frac{4\pi t}{3}\right), y = 400 \sin\left(\frac{4\pi t}{3}\right); 0 \leq t \leq 1.5$$

$$55. x = 50 \cos\left(\frac{\pi t}{12}\right), y(t) = 50 \sin\left(\frac{\pi t}{12}\right); 0 \leq t \leq 24$$

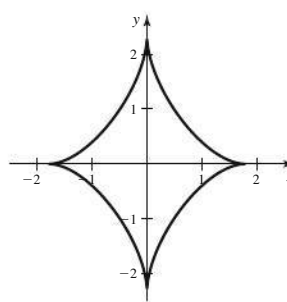
57.



59.



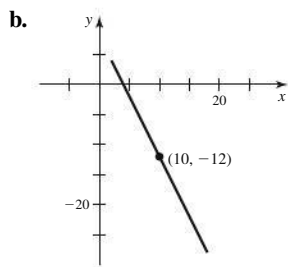
61.



$$63. \text{Plot } x = 1 + \cos^2 t - \sin^2 t, y = t.$$

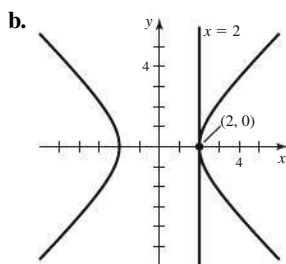
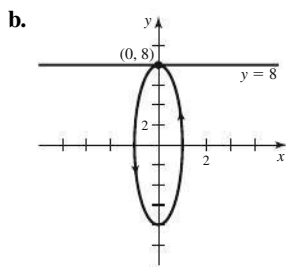
$$65. \text{Approx. } 2857 \text{ m}$$

67. a. $\frac{dy}{dx} = -2; -2$



71. a. $\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}, t \neq 0;$
undefined

69. a. $\frac{dy}{dx} = -8 \cot t; 0$

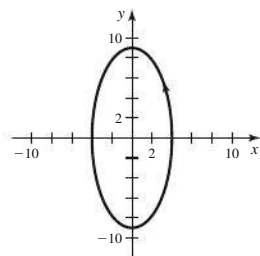


73. $y = \frac{13}{4}x + \frac{1}{4}$ 75. $y = x - \frac{\pi\sqrt{2}}{4}$ 77. $\left(-\frac{4}{\sqrt{5}}, \frac{8}{\sqrt{5}}\right)$ and $\left(\frac{4}{\sqrt{5}}, -\frac{8}{\sqrt{5}}\right)$ 79. There is no such point. 81. 10 83. $\pi\sqrt{2}$

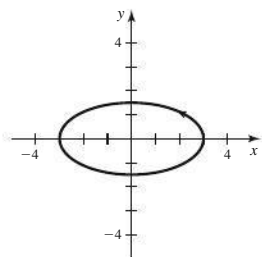
85. $\frac{1}{3}(5\sqrt{5} - 8)$ 87. $\frac{3}{2}$ 89. a. False b. True c. False

d. True e. True

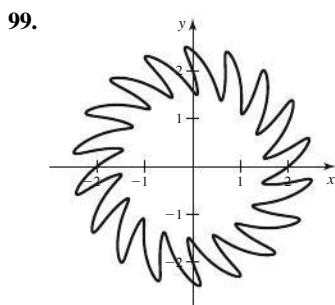
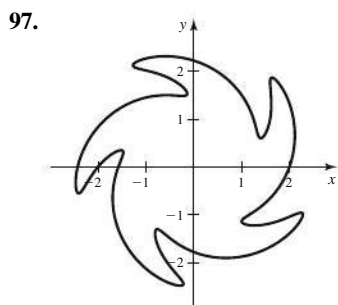
91. $0 \leq t \leq 2\pi$



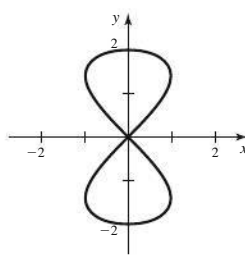
93. $x = 3 \cos t, y = \frac{3}{2} \sin t,$
 $0 \leq t \leq 2\pi; \left(\frac{x}{3}\right)^2 + \left(\frac{2y}{3}\right)^2 = 1;$
in the counterclockwise direction



95. a. Lines intersect at (1, 0). b. Lines are parallel.
c. Lines intersect at (4, 6).



101.



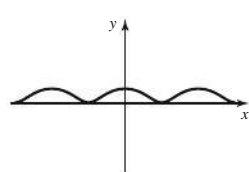
a. (0, 2) and (0, -2)
b. $(1, \sqrt{2}), (1, -\sqrt{2}),$
 $(-1, \sqrt{2}), (-1, -\sqrt{2})$

103. 27π 105. $\frac{3\pi}{8}$ 107. a. A circle of radius 3 centered at (0, 4)

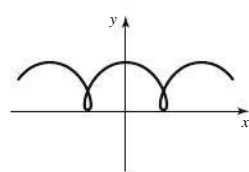
b. A torus (doughnut); $48\pi^2$ 109. $\frac{64\pi}{3}$

111. $\int_0^1 2\pi(e^{3t} + 1)\sqrt{4e^{4t} + 9e^{6t}} dt \approx 1445.9$

113. a.

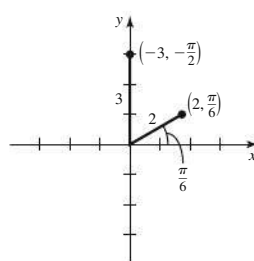


b.



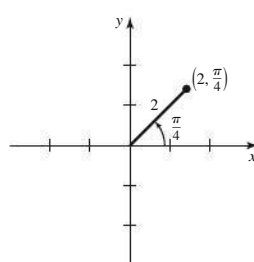
Section 12.2 Exercises, pp. 775–779

1.



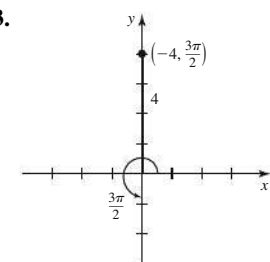
$(-2, -5\pi/6), (2, 13\pi/6);$
 $(3, \pi/2), (3, 5\pi/2)$

9.



$(-2, -3\pi/4), (2, 9\pi/4)$

13.



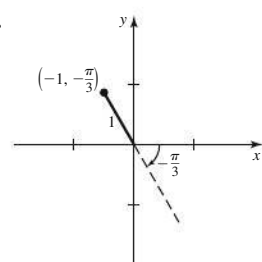
$(4, \pi/2), (4, 5\pi/2)$

3. $r^2 = x^2 + y^2, \tan \theta = \frac{y}{x}$

5. $r \cos \theta = 5$ or $r = 5 \sec \theta$

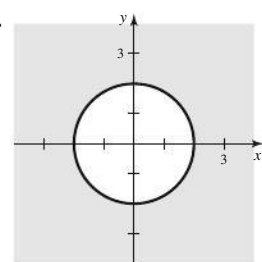
7. x -axis symmetry occurs if (r, θ) on the graph implies $(r, -\theta)$ is on the graph. y -axis symmetry occurs if (r, θ) on the graph implies $(r, \pi - \theta) = (-r, -\theta)$ is on the graph. Symmetry about the origin occurs if (r, θ) on the graph implies $(-r, \theta) = (r, \theta + \pi)$ is on the graph.

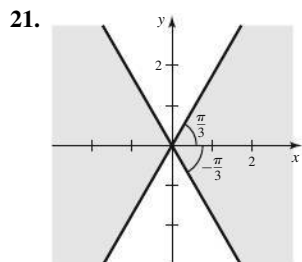
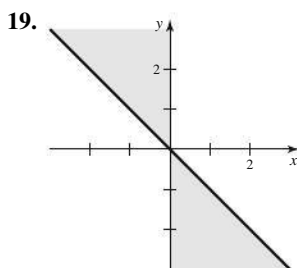
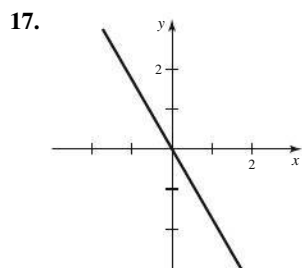
11.



$(1, 2\pi/3), (1, 8\pi/3)$

15.





23. $\left(100, -\frac{\pi}{4}\right)$

25. $(3\sqrt{2}/2, 3\sqrt{2}/2)$

27. $(1/2, -\sqrt{3}/2)$

29. $(2\sqrt{2}, -2\sqrt{2})$

31. $(2\sqrt{2}, \pi/4), (-2\sqrt{2}, 5\pi/4)$

33. $(2, \pi/3), (-2, 4\pi/3)$

35. $(8, 2\pi/3), (-8, -\pi/3)$

37. $x = -4$; vertical line passing through $(-4, 0)$

39. $x^2 + y^2 = 4$; circle of radius 2 centered at $(0, 0)$

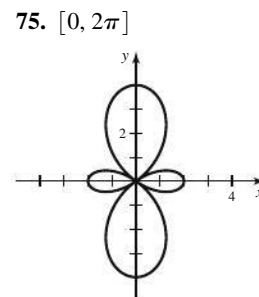
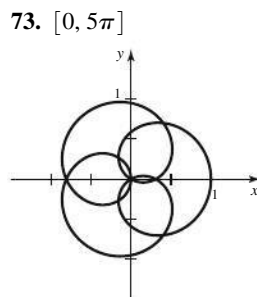
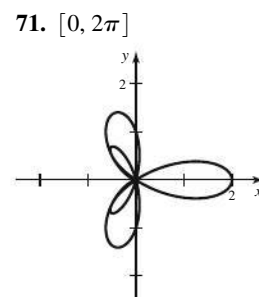
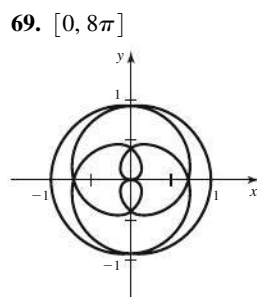
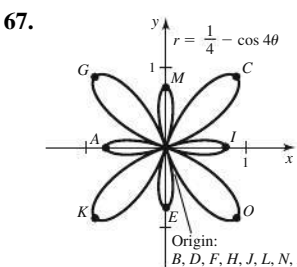
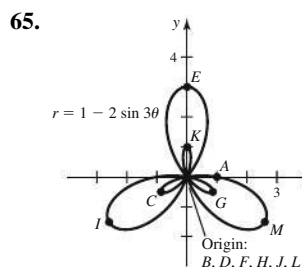
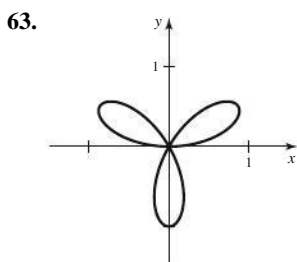
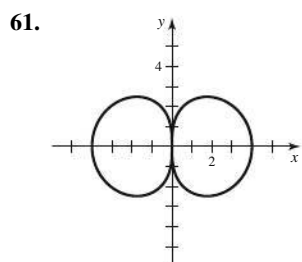
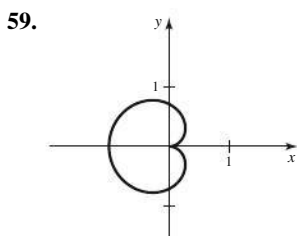
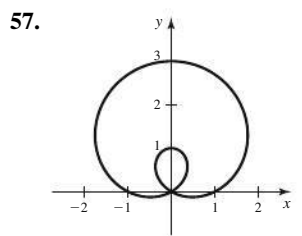
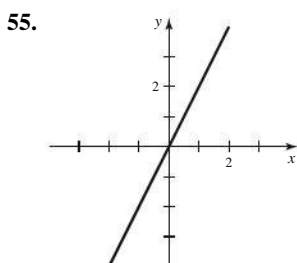
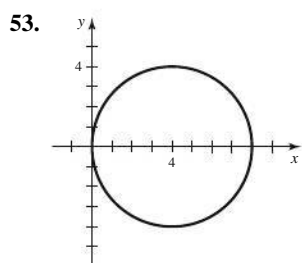
41. $(x - 1)^2 + (y - 1)^2 = 2$; circle of radius $\sqrt{2}$ centered at $(1, 1)$

43. $(x - 3)^2 + (y - 4)^2 = 25$; circle of radius 5 centered at $(3, 4)$

45. $x^2 + (y - 1)^2 = 1$; circle of radius 1 centered at $(0, 1)$ and

$x = 0$ 47. $x^2 + (y - 4)^2 = 16$; circle of radius 4 centered at $(0, 4)$

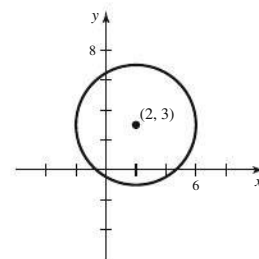
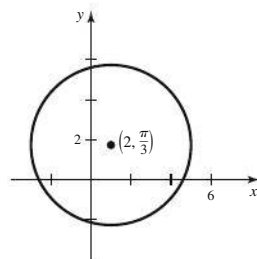
49. $r = \tan \theta \sec \theta$ 51. $r^2 = \sec \theta \csc \theta$ or $r^2 = 2 \csc 2\theta$



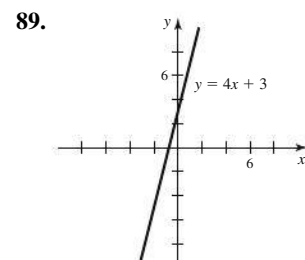
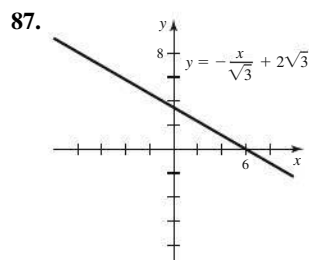
77. a. True b. True c. False d. True e. True

81. A circle of radius 4 and center $(2, \pi/3)$ (polar coordinates)

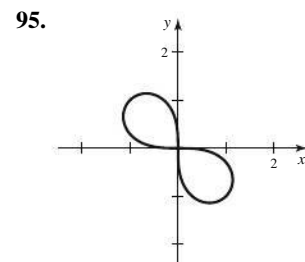
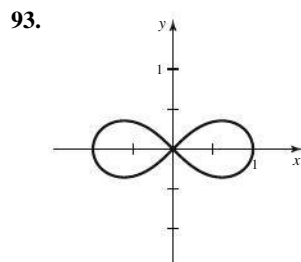
83. A circle of radius 4 centered at $(2, 3)$ (Cartesian coordinates)



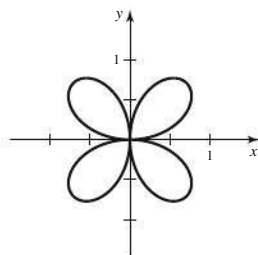
85. a. 132.3 miles b. 264.6 mi/hr



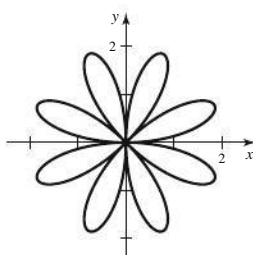
91. a. A b. C c. B d. D e. E f. F



97.

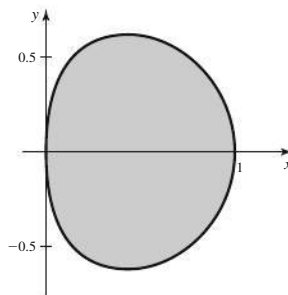
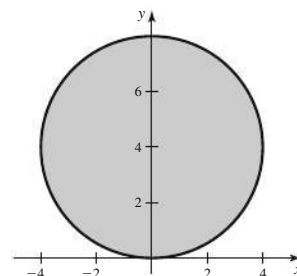


99.

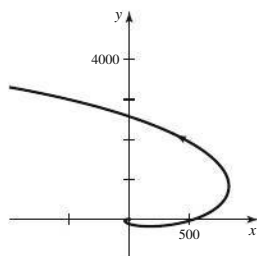
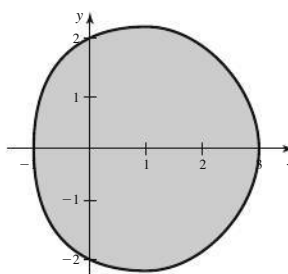
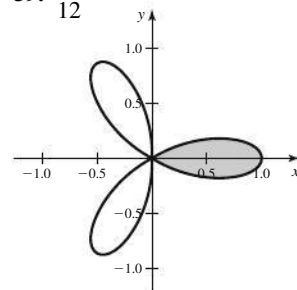


For $a = -1$, the spiral winds inward toward the origin.

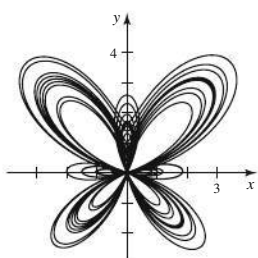
33. 1

35. 16π 

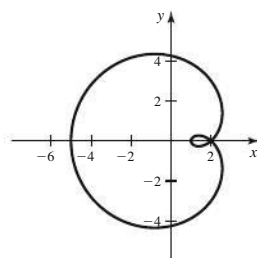
103.

37. $9\pi/2$ 39. $\frac{\pi}{12}$ 

105. a.



107. a.



109. Symmetry about the x -axis 111. $(x^2 + y^2)^2 = a^2(x^2 - y^2)$

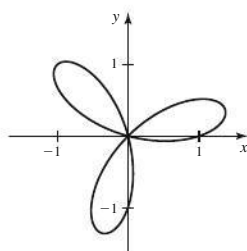
Section 12.3 Exercises, pp. 786–788

1. $x = f(\theta) \cos \theta$, $y = f(\theta) \sin \theta$ 3. The slope of the tangent line is the rate of change of the vertical coordinate with respect to the

horizontal coordinate. 5. $\sqrt{3}$ 7. $\frac{\pi^2}{4}$ 9. Both curves pass through the origin, but for different values of θ . 11. 0 13. $-\sqrt{3}$

15. Undefined, undefined 17. 0 at $(-4, \pi/2)$ and $(-4, 3\pi/2)$,

undefined at $(4, 0)$ and $(4, \pi)$ 19. ± 1 21. $\theta = \frac{3\pi}{4}$; $m = -1$

23. a. $[0, \pi]$ 

b. $\theta = \frac{\pi}{4}$, $m = 1$; $\theta = \frac{7\pi}{12}$, $m \approx -3.73$;

$\theta = \frac{11\pi}{12}$, $m \approx -0.27$

25. Horizontal: $(2\sqrt{2}, \pi/4)$, $(-2\sqrt{2}, 3\pi/4)$; vertical: $(0, \pi/2)$, $(4, 0)$

27. Horizontal: $(0, 0)$ (0.943, 0.955), $(-0.943, 2.186)$, $(0.943, 4.097)$, $(-0.943, 5.328)$; vertical: $(0, 0)$, $(0.943, 0.615)$, $(-0.943, 2.526)$, $(0.943, 3.757)$, $(-0.943, 5.668)$ 29. $(2, 0)$ and $(0, 0)$

31. $(1, \frac{\pi}{12})$, $(1, \frac{5\pi}{12})$, $(1, \frac{7\pi}{12})$, $(1, \frac{11\pi}{12})$,

$(1, \frac{13\pi}{12})$, $(1, \frac{17\pi}{12})$, $(1, \frac{19\pi}{12})$, and $(1, \frac{23\pi}{12})$

41. a. $(0, 0)$, $(\frac{3}{\sqrt{2}}, \frac{\pi}{4})$ b. $\frac{9}{8}(\pi - 2)$

43. a. $(1 + \frac{1}{\sqrt{2}}, \frac{\pi}{4})$, $(1 - \frac{1}{\sqrt{2}}, \frac{5\pi}{4})$, $(0, 0)$ b. $\frac{3\pi}{2} - 2\sqrt{2}$

45. $\frac{1}{24}(3\sqrt{3} + 2\pi)$ 47. $\frac{1}{4}(2 - \sqrt{3}) + \frac{\pi}{12}$ 49. $\pi/20$

51. $4(4\pi/3 - \sqrt{3})$ 53. $2\pi/3 - \sqrt{3}/2$ 55. $9\pi + 27\sqrt{3}$

57. 6 59. 18π 61. Intersection points: $(3, \pm \frac{\pi}{3})$; area of

region A = $6\sqrt{3} - 2\pi$; area of region B = $5\pi - 6\sqrt{3}$; area of region C = $4\pi + 6\sqrt{3}$ 63. πa 65. $\frac{8}{3}((1 + \pi^2)^{3/2} - 1)$

67. 32 69. $63\sqrt{5}$ 71. $\frac{2\pi - 3\sqrt{3}}{8}$ 73. 26.73

75. a. False b. False c. True

77. Horizontal: $(0, 0)$, $(4.05, 2.03)$, $(9.83, 4.91)$;

vertical: $(1.72, 0.86)$, $(6.85, 3.43)$, $(12.87, 6.44)$ 79. $\frac{\sqrt{1+a^2}}{a}$

81. a. $A_n = \frac{1}{4e^{(4n+2)\pi}} - \frac{1}{4e^{4n\pi}} - \frac{1}{4e^{(4n-2)\pi}} + \frac{1}{4e^{(4n-4)\pi}}$ b. 0

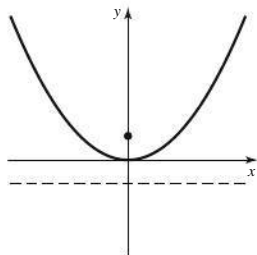
c. $e^{-4\pi}$ 85. $(a^2 - 2)\theta^* + \pi - \sin 2\theta^*$, where $\theta^* = \cos^{-1}(a/2)$

87. $a^2(\pi/2 + a/3)$

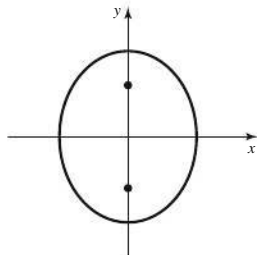
Section 12.4 Exercises, pp. 797–800

1. A parabola is the set of all points in a plane equidistant from a fixed point and a fixed line. 3. A hyperbola is the set of all points in a plane whose distances from two fixed points have a constant difference.

5. Parabola:

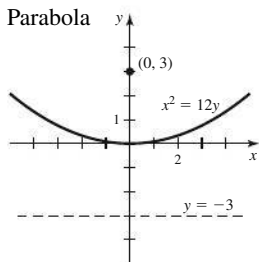


Ellipse:

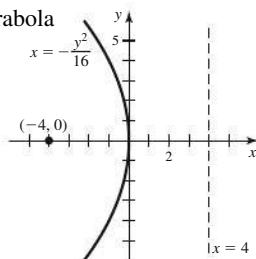


$$7. \left(\frac{x}{a}\right)^2 + \frac{y^2}{a^2 - c^2} = 1 \quad 9. (\pm ae, 0) \quad 11. y = \pm \frac{b}{a}x$$

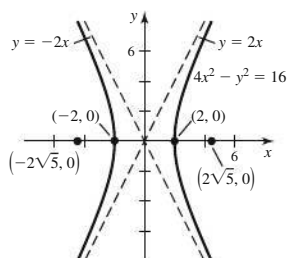
13. Parabola



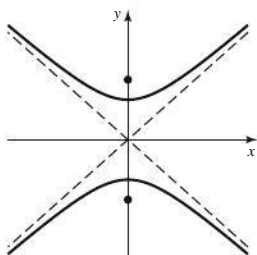
17. Parabola



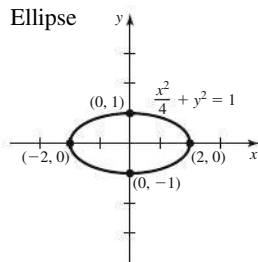
21. Hyperbola



Hyperbola:

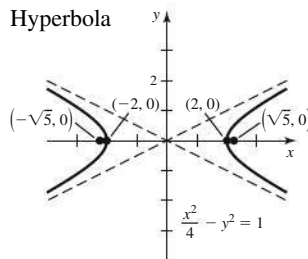


15. Ellipse



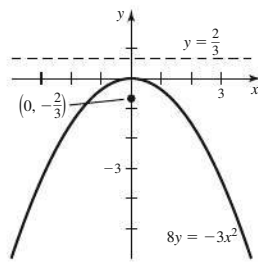
Vertices: $(\pm 2, 0)$; foci: $(\pm \sqrt{3}, 0)$;
major axis has length 4; minor axis
has length 2.

19. Hyperbola

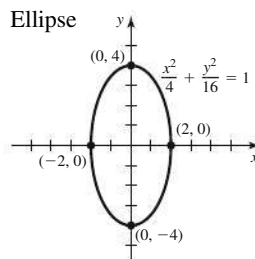


Vertices: $(\pm 2, 0)$;
foci: $(\pm \sqrt{5}, 0)$;
asymptotes: $y = \pm \frac{1}{2}x$

23. Parabola

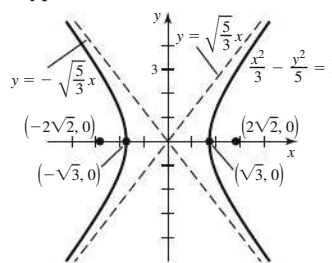


25. Ellipse



Vertices: $(0, \pm 4)$;
foci: $(0, \pm 2\sqrt{3})$; major axis has
length 8; minor axis has length 4.

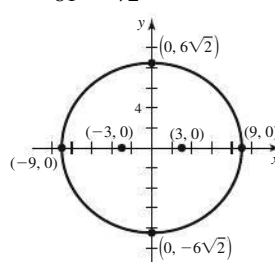
29. Hyperbola



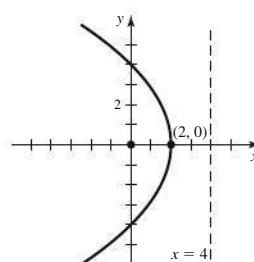
$$43. \frac{x^2}{25} + y^2 = 1 \quad 45. \frac{x^2}{4} - \frac{y^2}{9} = 1 \quad 47. \frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$49. \frac{x^2}{16} - \frac{y^2}{9} = 1 \quad 51. \text{ a. True } \text{ b. True } \text{ c. True } \text{ d. True}$$

$$53. \frac{x^2}{81} + \frac{y^2}{72} = 1$$

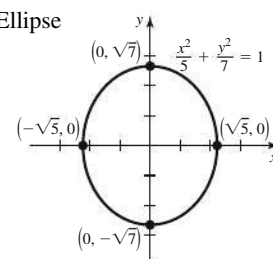
Directrices: $x = \pm 27$

57.



Vertex: $(2, 0)$; focus: $(0, 0)$;
directrix: $x = 4$

27. Ellipse



Vertices: $(0, \pm \sqrt{7})$; foci:
 $(0, \pm \sqrt{2})$; major axis has length
 $2\sqrt{7}$; minor axis has length $2\sqrt{5}$.

$$31. y^2 = 16x \quad 33. y^2 = 12x$$

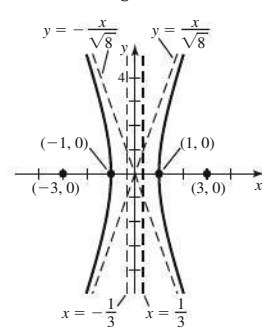
$$35. x^2 = -\frac{2}{3}y$$

$$37. y^2 = 4(x + 1)$$

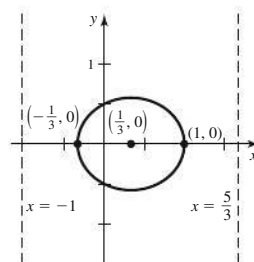
$$39. \frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$41. \frac{x^2}{16} - \frac{y^2}{20} = 1$$

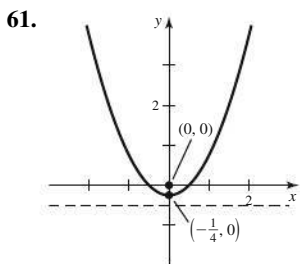
$$55. x^2 - \frac{y^2}{8} = 1$$



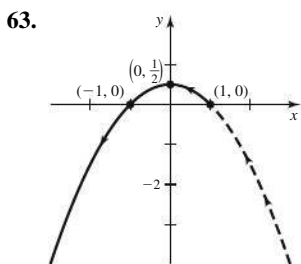
59.



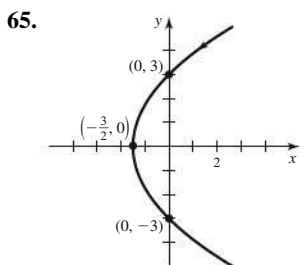
Vertices: $(1, 0)$, $(-\frac{1}{3}, 0)$;
center: $(\frac{1}{3}, 0)$; foci: $(0, 0)$, $(\frac{2}{3}, 0)$;
directrices: $x = -1$, $x = \frac{5}{3}$



Vertex: $(0, -\frac{1}{4})$; focus: $(0, 0)$;
directrix: $y = -\frac{1}{2}$



The parabola starts at $(1, 0)$ and goes through quadrants I, II, and III for θ in $[0, 3\pi/2]$; then it approaches $(1, 0)$ by traveling through quadrant IV on $(3\pi/2, 2\pi)$.



The parabola begins in the first quadrant and passes through the points $(0, 3)$, $(-\frac{3}{2}, 0)$, and $(0, -3)$ as θ ranges from 0 to 2π .

67. The parabolas open to the left due to the presence of a positive $\cos \theta$ term in the denominator. As d increases, the directrix $x = d$ moves to the right, resulting in wider parabolas.

69. $y = 2x + 6$ 71. $y = -\frac{3}{40}x - \frac{4}{5}$ 73. $\frac{dy}{dx} = -\frac{b^2x}{a^2y}$, so

$\frac{y - y_0}{x - x_0} = -\frac{b^2x_0}{a^2y_0}$, which is equivalent to the given equation.

75. $r = \frac{4}{1 - 2 \sin \theta}$ 79. $\frac{4\pi b^2 a}{3}$; $\frac{4\pi a^2 b}{3}$; yes, if $a \neq b$

81. a. $\frac{\pi b^2}{3a^2}(a - c)^2(2a + c)$ b. $\frac{4\pi b^4}{3a}$ 91. $2p$

97. a. $u(m) = \frac{2m^2 - \sqrt{3m^2 + 1}}{m^2 - 1}$; $v(m) = \frac{2m^2 + \sqrt{3m^2 + 1}}{m^2 - 1}$;

2 intersection points for $|m| > 1$ b. $\frac{5}{4}, \infty$ c. 2, 2

d. $2\sqrt{3} - \ln(\sqrt{3} + 2)$

Chapter 12 Review Exercises, pp. 800–803

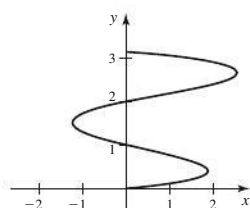
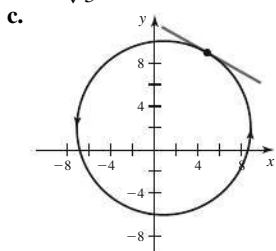
1. a. False b. False c. True d. False e. True f. True

3. $\frac{x^2}{16} + \frac{y^2}{9} = 1$; ellipse generated counterclockwise

5. Segment of the parabola $y = \sqrt{x}$ starting at $(4, 2)$ and ending at $(9, 3)$

7. a. $(x - 1)^2 + (y - 2)^2 = 64$ 9. $x = 5(t - 1)(t - 2) \sin t$,
 $y = t$

b. $-\frac{1}{\sqrt{3}}$



11. a. $x^2 + (y + 1)^2 = 9$ b. Lower half of a circle of radius 3 centered at $(0, -1)$, starting at $(3, -1)$ and ending at $(-3, -1)$ c. 0

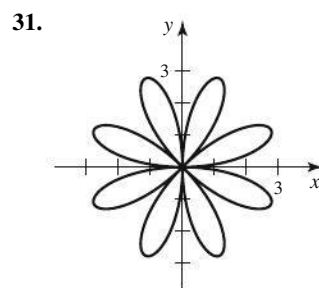
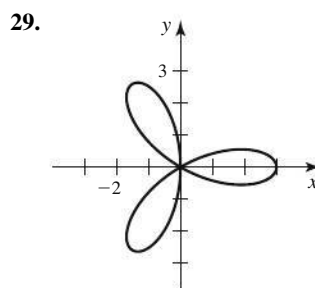
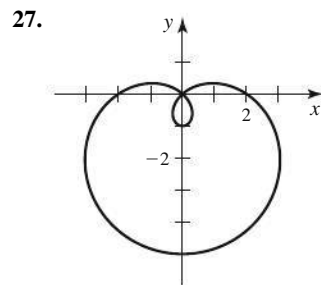
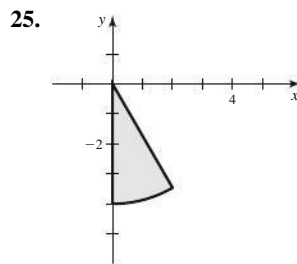
13. At $t = \pi/6$: $y = (2 + \sqrt{3})x + \left(2 - \frac{\pi}{3} - \frac{\pi\sqrt{3}}{6}\right)$; at

$t = \frac{2\pi}{3}$: $y = \frac{x}{\sqrt{3}} + 2 - \frac{2\pi}{3\sqrt{3}}$ 15. $x = -1 + 2t$, $y = t$,

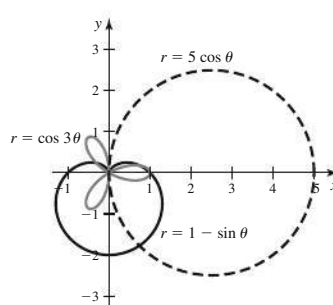
for $0 \leq t \leq 1$; $x = 1 - 2t$, $y = 1 - t$, for $0 \leq t \leq 1$

17. $x = 3 \sin t$, $y = 3 \cos t$, for $0 \leq t \leq 2\pi$

19. $\frac{4}{15}$ 21. 9.1 23. $4 - 2\sqrt{2}$



33. Liz should choose
 $r = 1 - \sin \theta$.



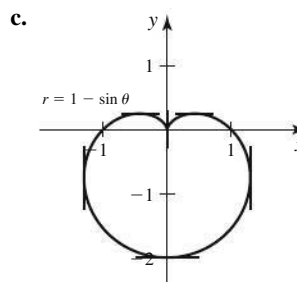
35. $(x - 3)^2 + (y + 1)^2 = 10$; a circle of radius $\sqrt{10}$ centered at $(3, -1)$

37. $r = 1 + \cos \theta$; a cardioid

39. $r = 8 \cos \theta$, $0 \leq \theta \leq \pi$

41. a. Horizontal: $\left(\frac{1}{2}, \frac{\pi}{6}\right), \left(\frac{1}{2}, \frac{5\pi}{6}\right), \left(2, \frac{3\pi}{2}\right)$;

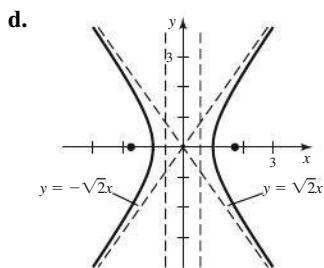
vertical: $\left(\frac{3}{2}, \frac{7\pi}{6}\right), \left(\frac{3}{2}, \frac{11\pi}{6}\right), \left(0, \frac{\pi}{2}\right)$ b. Undefined



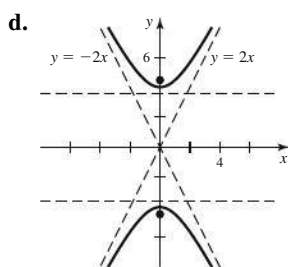
43. $\left(\frac{\pi}{12}, \frac{1}{2^{1/4}}\right), \left(\frac{3\pi}{4}, \frac{1}{2^{1/4}}\right), \left(\frac{17\pi}{12}, \frac{1}{2^{1/4}}\right), (0, 0)$

45. $\pi - \frac{3\sqrt{3}}{2}$ 47. $2\sqrt{3} - \frac{2\pi}{3}$ 49. 4 51. 40.09

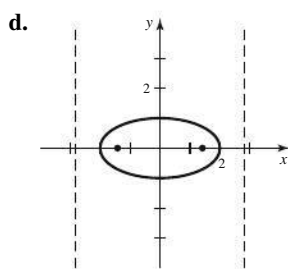
53. a. Hyperbola
 b. Foci $(\pm\sqrt{3}, 0)$, vertices $(\pm 1, 0)$, directrices $x = \pm \frac{1}{\sqrt{3}}$
 c. $e = \sqrt{3}$



55. a. Hyperbola
 b. Foci $(0, \pm 2\sqrt{5})$, vertices $(0, \pm 4)$, directrices $y = \pm \frac{8}{\sqrt{5}}$ c. $e = \frac{\sqrt{5}}{2}$

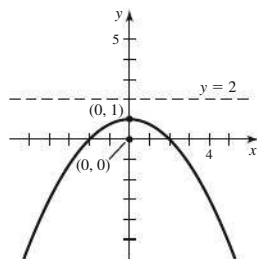


57. a. Ellipse
 b. Foci $(\pm\sqrt{2}, 0)$, vertices $(\pm 2, 0)$, directrices $x = \pm 2\sqrt{2}$ c. $e = \frac{\sqrt{2}}{2}$

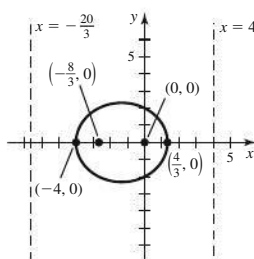


59. $y = \frac{3}{2}x - 2$

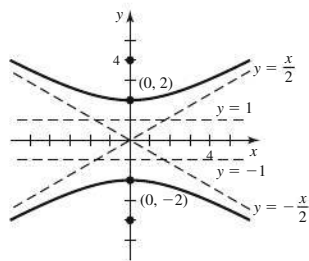
61. $e = 1$



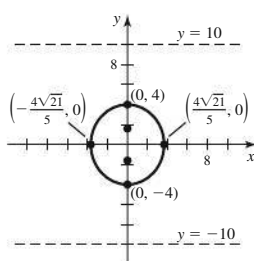
63. $e = \frac{1}{2}$



65. $\frac{y^2}{4} - \frac{x^2}{12} = 1$



67. $\frac{y^2}{16} + \frac{25x^2}{336} = 1$; foci: $(0, \pm \frac{8}{5})$



69. $e = 2/3$, $y = \pm 9$, $(\pm 2\sqrt{5}, 0)$ 71. $m = \frac{b}{a}$

75. a. $x = \pm a \cos^{2/n} t$, $y = \pm b \sin^{2/n} t$

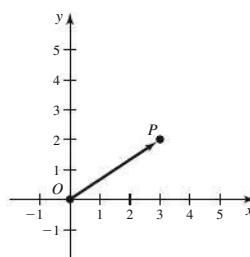
c. The curve becomes more rectangular as n increases.

CHAPTER 13

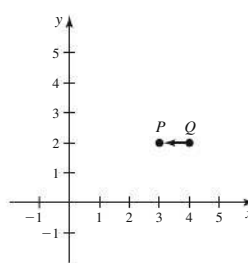
Section 13.1 Exercises, pp. 813–816

3. There are infinitely many vectors with the same direction and length as $\mathbf{v}$. 5. $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$ 7. No
 9. $|\langle v_1, v_2 \rangle| = \sqrt{v_1^2 + v_2^2}$ 11. If P has coordinates (x_1, y_1) and Q has coordinates (x_2, y_2) , then the magnitude of $\overrightarrow{PQ}$ is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. 13. a, c, e 15. a. $3\mathbf{v}$ b. $2\mathbf{u}$
 c. $-3\mathbf{u}$ d. $-2\mathbf{u}$ e. $\mathbf{v}$ 17. a. $3\mathbf{u} + 3\mathbf{v}$ b. $\mathbf{u} + 2\mathbf{v}$ c. $2\mathbf{u} + 5\mathbf{v}$
 d. $-2\mathbf{u} + 3\mathbf{v}$ e. $3\mathbf{u} + 2\mathbf{v}$ f. $-3\mathbf{u} - 2\mathbf{v}$ g. $-2\mathbf{u} - 4\mathbf{v}$
 h. $\mathbf{u} - 4\mathbf{v}$ i. $-\mathbf{u} - 6\mathbf{v}$

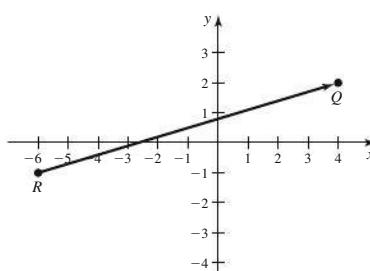
19. a. $\overrightarrow{OP} = \langle 3, 2 \rangle = 3\mathbf{i} + 2\mathbf{j}$
 $|\overrightarrow{OP}| = \sqrt{13}$



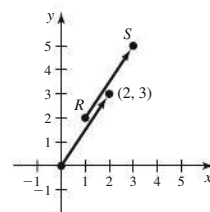
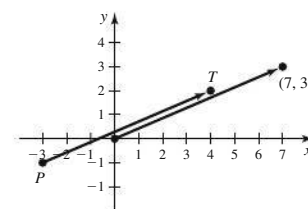
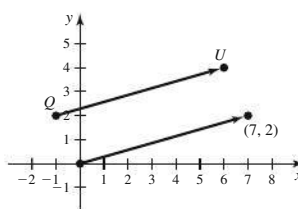
b. $\overrightarrow{QP} = \langle -1, 0 \rangle = -\mathbf{i}$
 $|\overrightarrow{QP}| = 1$



c. $\overrightarrow{RQ} = \langle 10, 3 \rangle = 10\mathbf{i} + 3\mathbf{j}$
 $|\overrightarrow{RQ}| = \sqrt{109}$



21. $\overrightarrow{QU} = \langle 7, 2 \rangle$, $\overrightarrow{PT} = \langle 7, 3 \rangle$, $\overrightarrow{RS} = \langle 2, 3 \rangle$



23. $\vec{QT}$ 25. $\langle -4, 10 \rangle$ 27. $\langle 52, -30 \rangle$ 29. $2\sqrt{2}$

31. $\mathbf{w} - \mathbf{u}$ 33. $13\left\langle -\frac{5}{13}, \frac{12}{13} \right\rangle$ 35. $\langle 3, 3\sqrt{3} \rangle$

37. $\left\langle \frac{15}{13}, -\frac{36}{13} \right\rangle$ 39. $\left\langle \frac{30}{\sqrt{13}}, -\frac{20}{\sqrt{13}} \right\rangle$ 41. $-\mathbf{i} + 10\mathbf{j}$

43. $\pm \frac{1}{\sqrt{61}} \langle 6, 5 \rangle$ 45. $\left\langle -\frac{28}{\sqrt{74}}, \frac{20}{\sqrt{74}} \right\rangle, \left\langle \frac{28}{\sqrt{74}}, -\frac{20}{\sqrt{74}} \right\rangle$

47. a. $\left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle, \left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle$ b. $b = \pm \frac{2\sqrt{2}}{3}$ c. $a = \pm \frac{3}{\sqrt{10}}$

49. $\langle -4\sqrt{3}, 4 \rangle$ 51. $\langle 15\sqrt{3}, -15 \rangle$ 53. a. $\mathbf{v}_a = \langle -320, 0 \rangle$;
 $\mathbf{w} = \langle -20\sqrt{2}, -20\sqrt{2} \rangle$; $\mathbf{v}_g = \langle -320 - 20\sqrt{2}, -20\sqrt{2} \rangle$

b. Approx. 349.4 mi/hr; approx. 4.6° south of west55. Approx. 490.3 mi/hr with a heading of about 1.2° west of north57. $5\sqrt{65}$ km/hr ≈ 40.3 km/hr 59. 1 m/s in the direction 30° eastof north 61. a. $\langle 20, 20\sqrt{3} \rangle$ b. Yes c. No 63. $250\sqrt{2}$ lb

65. a. True b. True c. False d. False e. False f. False

g. False h. True 67. $\mathbf{x} = \left\langle \frac{1}{5}, -\frac{3}{10} \right\rangle$ 69. $\mathbf{x} = \left\langle \frac{4}{3}, -\frac{11}{3} \right\rangle$

71. $4\mathbf{i} - 8\mathbf{j}$ 73. $\langle a, b \rangle = \left(\frac{a+b}{2} \right) \mathbf{u} + \left(\frac{b-a}{2} \right) \mathbf{v}$

75. a. $\mathbf{0}$ b. The 6:00 vector c. Sum any six consecutive vectors.

d. A vector pointing from 12:00 to 6:00 with a length 12 times the radius of the clock

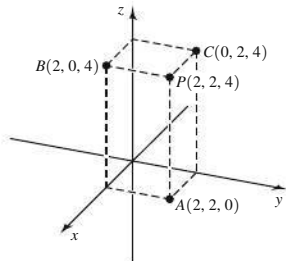
77. $\mathbf{u} + \mathbf{v} = \langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle u_1 + v_1, u_2 + v_2 \rangle$
 $= \langle v_1 + u_1, v_2 + u_2 \rangle = \langle v_1, v_2 \rangle + \langle u_1, u_2 \rangle$
 $= \mathbf{v} + \mathbf{u}$

79. $a(c\mathbf{v}) = a(c\langle v_1, v_2 \rangle) = a\langle cv_1, cv_2 \rangle$
 $= \langle acv_1, acv_2 \rangle = \langle (ac)v_1, (ac)v_2 \rangle$
 $= ac\langle v_1, v_2 \rangle = (ac)\mathbf{v}$

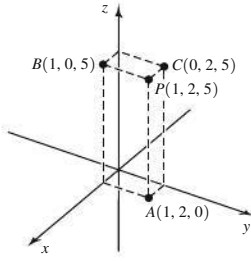
81. $(a+c)\mathbf{v} = (a+c)\langle v_1, v_2 \rangle$
 $= \langle (a+c)v_1, (a+c)v_2 \rangle$
 $= \langle av_1 + cv_1, av_2 + cv_2 \rangle$
 $= \langle av_1, av_2 \rangle + \langle cv_1, cv_2 \rangle$
 $= a\langle v_1, v_2 \rangle + c\langle v_1, v_2 \rangle$
 $= a\mathbf{v} + c\mathbf{v}$

85. a. $\{\mathbf{u}, \mathbf{v}\}$ are linearly dependent. $\{\mathbf{u}, \mathbf{w}\}$ and $\{\mathbf{v}, \mathbf{w}\}$ are linearly independent. b. Two linearly dependent vectors are parallel. Two linearly independent vectors are not parallel. 87. a. $\frac{5}{3}$ b. -15**Section 13.2 Exercises, pp. 823–827**1. Move 3 units from the origin in the direction of the positive x -axis, then 2 units in the direction of the negative y -axis, and then 1 unit in the direction of the positive z -axis. 3. It is parallel to the yz -plane and contains the point $(4, 0, 0)$. 5. $\mathbf{u} + \mathbf{v} = \langle 9, 0, -6 \rangle$; $3\mathbf{u} - \mathbf{v} = \langle 3, 20, -22 \rangle$ 7. $(0, 0, -4)$ 9. $A(3, 0, 5), B(3, 4, 0), C(0, 4, 5)$ 11. $A(3, -4, 5), B(0, -4, 0), C(0, -4, 5)$

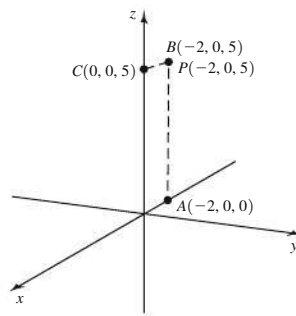
13. a.



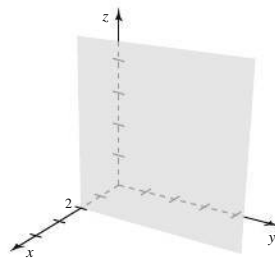
b.



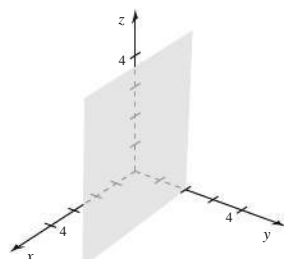
c.



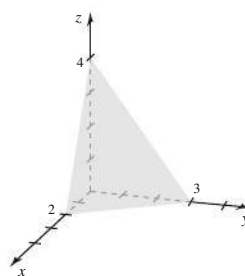
15.



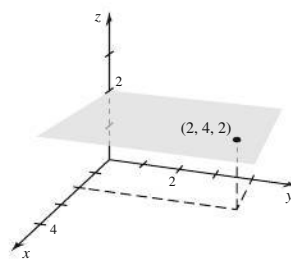
17.



19.



21.



23. $(x-1)^2 + (y-2)^2 + (z-3)^2 = 16$

25. $(x+2)^2 + y^2 + (z-4)^2 \leq 1$

27. $(x - \frac{3}{2})^2 + (y - \frac{3}{2})^2 + (z-7)^2 = \frac{13}{2}$ 29. A sphere centered

at $(1, 0, 0)$ with radius 3 31. A sphere centered at $(0, 1, 2)$ with radius 3 33. All points on or outside the sphere with center $(0, 7, 0)$ and radius 6 35. The ball centered at $(4, 7, 9)$ with radius 1537. The single point $(1, -3, 0)$ 39. a. $\langle 12, -7, 2 \rangle$ b. $\langle 16, -13, -1 \rangle$ c. 5 41. a. $\langle -4, 5, -4 \rangle$ b. $\langle -9, 3, -9 \rangle$ c. $3\sqrt{2}$ 43. a. $\langle -15, 23, 22 \rangle$ b. $\langle -31, 49, 33 \rangle$ c. $3\sqrt{5}$

45. a. $\vec{PQ} = \langle 2, 6, 2 \rangle = 2\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ b. $|\vec{PQ}| = 2\sqrt{11}$

c. $\left\langle \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right\rangle$ and $\left\langle -\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}}, -\frac{1}{\sqrt{11}} \right\rangle$

47. a. $\vec{PQ} = \langle 0, -5, 1 \rangle = -5\mathbf{j} + \mathbf{k}$ b. $|\vec{PQ}| = \sqrt{26}$

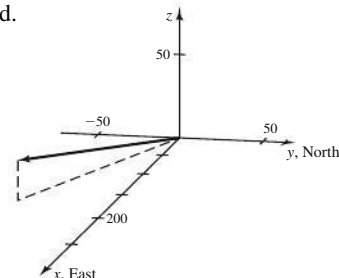
c. $\left\langle 0, -\frac{5}{\sqrt{26}}, \frac{1}{\sqrt{26}} \right\rangle$ and $\left\langle 0, \frac{5}{\sqrt{26}}, -\frac{1}{\sqrt{26}} \right\rangle$

49. a. $\vec{PQ} = \langle -2, 4, -2 \rangle = -2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ b. $|\vec{PQ}| = 2\sqrt{6}$

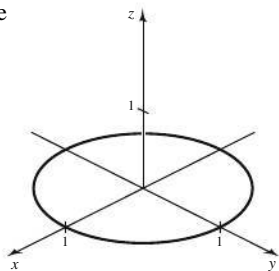
c. $\left\langle -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right\rangle$ and $\left\langle \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$

51. a. $20\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}$; b. 30 mi/hr

53. The speed of the plane is approximately 220 mi/hr; the direction is slightly south of east and upward.



55. $5\sqrt{6}$ knots to the east, $5\sqrt{6}$ knots to the north, 10 knots upward
 57. a. False b. False c. False d. True 59. All points in $\mathbb{R}^3$ except those on the coordinate axes 61. A circle of radius 1 centered at $(0, 0, 0)$ in the xy -plane



63. A circle of radius 2 centered at $(0, 0, 1)$ in the horizontal plane $z = 1$ 65. $(x - 2)^2 + (z - 1)^2 = 9, y = 4$ 67. $6\left\langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \right\rangle$
 69. $\left\langle -\frac{15}{4}, \frac{5}{2}, -\frac{5\sqrt{3}}{4} \right\rangle$ 71. $\langle 12, -16, 0 \rangle, \langle -12, 16, 0 \rangle$
 73. $\langle -\sqrt{3}, -\sqrt{3}, \sqrt{3} \rangle, \langle \sqrt{3}, \sqrt{3}, -\sqrt{3} \rangle$ 75. a. Collinear; Q is between P and R . b. Collinear; P is between Q and R .
 c. Noncollinear d. Noncollinear 77. $\left\langle \frac{500\sqrt{3}}{9}, 0, -\frac{500}{3} \right\rangle,$
 $\left\langle -\frac{250\sqrt{3}}{9}, -\frac{250}{3}, -\frac{500}{3} \right\rangle, \left\langle -\frac{250\sqrt{3}}{9}, \frac{250}{3}, -\frac{500}{3} \right\rangle$
 79. $(3, 8, 9), (-1, 0, 3), (1, 0, -3)$

Section 13.3 Exercises, pp. 833–837

1. $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta$ 3. -40 5. $\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$, so
 $\theta = \cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$ 7. $\left\langle -\frac{4}{3}, \frac{2}{3}, \frac{4}{3} \right\rangle$ 9. -1 11. 2
 13. $\frac{\pi}{2}; 0$ 15. 100; $\frac{\pi}{4}$ 17. $\frac{1}{2}$ 19. 0; $\frac{\pi}{2}$ 21. 1; $\pi/3$
 23. $-2; 93.2^\circ$ 25. 2; 87.2° 27. $-4; 104^\circ$ 29. $\angle P = 78.8^\circ,$
 $\angle Q = 47.2^\circ, \angle R = 54.0^\circ$ 31. $\langle 3, 0 \rangle; 3$ 33. $\langle 0, 3 \rangle; 3$
 35. $\frac{6}{5}\langle -2, 1 \rangle; \frac{6}{\sqrt{5}}$ 37. $\frac{14}{19}\langle -1, -3, 3 \rangle; -\frac{14}{\sqrt{19}}$
 39. $-\mathbf{i} + \mathbf{j} - 2\mathbf{k}; \sqrt{6}$ 41. $750\sqrt{3}$ ft-lb 43. $25\sqrt{2}$ J
 45. 400 J 47. $\frac{1}{2}\langle 5\sqrt{3}, -15 \rangle, \frac{1}{2}\langle -5\sqrt{3}, -5 \rangle$ 49. $\langle 490, -490 \rangle,$
 $\langle -490, -490 \rangle$ 51. a. False b. True c. True d. False
 e. False f. True 53. $c = \frac{4}{9}$ 55. $\langle 1, a, 4a - 2 \rangle, a$ real
 57. a. $\text{proj}_{\mathbf{k}} \mathbf{u} = |\mathbf{u}| \cos 60^\circ \left(\frac{\mathbf{k}}{|\mathbf{k}|}\right) = \frac{1}{2}\mathbf{k}$, for all such $\mathbf{u}$ b. Yes
 59. The heads of the vectors lie on the line $y = 3 - x$.
 61. The heads of the vectors lie on the plane $z = 3$.
 63. $\mathbf{u} = \left\langle -\frac{4}{5}, -\frac{2}{5} \right\rangle + \left\langle -\frac{6}{5}, \frac{12}{5} \right\rangle$
 65. $\mathbf{u} = \left\langle 1, \frac{1}{2}, \frac{1}{2} \right\rangle + \left\langle -2, \frac{3}{2}, \frac{5}{2} \right\rangle$ 67. $3x - 7y = -36$
 69. $-\frac{5}{3}$ 71. $\mathbf{I} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}, \mathbf{J} = -\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j};$
 $\mathbf{i} = \frac{1}{\sqrt{2}}(\mathbf{I} - \mathbf{J}), \mathbf{j} = \frac{1}{\sqrt{2}}(\mathbf{I} + \mathbf{J})$ 73. a. $|\mathbf{I}| = |\mathbf{J}| = |\mathbf{K}| = 1$
 b. $\mathbf{I} \cdot \mathbf{J} = 0, \mathbf{I} \cdot \mathbf{K} = 0, \mathbf{J} \cdot \mathbf{K} = 0$ c. $\langle 1, 0, 0 \rangle = \frac{1}{2}\mathbf{I} - \frac{1}{\sqrt{2}}\mathbf{J} + \frac{1}{2}\mathbf{K}$
 75. a. The faces on $y = 0$ and $z = 0$ b. The faces on $y = 1$ and
 $z = 1$ c. The faces on $x = 0$ and $x = 1$ d. 0 e. 1 f. 2

77. a. $\left(\frac{2}{\sqrt{3}}, 0, \frac{2\sqrt{2}}{\sqrt{3}}\right)$ b. $\mathbf{r}_{OP} = \langle \sqrt{3}, -1, 0 \rangle, \mathbf{r}_{OQ} = \langle \sqrt{3}, 1, 0 \rangle,$
 $\mathbf{r}_{PQ} = \langle 0, 2, 0 \rangle, \mathbf{r}_{OR} = \left\langle \frac{2}{\sqrt{3}}, 0, \frac{2\sqrt{2}}{\sqrt{3}} \right\rangle, \mathbf{r}_{PR} = \left\langle -\frac{\sqrt{3}}{3}, 1, \frac{2\sqrt{2}}{\sqrt{3}} \right\rangle$

$$83. \text{ a. } \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \left(\frac{\mathbf{v} \cdot \mathbf{i}}{|\mathbf{v}||\mathbf{i}|}\right)^2 + \left(\frac{\mathbf{v} \cdot \mathbf{j}}{|\mathbf{v}||\mathbf{j}|}\right)^2 + \left(\frac{\mathbf{v} \cdot \mathbf{k}}{|\mathbf{v}||\mathbf{k}|}\right)^2$$

$$= \frac{a^2}{a^2 + b^2 + c^2} + \frac{b^2}{a^2 + b^2 + c^2} + \frac{c^2}{a^2 + b^2 + c^2} = 1$$

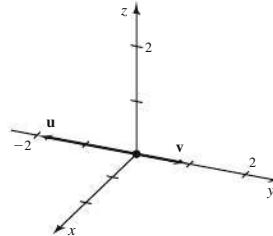
- b. $\langle 1, 1, 0 \rangle, 90^\circ$ c. $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1 \right\rangle, 45^\circ$ d. No. If so,
 $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \cos^2 \gamma = 1$, which has no solution. e. 54.7°
 85. $|\mathbf{u} \cdot \mathbf{v}| = 33 = \sqrt{33} \cdot \sqrt{33} < \sqrt{70} \cdot \sqrt{74} = |\mathbf{u}||\mathbf{v}|$

Section 13.4 Exercises, pp. 842–844

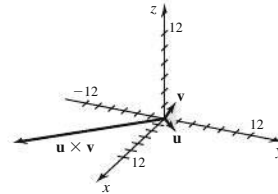
1. 0 3. a. $\mathbf{u}$ is orthogonal to $\mathbf{v}$. b. $\mathbf{u}$ is parallel to $\mathbf{v}$. 5. $\sqrt{2}/2$

$$7. -3\mathbf{i} - 2\mathbf{j} + 7\mathbf{k} \quad 9. \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \quad 11. 15\mathbf{k}$$

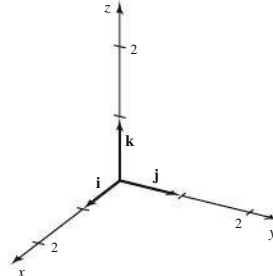
13. 0



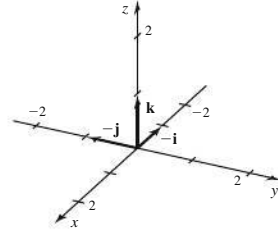
15. 18



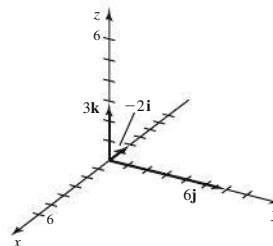
17. i



19. $-\mathbf{i}$

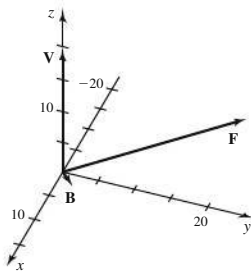


21. $6\mathbf{j}$



23. $\mathbf{u} \times \mathbf{v} = \langle -30, 18, 9 \rangle, \mathbf{v} \times \mathbf{u} = \langle 30, -18, -9 \rangle$
 25. $\mathbf{u} \times \mathbf{v} = \langle 6, 11, 5 \rangle, \mathbf{v} \times \mathbf{u} = \langle -6, -11, -5 \rangle$
 27. $\mathbf{u} \times \mathbf{v} = \langle 8, 4, 10 \rangle, \mathbf{v} \times \mathbf{u} = \langle -8, -4, -10 \rangle$ 29. 11
 31. $3\sqrt{10}$ 33. $\sqrt{11}/2$ 35. $4\sqrt{2}$ 37. $9\sqrt{2}$ 41. Not collinear
 43. $\langle 3, -4, 2 \rangle$ 45. $\langle 0, 20, -20 \rangle$ 47. The force $\mathbf{F} = 5\mathbf{i} - 5\mathbf{k}$
 produces the greater torque. 49. $5/\sqrt{2}$ N-m 51. $|\tau| = 13.2$ N-m;
 direction: into the page

53. The magnitude is $20\sqrt{2}$ at a 135° angle with the positive x -axis in the xy -plane.



55. $4.53 \times 10^{-14} \text{ kg}\cdot\text{m/s}^2$ 57. a. False b. False c. False

d. True e. False 59. $\langle u_1, u_1 + 2, u_1 + 1 \rangle, u_1$ real

$$61. \frac{\sqrt{(ab)^2 + (ac)^2 + (bc)^2}}{2}$$

63. $|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = |\mathbf{u}| |\mathbf{v} \times \mathbf{w}| |\cos \theta|$, where $|\mathbf{v} \times \mathbf{w}|$ is the area of the base of the parallelepiped and $|\mathbf{u}| |\cos \theta|$ is its height.

67. $1.76 \times 10^7 \text{ m/s}$

Section 13.5 Exercises, pp. 852–855

1. $\langle 4, -8, 9 \rangle$ 3. $\langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$ 5. Perpendicular

7. A point and a normal vector 9. $(-6, 0, 0), (0, -4, 0), (0, 0, 3)$

11. $x = 4t, y = 7t, z = 1; \mathbf{r} = \langle 0, 0, 1 \rangle + t\langle 4, 7, 0 \rangle$

13. $x = 0, y = t, z = 1; \mathbf{r} = \langle 0, 0, 1 \rangle + t\langle 0, 1, 0 \rangle$

15. $x = t, y = 2t, z = 3t; \mathbf{r} = t\langle 1, 2, 3 \rangle$

17. $x = -2t, y = 8t, z = -4t; \mathbf{r} = t\langle -2, 8, -4 \rangle$

19. $x = -2t, y = -t, z = t; \mathbf{r} = t\langle -2, -1, 1 \rangle$

21. $x = -2, y = 5 - 2t, z = 3 - t; \mathbf{r} = \langle -2, 5, 3 \rangle + t\langle 0, -2, -1 \rangle$ 23. $x = 1 - 4t, y = 2 + 6t, z = 3 + 14t;$

$\mathbf{r} = \langle 1, 2, 3 \rangle + t\langle -4, 6, 14 \rangle$ 25. $x = 4, y = 3 - 9t, z = 3 + 6t;$

$\mathbf{r} = \langle 4, 3, 3 \rangle + t\langle 0, -9, 6 \rangle$ 27. $x = t, y = 2t, z = 3t, 0 \leq t \leq 1$

29. $x = 2 + 5t, y = 4 + t, z = 8 - 5t, 0 \leq t \leq 1$ 31. Intersect

at $(1, 3, 2)$ 33. Skew 35. Same line 37. Parallel, distinct lines

39. 13 41. a. Yes b. No c. $13.16^\circ < \theta < 18.12^\circ$

43. $x + y - z = 4$ 45. $2x + y - 2z = -2$

47. $x + 4y + 7z = 0$ 49. $7x + 2y + z = 10$

51. $-x + 2y - 4z = -17$ 53. $3y - 2z = 0$

55. $8x - 7y + 2z = 0$ 57. $x + 3y - z = -3$

59. Yes; $2x - y = -1$

61. Intercepts

$x = 2, y = -3, z = 6;$

$3x - 2y = 6, z = 0;$

$-2y + z = 6, x = 0;$ and

$3x + z = 6, y = 0$

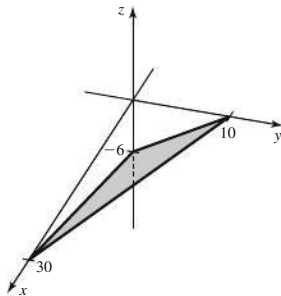
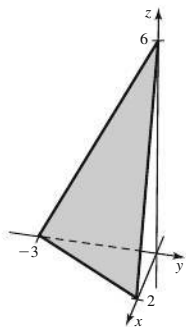
63. Intercepts

$x = 30, y = 10, z = -6;$

$x + 3y = 30, z = 0;$

$x - 5z = 30, y = 0;$ and

$3y - 5z = 30, x = 0$



65. Orthogonal 67. Neither 69. Q and T are identical; Q, R , and T are parallel; S is orthogonal to Q, R , and T .

71. $\mathbf{r} = \langle 2 + 2t, 1 - 4t, 3 + t \rangle$

73. $x = t, y = 1 + 2t, z = -1 - 3t$

75. $x = \frac{7}{5} + 2t, y = \frac{9}{5} + t, z = -t$ 77. $(3, 3, 3)$ 79. $(1, 1, 2)$

81. a. True b. False c. False d. True e. False f. False

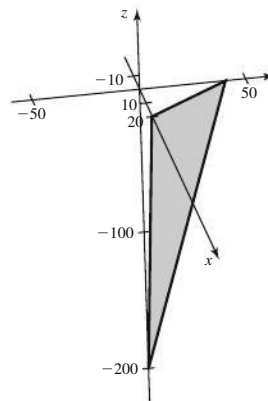
g. True 83. 6 85. $\frac{x-1}{4} = \frac{y-2}{7} = \frac{z}{2}$ 87. Approx. 43°

89. $6x - 4y + z = d$ 91. The planes intersect in the point $(3, 6, 0)$.

93. a.

b. Positive

c. $2x + y = 40$, line in the xy -plane



Section 13.6 Exercises, pp. 863–865

1. z -axis; x -axis; y -axis 3. Intersection of the surface with a plane

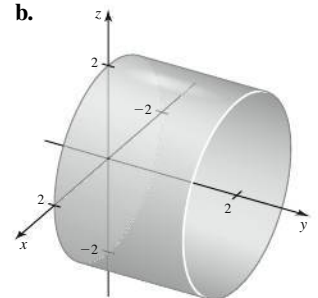
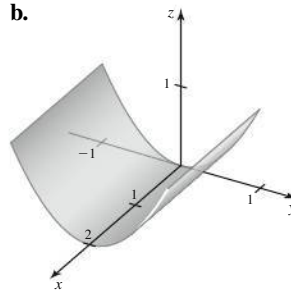
parallel to one of the coordinate planes 5. Ellipsoid

7. a. x -axis

9. a. y -axis

b.

b.

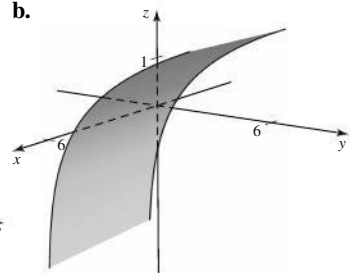
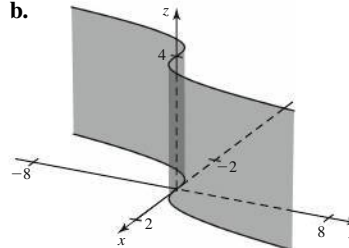


11. a. z -axis

13. a. x -axis

b.

b.



15. Ellipsoid; xy -trace: $x^2 + y^2 = 1$ (circle); xz -trace:

$x^2 + \frac{z^2}{25} = 1$ (ellipse); yz -trace: $y^2 + \frac{z^2}{25} = 1$ (ellipse)

17. Paraboloid; xy -trace: $(0, 0, 0)$ (a single point); xz -trace:

$z = 25x^2$ (parabola); yz -trace: $z = 25y^2$ (parabola) 19. Hyperboloid

of two sheets; xz -trace: $z^2 - 25x^2 = 25$ (hyperbola);

yz -trace: $z^2 - 25y^2 = 25$ (hyperbola) 21. Hyperbolic paraboloid

23. Elliptic paraboloid 25. Hyperbolic cylinder

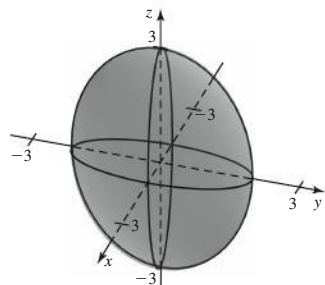
27. Elliptic paraboloid

29. a. $x = \pm 1, y = \pm 2, z = \pm 3$

b. $x^2 + \frac{y^2}{4} = 1, x^2 + \frac{z^2}{9} = 1,$

$\frac{y^2}{4} + \frac{z^2}{9} = 1$

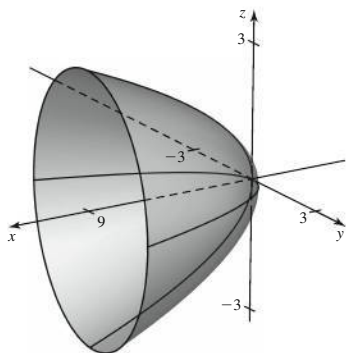
c. Ellipsoid



31. a. $x = y = z = 0$

b. $x = y^2, x = z^2$, origin

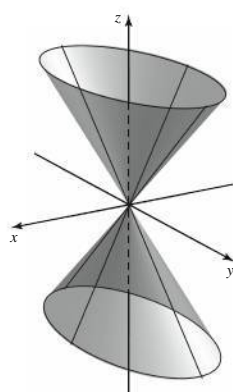
c. Elliptic paraboloid



37. a. $x = y = z = 0$

b. Origin, $\frac{y^2}{4} = z^2, x^2 = z^2$

c. Elliptic cone

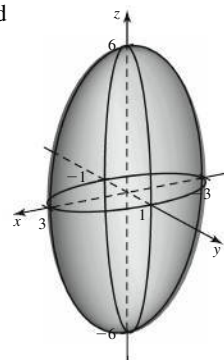


39. a. $x = \pm 3, y = \pm 1, z = \pm 6$

b. $\frac{x^2}{3} + 3y^2 = 3, \frac{x^2}{3} + \frac{z^2}{12} = 3,$

$3y^2 + \frac{z^2}{12} = 3$

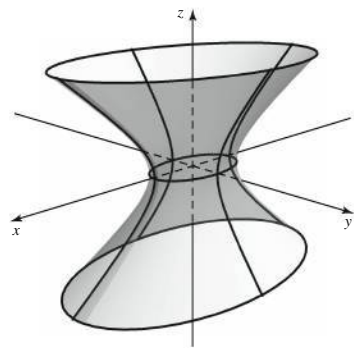
c. Ellipsoid



33. a. $x = \pm 5, y = \pm 3$, no z -intercept

b. $\frac{x^2}{25} + \frac{y^2}{9} = 1, \frac{x^2}{25} - z^2 = 1, \frac{y^2}{9} - z^2 = 1$

c. Hyperboloid of one sheet

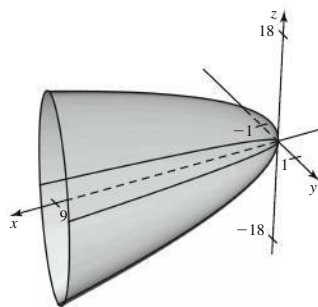


41. a. $x = y = z = 0$

b. Origin,

$x - 9y^2 = 0, 9x - \frac{z^2}{4} = 0$

c. Elliptic paraboloid



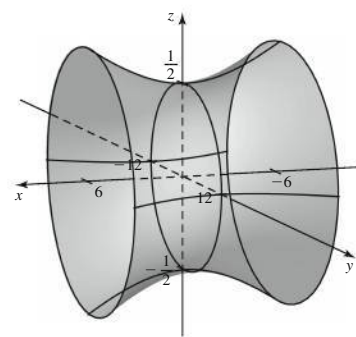
43. a. No x -intercept,

$y = \pm 12, z = \pm \frac{1}{2}$

b. $-\frac{x^2}{4} + \frac{y^2}{16} = 9,$

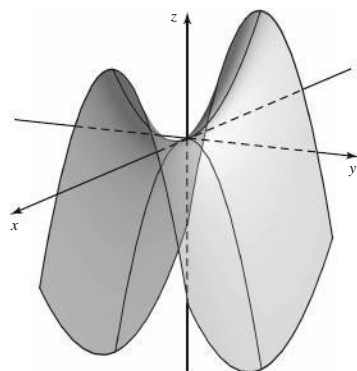
$-\frac{x^2}{4} + 36z^2 = 9, \frac{y^2}{16} + 36z^2 = 9$

c. Hyperboloid of one sheet



35. a. $x = y = z = 0$ b. $\frac{x^2}{9} - y^2 = 0, z = \frac{x^2}{9}, z = -y^2$

c. Hyperbolic paraboloid

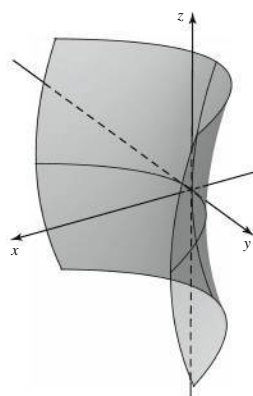


45. a. $x = y = z = 0$

b. $5x - \frac{y^2}{5} = 0, 5x + \frac{z^2}{20} = 0,$

$-\frac{y^2}{5} + \frac{z^2}{20} = 0$

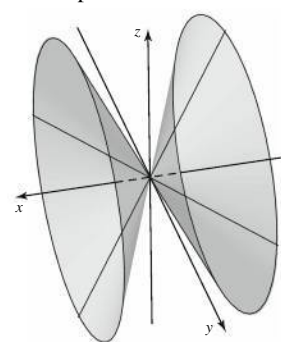
c. Hyperbolic paraboloid



47. a. $x = y = z = 0$

b. $\frac{y^2}{18} = 2x^2, \frac{z^2}{32} = 2x^2$, origin

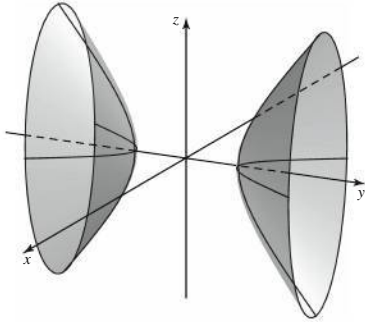
c. Elliptic cone



49. a. No x -intercept, $y = \pm 2$, no z -intercept

b. $-x^2 + \frac{y^2}{4} = 1$, no xz -trace, $\frac{y^2}{4} - \frac{z^2}{9} = 1$

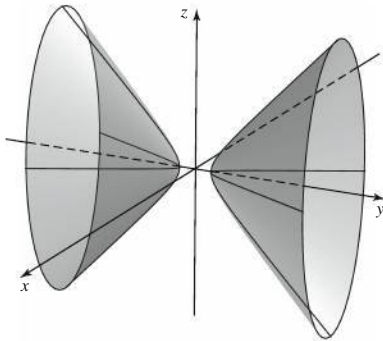
c. Hyperboloid of two sheets



51. a. No x -intercept, $y = \pm \frac{\sqrt{3}}{3}$, no z -intercept

b. $-\frac{x^2}{3} + 3y^2 = 1$, no xz -trace, $3y^2 - \frac{z^2}{12} = 1$

c. Hyperboloid of two sheets



53. The graph of the ellipsoid $x^2 + 4y^2 + 9z^2 + 54z = 19$ is obtained by shifting the graph of the ellipsoid $x^2 + 4y^2 + 9z^2 = 100$ down 3 units.

55. Hyperboloid of one sheet 57. Hyperboloid of two sheets

59. a. True b. True c. True d. False e. False 61. All except

the hyperbolic paraboloid 63. 8 65. b. $\frac{x^2 + z^2}{(10.55/\pi)^2} + \frac{y^2}{(5.55)^2} = 1$

67. $4x^2 + 8y^2 + 4(z - 3)^2 = 9$, $3 \leq z \leq 4.5$

Chapter 13 Review Exercises, pp. 865–867

1. a. True b. False c. True d. False e. True f. True

3. $\langle 3, -6 \rangle$ 5. $\langle -5, 8 \rangle$ 7. $\sqrt{221}$ 9. $12\langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \rangle$

11. $\langle \frac{10}{3}, -\frac{20}{3}, \frac{20}{3} \rangle$ 13. $\langle 58, 26, 44 \rangle$ 15. $a = -3$

17. a. $\mathbf{v} = -275\sqrt{2}\mathbf{i} + 275\sqrt{2}\mathbf{j}$ b. $-275\sqrt{2}\mathbf{i} + (275\sqrt{2} + 40)\mathbf{j}$

19. $\{(x, y, z): (x - 1)^2 + y^2 + (z + 1)^2 = 16\}$

21. $\{(x, y, z): x^2 + (y - 1)^2 + z^2 > 4\}$ 23. A ball centered at $(\frac{1}{2}, -2, 3)$ of radius $\frac{3}{2}$

25. All points outside a sphere of radius 10 centered at $(3, 0, 10)$ 27. 50.15 m/s; 85.4° below the horizontal in the northerly horizontal direction

29. 50 lb; 36.9° north of east

31. A circle of radius 1 centered at $(0, 2, 0)$ in the vertical plane $y = 2$

33. a. 0.68 radian b. $\frac{7}{9}\langle 1, 2, 2 \rangle$; $\frac{7}{3}$ c. $\frac{7}{3}\langle -1, 2, 2 \rangle$; 7

35. $250\sqrt{2}$ ft-lb 37. $90\sqrt{3}$ lb; 90 lb 39. 11

41. $\pm \langle \frac{12}{\sqrt{197}}, \frac{7}{\sqrt{197}}, \frac{2}{\sqrt{197}} \rangle$ 43. $\langle -10, 10, 10 \rangle$

45. $|\tau|(\theta) = 39.2 \sin \theta$ has a maximum value of 39.2 N-m (when $\theta = \pi/2$) and a minimum value of 0 N-m (when $\theta = 0$). Direction does not change. 47. $\mathbf{r} = \langle 0, -3, 9 \rangle + t\langle 2, -5, -8 \rangle$, $0 \leq t \leq 1$

49. $\mathbf{r} = \langle t, 1 + 6t, 1 + 2t \rangle$

51. a. $18x - 9y + 2z = 6$ b. $x = \frac{1}{3}$, $y = -\frac{2}{3}$, $z = 3$

c.



53. $x = t$, $y = 12 - 9t$, $z = -6 + 6t$ 55. $4x + 2y + 13z = 39$

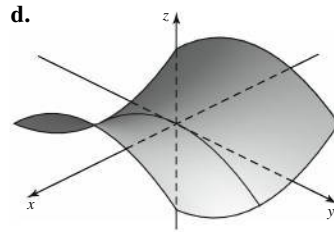
57. $3x + y + 7z = 4$ 59. 3

61. a. Hyperbolic paraboloid

b. $y^2 = 4x^2$, $z = \frac{x^2}{36}$, $z = -\frac{y^2}{144}$

c. $x = y = z = 0$

d.

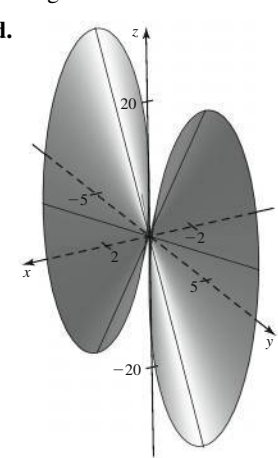


63. a. Elliptic cone

b. $y^2 = 4x^2$, origin, $y^2 = \frac{z^2}{25}$

c. Origin

d.

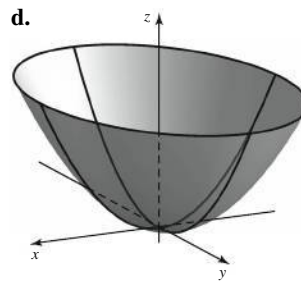


65. a. Elliptic paraboloid

b. Origin, $z = \frac{x^2}{16}$, $z = \frac{y^2}{36}$

c. Origin

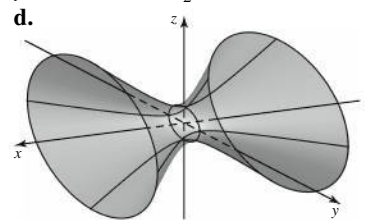
d.



67. a. Hyperboloid of one sheet

b. $y^2 - 2x^2 = 1$, $4z^2 - 2x^2 = 1$, $y^2 + 4z^2 = 1$ c. No x -intercept, $y = \pm 1$, $z = \pm \frac{1}{2}$

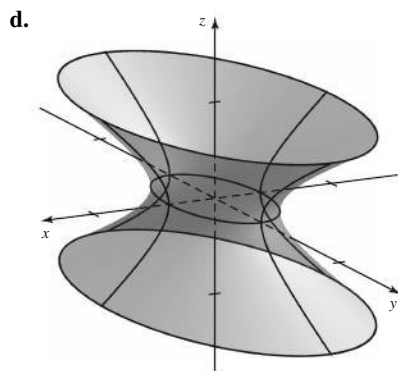
d.



69. a. Hyperboloid of one sheet

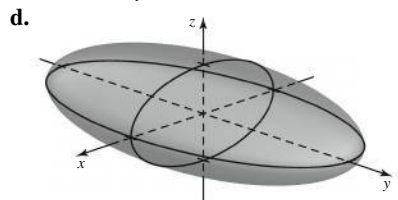
b. $\frac{x^2}{4} + \frac{y^2}{16} = 4$, $\frac{x^2}{4} - z^2 = 4$, $\frac{y^2}{16} - z^2 = 4$

c. $x = \pm 4$, $y = \pm 8$, no z -intercept

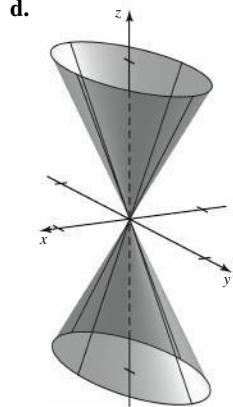


71. a. Ellipsoid b. $\frac{x^2}{4} + \frac{y^2}{16} = 4, \frac{x^2}{4} + z^2 = 4, \frac{y^2}{16} + z^2 = 4$

c. $x = \pm 4, y = \pm 8, z = \pm 2$



73. a. Elliptic cone b. Origin, $\frac{x^2}{9} = \frac{z^2}{64}, \frac{y^2}{49} = \frac{z^2}{64}$ c. Origin



75. a. A b. D c. C d. B

CHAPTER 14

Section 14.1 Exercises, pp. 873–875

1. One 3. Its output is a vector.

5. $\lim_{t \rightarrow a} \mathbf{r}(t) = \langle \lim_{t \rightarrow a} f(t), \lim_{t \rightarrow a} g(t), \lim_{t \rightarrow a} h(t) \rangle$

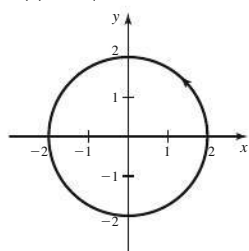
7. $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + 3t\mathbf{k}$

9. $\mathbf{r}(t) = \langle 2 + 2t, 3 + 3t, 7 - 4t \rangle$

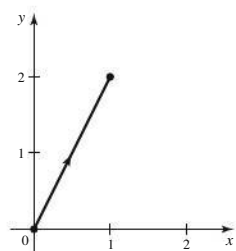
11. $\mathbf{r}(t) = \langle 3 + 2t, 4, 5 - t \rangle$

13. $\mathbf{r}(t) = \langle 1 - t, 2, 1 + 2t \rangle$, for $0 \leq t \leq 1$

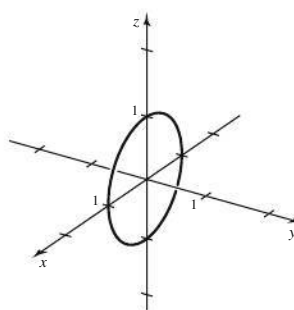
15.



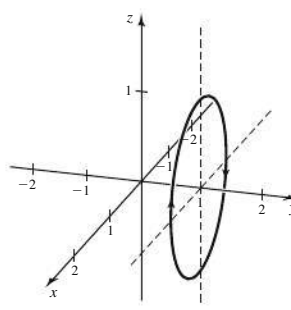
17.



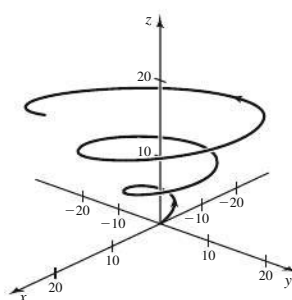
19.



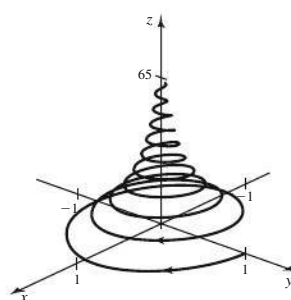
21.



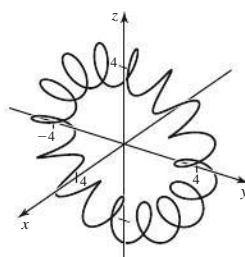
23.



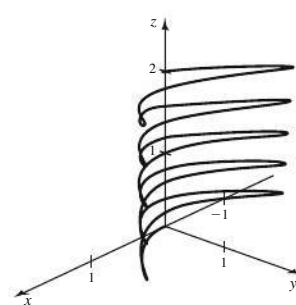
25.



27.



29. When viewed from above, the curve is a portion of the parabola $y = x^2$.



31. $-\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ 33. $-2\mathbf{j} + \frac{\pi}{2}\mathbf{k}$ 35. $\mathbf{i}$ 37. a. True b. False

c. True d. True 39. $\{t : |t| \leq 2\}$ 41. $\{t : 0 \leq t \leq 2\}$

43. (4, 8, 16) 45. a. E b. D c. F d. C e. A f. B

47. $\mathbf{r}(t) = \langle 2 \cos t, 2 \sin t, 4 \rangle$

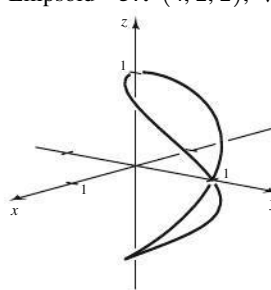
49. $\mathbf{r}(t) = \langle 5 \cos t, 5 \sin t, 10 \cos t + 10 \sin t \rangle$

51. a. Ball has a parabolic trajectory in the yz-plane; 1200 ft

b. Approx. 1199.7 ft c. 1196 ft 53. Hyperboloid of one sheet

55. Ellipsoid 57. (4, 2, 2); $\sqrt{179}$

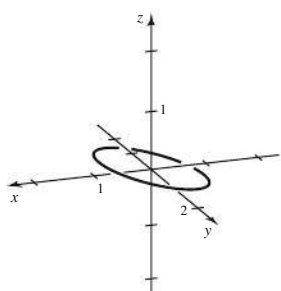
59.



The curve lies on the sphere $x^2 + y^2 + z^2 = 1$.

61. $\frac{2\pi}{(m, n)}$, where (m, n) = greatest common factor of m and n

63. a.



b. Curve is a tilted circle of radius 1 centered at the origin.

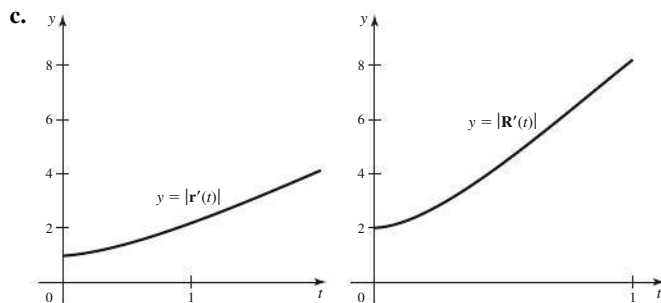
65. $\langle cf - ed, be - af, ad - bc \rangle$ or any scalar multiple

Section 14.2 Exercises, pp. 881–883

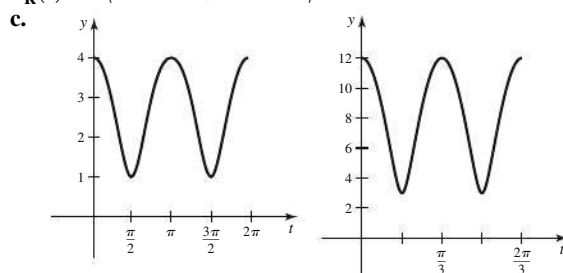
1. $\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$ 3. $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$
5. $\int \mathbf{r}(t) dt = \left(\int f(t) dt \right) \mathbf{i} + \left(\int g(t) dt \right) \mathbf{j} + \left(\int h(t) dt \right) \mathbf{k}$
7. $\mathbf{C} = \langle -1, -3, -10 \rangle$ 9. $\langle -\sin t, 2t, \cos t \rangle$
11. $\left\langle 6t^2, \frac{3}{\sqrt{t}}, -\frac{3}{t^2} \right\rangle$ 13. $e^t \mathbf{i} - 2e^{-t} \mathbf{j} - 8e^{2t} \mathbf{k}$
15. $\langle e^{-t}(1-t), 1 + \ln t, \cos t - t \sin t \rangle$
17. $\langle 1, 6, 3 \rangle$ 19. $\langle 1, 0, 0 \rangle$ 21. $8\mathbf{i} + 9\mathbf{j} - 10\mathbf{k}$
23. $\langle 2/3, 2/3, 1/3 \rangle$
25. $\frac{\langle 0, -\sin 2t, 2 \cos 2t \rangle}{\sqrt{1 + 3 \cos^2 2t}}$ 27. $\frac{t^2}{\sqrt{t^4 + 4}} \left\langle 1, 0, -\frac{2}{t^2} \right\rangle$
29. $\langle 0, 0, -1 \rangle$ 31. $\left\langle \frac{2}{\sqrt{5}}, 0, -\frac{1}{\sqrt{5}} \right\rangle$
33. $\langle 30t^{14} + 24t^3, 14t^{13} - 12t^{11} + 9t^2 - 3, -96t^{11} - 24 \rangle$
35. $4t(2t^3 - 1)(t^3 - 2) \langle 3t(t^3 - 2), 1, 0 \rangle$
37. $e^t(2t^3 + 6t^2) - 2e^{-t}(t^2 - 2t - 1) - 16e^{-2t}$
39. 11 41. $\langle 0, 7, 1 \rangle$ 43. $\langle 2e^{2t}, -2e^t, 0 \rangle$ 45. $\left\langle 4, -\frac{2}{\sqrt{t}}, 0 \right\rangle$
47. $\langle 1 + 6t^2, 4t^3, -2 - 3t^2 \rangle$ 49. $5te^t(t + 2) - 6t^2e^{-t}(t - 3)$
51. $-3t^2 \sin t + 6t \cos t + 2\sqrt{t} \cos 2t + \frac{1}{2\sqrt{t}} \sin 2t$
53. $\langle 2, 0, 0 \rangle, \langle 0, 0, 0 \rangle$ 55. $\langle -9 \cos 3t, -16 \sin 4t, -36 \cos 6t \rangle, \langle 27 \sin 3t, -64 \cos 4t, 216 \sin 6t \rangle$
57. $\left\langle -\frac{1}{4}(t + 4)^{-3/2}, -2(t + 1)^{-3}, 2e^{-t}(1 - 2t^2) \right\rangle, \left\langle \frac{3}{8}(t + 4)^{-5/2}, 6(t + 1)^{-4}, -4te^{-t}(3 - 2t^2) \right\rangle$
59. $\left\langle \frac{t^5}{5} - \frac{3t^2}{2}, t^2 - t, 10t \right\rangle + \mathbf{C}$
61. $\left\langle 2 \sin t, -\frac{2}{3} \cos 3t, \frac{1}{2} \sin 8t \right\rangle + \mathbf{C}$
63. $\frac{1}{3}e^{3t} \mathbf{i} + \tan^{-1} t \mathbf{j} - \sqrt{2t} \mathbf{k} + \mathbf{C}$
65. $\mathbf{r}(t) = \langle e^t + 1, 3 - \cos t, \tan t + 2 \rangle$
67. $\mathbf{r}(t) = \langle t + 3, t^2 + 2, t^3 - 6 \rangle$
69. $\mathbf{r}(t) = \left\langle \frac{1}{2}e^{2t} + \frac{1}{2}, 2e^{-t} + t - 1, t - 2e^t + 3 \right\rangle$
71. $\langle 2, 0, 2 \rangle$ 73. $\mathbf{i}$ 75. $\langle 0, 0, 0 \rangle$
77. $(e^2 + 1) \langle 1, 2, -1 \rangle$ 79. a. False b. True c. True
81. $\langle 2 - t, 3 - 2t, \pi/2 + t \rangle$ 83. $\langle 2 + 3t, 9 + 7t, 1 + 2t \rangle$
85. $(1, 0)$ 87. $(1, 0, 0)$ 89. $\mathbf{r}(t) = \langle a_1 t, a_2 t, a_3 t \rangle$ or $\mathbf{r}(t) = \langle a_1 e^{kt}, a_2 e^{kt}, a_3 e^{kt} \rangle$, where a_i and k are real numbers

Section 14.3 Exercises, pp. 892–896

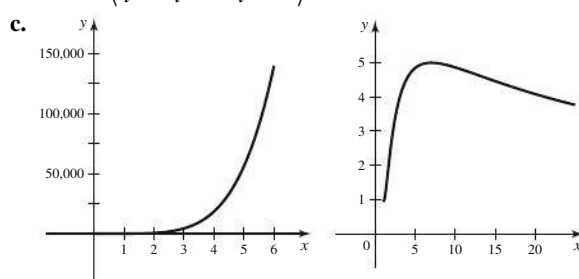
1. $\mathbf{v}(t) = \mathbf{r}'(t)$, speed $= |\mathbf{r}'(t)|$, $\mathbf{a}(t) = \mathbf{r}''(t)$ 3. $m\mathbf{a}(t) = \mathbf{F}$
5. $\mathbf{v}(t) = \int \mathbf{a}(t) dt = \langle v_1(t), v_2(t) \rangle + \mathbf{C}$. Use initial conditions to find $\mathbf{C}$.
7. a. $t = 3$ s b. $\mathbf{r}(t) = \langle 60t, -16t^2 + 96t + 3 \rangle$
9. a. $\langle 6t, 8t \rangle, 10t$ b. $\langle 6, 8 \rangle$ 11. a. $\mathbf{v}(t) = \langle 2, -4 \rangle$, $|\mathbf{v}(t)| = 2\sqrt{5}$ b. $\mathbf{a}(t) = \langle 0, 0 \rangle$ 13. a. $\mathbf{v}(t) = \langle 8 \cos t, -8 \sin t \rangle$, $|\mathbf{v}(t)| = 8$ b. $\mathbf{a}(t) = \langle -8 \sin t, -8 \cos t \rangle$ 15. a. $\langle 2t, 2t, t \rangle, 3t$ b. $\langle 2, 2, 1 \rangle$ 17. a. $\mathbf{v}(t) = \langle 1, -4, 6 \rangle$, $|\mathbf{v}(t)| = \sqrt{53}$ b. $\mathbf{a}(t) = \langle 0, 0, 0 \rangle$ 19. a. $\mathbf{v}(t) = \langle 0, 2t, -e^{-t} \rangle$, $|\mathbf{v}(t)| = \sqrt{4t^2 + e^{-2t}}$ b. $\mathbf{a}(t) = \langle 0, 2, e^{-t} \rangle$
21. a. $[c, d] = [0, 1]$ b. $\langle 1, 2t \rangle, \langle 2, 8t \rangle$



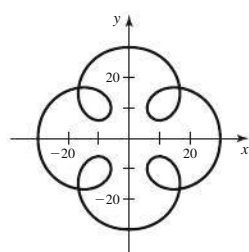
23. a. $[0, \frac{2\pi}{3}]$ b. $\mathbf{V}_r(t) = \langle -\sin t, 4 \cos t \rangle$, $\mathbf{V}_R(t) = \langle -3 \sin 3t, 12 \cos 3t \rangle$



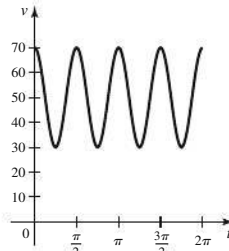
25. a. $[1, e^{36}]$ b. $\mathbf{V}_r(t) = \langle 2t, -8t^3, 18t^5 \rangle$, $\mathbf{V}_R(t) = \left\langle \frac{1}{t}, -\frac{4}{t} \ln t, \frac{9}{t} \ln^2 t \right\rangle$



27. a.



- b. $\langle -20 \sin t - 50 \sin 5t, 20 \cos t + 50 \cos 5t \rangle$

- c.  d. 70 ft/s; 30 ft/s

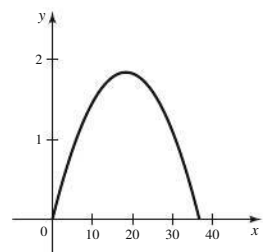
29. $\mathbf{r}(t)$ lies on a circle of radius 8;

$\langle -16 \sin 2t, 16 \cos 2t \rangle \cdot \langle 8 \cos 2t, 8 \sin 2t \rangle = 0$. 31. $\mathbf{r}(t)$ lies on a sphere of radius 2; $\langle \cos t - \sqrt{3} \sin t, \sqrt{3} \cos t + \sin t \rangle \cdot \langle \sin t + \sqrt{3} \cos t, \sqrt{3} \sin t - \cos t \rangle = 0$. 33. 5

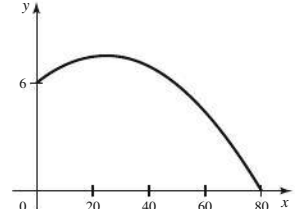
35. $\mathbf{v}(t) = \langle 2, t + 3 \rangle$, $\mathbf{r}(t) = \left\langle 2t, \frac{t^2}{2} + 3t \right\rangle$

37. $\mathbf{v}(t) = \langle 0, 10t + 5 \rangle$, $\mathbf{r}(t) = \langle 1, 5t^2 + 5t - 1 \rangle$

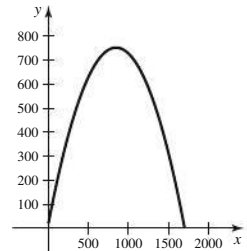
39. $\mathbf{v}(t) = \langle \sin t, -2 \cos t + 3 \rangle$,
 $\mathbf{r}(t) = \langle -\cos t + 2, -2 \sin t + 3t \rangle$

41. a. $\mathbf{v}(t) = \langle 30, -9.8t + 6 \rangle$, $\mathbf{r}(t) = \langle 30t, -4.9t^2 + 6t \rangle$
b.  c. $T \approx 1.22$ s, range ≈ 36.7 m
d. 1.84 m

43. a. $\mathbf{v}(t) = \langle 80, 10 - 32t \rangle$, $\mathbf{r}(t) = \langle 80t, -16t^2 + 10t + 6 \rangle$

b.  c. 1 s, 80 ft
d. Max height ≈ 7.56 ft

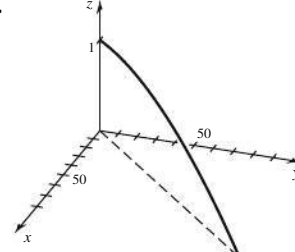
45. a. $\mathbf{v}(t) = \langle 125, -32t + 125\sqrt{3} \rangle$,
 $\mathbf{r}(t) = \langle 125t, -16t^2 + 125\sqrt{3}t + 20 \rangle$

b.  c. 13.6 s, 1702.5 ft d. 752.4 ft

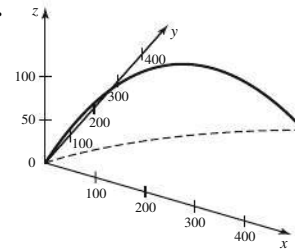
47. $\mathbf{v}(t) = \langle 1, 5, 10t \rangle$, $\mathbf{r}(t) = \langle t, 5t + 5, 5t^2 \rangle$

49. $\mathbf{v}(t) = \langle -\cos t + 1, \sin t + 2, t \rangle$,
 $\mathbf{r}(t) = \left\langle -\sin t + t, -\cos t + 2t + 1, \frac{t^2}{2} \right\rangle$

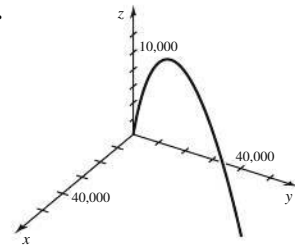
51. a. $\mathbf{v}(t) = \langle 200, 200, -9.8t \rangle$, $\mathbf{r}(t) = \langle 200t, 200t, -4.9t^2 + 1 \rangle$

b.  c. 0.452 s, 127.8 m d. 1 m

53. a. $\mathbf{v}(t) = \langle 60 + 10t, 80, 80 - 32t \rangle$,
 $\mathbf{r}(t) = \langle 60t + 5t^2, 80t, 80t - 16t^2 + 3 \rangle$

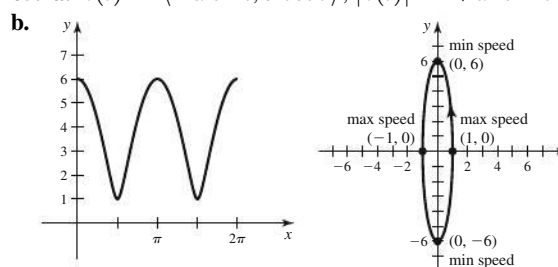
b.  c. 5.04 s, 589 ft
d. 103 ft

55. a. $\mathbf{v}(t) = \langle 300, 2.5t + 400, -9.8t + 500 \rangle$,
 $\mathbf{r}(t) = \langle 300t, 1.25t^2 + 400t, -4.9t^2 + 500t + 10 \rangle$

b.  c. 102.1 s, 61,941.5 m
d. 12,765.1 m

57. a. False b. True c. False d. True e. False f. True
g. True 59. 15.3 s, 1988.3 m, 287.0 m 61. 21.7 s, 4330.1 ft, 1875 ft
63. Approx. 27.4° and 62.6°

65. a. $\mathbf{v}(t) = \langle -a \sin t, b \cos t \rangle$; $|\mathbf{v}(t)| = \sqrt{a^2 \sin^2 t + b^2 \cos^2 t}$



c. Yes d. Max $\left\{ \frac{a}{b}, \frac{b}{a} \right\}$ 67. Approx. 23.5° or 59.6°

69. 113.4 ft/s 71. a. 1.2 ft, 0.46 s b. 0.88 ft/s c. 0.85 ft

d. More curve in the second half e. $c = 28.17$ ft/s²

73. $T = \frac{|\mathbf{v}_0| \sin \alpha + \sqrt{|\mathbf{v}_0|^2 \sin^2 \alpha + 2gy_0}}{g}$,

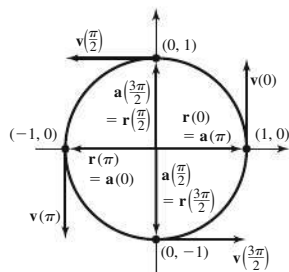
range = $|\mathbf{v}_0| (\cos \alpha) T$, max height = $y_0 + \frac{|\mathbf{v}_0|^2 \sin^2 \alpha}{2g}$

75. a. $\left[0, \frac{2\pi}{\omega} \right]$ b. $\mathbf{v}(t) = \langle -A\omega \sin \omega t, A\omega \cos \omega t \rangle$ is not constant;

$|\mathbf{v}(t)| = |A\omega|$ is constant. c. $\mathbf{a}(t) = \langle -A\omega^2 \cos \omega t, -A\omega^2 \sin \omega t \rangle$

d. $\mathbf{r}$ and $\mathbf{v}$ are orthogonal; $\mathbf{r}$ and $\mathbf{a}$ are in opposite directions.

e.



77. **a.** $\mathbf{r}(t) = \langle 5 \sin(\pi t/6), 5 \cos(\pi t/6) \rangle$

b. $\mathbf{r}(t) = \langle 5 \sin(\frac{1-e^{-t}}{5}), 5 \cos(\frac{1-e^{-t}}{5}) \rangle$

79. $\{(\cos t, \sin t, c \sin t) : t \in \mathbb{R}\}$ satisfies the equations $x^2 + y^2 = 1$ and $z - cy = 0$ so that $\langle \cos t, \sin t, c \sin t \rangle$ lies on the intersection of a right circular cylinder and a plane, which is an ellipse.

83. **a.** The direction of $\mathbf{r}$ does not change. **b.** Constant in direction, not in magnitude

Section 14.4 Exercises, pp. 900–902

1. $\sqrt{5}(b-a)$ 3. $\int_a^b |\mathbf{v}(t)| dt$ 5. 20π 7. If the parameter t used to describe a trajectory also measures the arc length s of the curve that is generated, we say the curve has been parameterized by its arc length.

9. 5 11. 3π 13. $\frac{\pi^2}{8}$ 15. $5\sqrt{34}$ 17. $4\pi\sqrt{65}$ 19. 9 21. $\frac{3}{2}$

23. $3t^2\sqrt{30}$; $64\sqrt{30}$ 25. 26; 26π

27. Approx. 66,626 mi/hr 29. 19.38

31. 32.50 33. Yes 35. No; $\mathbf{r}(s) = \left\langle \frac{s}{\sqrt{5}}, \frac{2s}{\sqrt{5}} \right\rangle, 0 \leq s \leq 3\sqrt{5}$

37. No; $\mathbf{r}(s) = \left\langle 2 \cos \frac{s}{2}, 2 \sin \frac{s}{2} \right\rangle, 0 \leq s \leq 4\pi$

39. No; $\mathbf{r}(s) = \langle \cos s, \sin s \rangle, 0 \leq s \leq \pi$

41. No; $\mathbf{r}(s) = \left\langle \frac{s}{\sqrt{3}} + 1, \frac{s}{\sqrt{3}} + 1, \frac{s}{\sqrt{3}} + 1 \right\rangle, s \geq 0$

43. **a.** True **b.** True **c.** True **d.** False 45. **a.** If $a^2 = b^2 + c^2$, then $|\mathbf{r}(t)|^2 = (a \cos t)^2 + (b \sin t)^2 + (c \sin t)^2 = a^2$ so that $\mathbf{r}(t)$ is a circle centered at the origin of radius $|a|$. **b.** $2\pi a$

c. If $a^2 + c^2 + e^2 = b^2 + d^2 + f^2$ and $ab + cd + ef = 0$, then $\mathbf{r}(t)$ is a circle of radius $\sqrt{a^2 + c^2 + e^2}$ and its arc length is $2\pi\sqrt{a^2 + c^2 + e^2}$. 47. **a.** $\int_a^b \sqrt{(Ah'(t))^2 + (Bh'(t))^2} dt$

$$= \int_a^b \sqrt{(A^2 + B^2)(h'(t))^2} dt = \sqrt{A^2 + B^2} \int_a^b |h'(t)| dt$$

b. $64\sqrt{29}$ **c.** $\frac{7\sqrt{29}}{4}$ 49. **a.** 5.102 s

b. $\int_0^{5.102} \sqrt{400 + (25 - 9.8t)^2} dt$ **c.** 124.43 m **d.** 102.04 m

51. $|\mathbf{v}(t)| = \sqrt{a^2 + b^2 + c^2} = 1$, if $a^2 + b^2 + c^2 = 1$

$$53. \int_a^b |\mathbf{r}'(t)| dt = \int_a^b \sqrt{(cf'(t))^2 + (cg'(t))^2} dt$$

$$= |c| \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2} dt = |c|L$$

Section 14.5 Exercises, pp. 913–915

1. 0 3. $\kappa = \frac{1}{|\mathbf{v}|} \left| \frac{d\mathbf{T}}{dt} \right|$ or $\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$ 5. $\mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

7. These three unit vectors are mutually orthogonal at all points of the curve. 9. The torsion measures the rate at which the curve rises or

twists out of the $\mathbf{TN}$ -plane at a point. 11. $\mathbf{T} = \frac{\langle 1, 2, 3 \rangle}{\sqrt{14}}, \kappa = 0$

13. $\mathbf{T} = \frac{\langle 1, 2 \cos t, -2 \sin t \rangle}{\sqrt{5}}, \kappa = \frac{1}{5}$

15. $\mathbf{T} = \frac{\langle \sqrt{3} \cos t, \cos t, -2 \sin t \rangle}{2}, \kappa = \frac{1}{2}$

17. $\mathbf{T} = \frac{\langle 1, 4t \rangle}{\sqrt{1 + 16t^2}}, \kappa = \frac{4}{(1 + 16t^2)^{3/2}}$

19. $\mathbf{T} = \left\langle \cos\left(\frac{\pi t^2}{2}\right), \sin\left(\frac{\pi t^2}{2}\right) \right\rangle, \kappa = \pi t$

21. $\frac{1}{3}$ 23. $\frac{2}{(4t^2 + 1)^{3/2}}$ 25. $\frac{2\sqrt{5}}{(20 \sin^2 t + \cos^2 t)^{3/2}}$

27. $\mathbf{T} = \langle \cos t, -\sin t \rangle, \mathbf{N} = \langle -\sin t, -\cos t \rangle$

29. $\mathbf{T} = \frac{\langle t, -3, 0 \rangle}{\sqrt{t^2 + 9}}, \mathbf{N} = \frac{\langle 3, t, 0 \rangle}{\sqrt{t^2 + 9}}$

31. $\mathbf{T} = \langle -\sin t^2, \cos t^2 \rangle, \mathbf{N} = \langle -\cos t^2, -\sin t^2 \rangle$

33. $\mathbf{T} = \frac{\langle 2t, 1 \rangle}{\sqrt{4t^2 + 1}}, \mathbf{N} = \frac{\langle 1, -2t \rangle}{\sqrt{4t^2 + 1}}$ 35. $a_N = a_T = 0$

37. $a_T = \sqrt{3}e^t; a_N = \sqrt{2}e^t$ 39. $\mathbf{a} = \frac{6t}{\sqrt{9t^2 + 4}}\mathbf{N} + \frac{18t^2 + 4}{\sqrt{9t^2 + 4}}\mathbf{T}$

41. $\mathbf{B}(t) = \langle 0, 0, -1 \rangle, \tau = 0$ 43. $\mathbf{B}(t) = \langle 0, 0, 1 \rangle, \tau = 0$

45. $\mathbf{B}(t) = \frac{\langle -\sin t, \cos t, 2 \rangle}{\sqrt{5}}, \tau = -\frac{1}{5}$

47. $\mathbf{B}(t) = \frac{\langle 5, 12 \sin t, -12 \cos t \rangle}{13}, \tau = \frac{12}{169}$ 49. **a.** False

b. False **c.** False **d.** True **e.** False **f.** False **g.** False

51. $\kappa = \frac{2}{(1 + 4x^2)^{3/2}}$ 53. $\kappa = \frac{x}{(x^2 + 1)^{3/2}}$

57. $\kappa = \frac{|ab|}{(a^2 \cos^2 t + b^2 \sin^2 t)^{3/2}}$ 59. $\kappa = \frac{2|a|}{(1 + 4a^2 t^2)^{3/2}}$

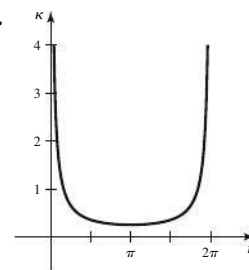
61. **b.** $\mathbf{v}_A(t) = \langle 1, 2, 3 \rangle, \mathbf{a}_A(t) = \langle 0, 0, 0 \rangle$ and $\mathbf{v}_B(t) = \langle 2t, 4t, 6t \rangle, \mathbf{a}_B(t) = \langle 2, 4, 6 \rangle$; A has constant velocity and zero acceleration, while B has increasing speed and constant acceleration.

c. $\mathbf{a}_A(t) = 0\mathbf{N} + 0\mathbf{T}, \mathbf{a}_B(t) = 0\mathbf{N} + 2\sqrt{14}\mathbf{T}$; both normal components are zero since the path is a straight line ($\kappa = 0$).

63. **b.** $\mathbf{v}_A(t) = \langle -\sin t, \cos t \rangle, \mathbf{a}_A(t) = \langle -\cos t, -\sin t \rangle$
 $\mathbf{v}_B(t) = \langle -2t \sin t^2, 2t \cos t^2 \rangle$
 $\mathbf{a}_B(t) = \langle -4t^2 \cos t^2 - 2 \sin t^2, -4t^2 \sin t^2 + 2 \cos t^2 \rangle$

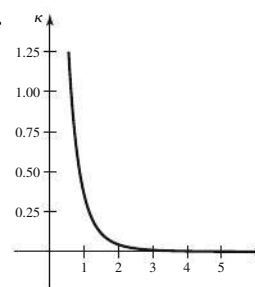
c. $\mathbf{a}_A(t) = \mathbf{N} + 0\mathbf{T}, \mathbf{a}_B(t) = 4t^2\mathbf{N} + 2\mathbf{T}$; for A , the acceleration is always normal to the curve, but this is not true for B .

65. **b.** $\kappa = \frac{1}{2\sqrt{2}(1 - \cos t)}$ **c.**



d. Minimum curvature at $t = \pi$

67. **b.** $\kappa = \frac{1}{t(1 + t^2)^{3/2}}$ **c.**



d. No maximum or minimum curvature

$$69. \kappa = \frac{e^x}{(1 + e^{2x})^{3/2}}, \left(-\frac{\ln 2}{2}, \frac{1}{\sqrt{2}} \right), \frac{2\sqrt{3}}{9}$$

$$71. \frac{1}{\kappa} = \frac{1}{2}; x^2 + \left(y - \frac{1}{2} \right)^2 = \frac{1}{4}$$

$$73. \frac{1}{\kappa} = 4; (x - \pi)^2 + (y + 2)^2 = 16$$

$$75. \kappa\left(\frac{\pi}{2n}\right) = n^2; \kappa \text{ increases as } n \text{ increases.}$$

$$77. \text{a. Speed} = \sqrt{V_0^2 - 2V_0 g t \sin \alpha + g^2 t^2}$$

$$\text{b. } \kappa(t) = \frac{g V_0 \cos \alpha}{(V_0^2 - 2V_0 g t \sin \alpha + g^2 t^2)^{3/2}}$$

$$\text{c. Speed has a minimum at } t = \frac{V_0 \sin \alpha}{g} \text{ and } \kappa(t) \text{ has a maximum at}$$

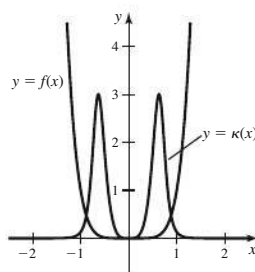
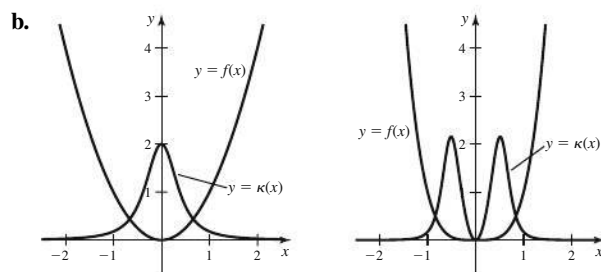
$$t = \frac{V_0 \sin \alpha}{g}. \quad 79. \kappa = \frac{1}{|\mathbf{v}|} \cdot \left| \frac{d\mathbf{T}}{dt} \right|, \text{ where } \mathbf{T} = \frac{\langle b, d, f \rangle}{\sqrt{b^2 + d^2 + f^2}}$$

and b, d , and f are constant. Therefore, $\frac{d\mathbf{T}}{dt} = \mathbf{0}$ so $\kappa = 0$.

$$81. \text{a. } \kappa_1(x) = \frac{2}{(1 + 4x^2)^{3/2}}$$

$$\kappa_2(x) = \frac{12x^2}{(1 + 16x^6)^{3/2}}$$

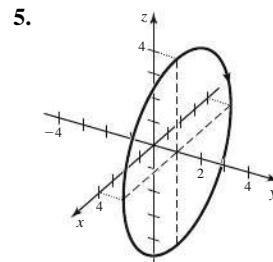
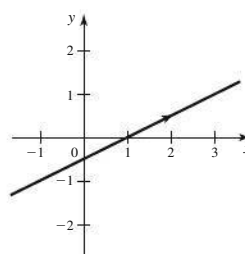
$$\kappa_3(x) = \frac{30x^4}{(1 + 36x^{10})^{3/2}}$$



c. κ_1 has its maximum at $x = 0$, κ_2 has its maxima at $x = \pm \sqrt[6]{\frac{1}{56}}$, and κ_3 has its maxima at $x = \pm \sqrt[10]{\frac{1}{99}}$. d. $\lim_{n \rightarrow \infty} z_n = 1$; the graphs of $y = f_n(x)$ show that as $n \rightarrow \infty$, the point corresponding to maximum curvature gets arbitrarily close to the point $(1, 0)$.

Chapter 14 Review Exercises, pp. 916–918

1. a. False b. True c. True d. True e. False f. False
3.



7. $x^2 + y^2 + z^2 = 2$; $y = z$; a tilted circle of radius $\sqrt{2}$

9. $\mathbf{r}(t) = \langle 4 + 15t, -2 - t, 3 - 5t \rangle$

11. $\mathbf{r}(t) = \langle 2, 3 \cos t, 4 \sin t \rangle$, for $0 \leq t \leq 2\pi$

13. $\mathbf{r}(t) = \langle \cos t, \sin t, \sin t \rangle$, for $0 \leq t \leq 2\pi$

15. $\mathbf{r}(t) = \langle 3 \cos t, \sin t, \sin t \rangle$, for $0 \leq t \leq 2\pi$

17. a. $\langle 1, 0 \rangle$; $\langle 0, 1 \rangle$ b. $\left\langle -\frac{2}{(2t+1)^2}, \frac{1}{(t+1)^2} \right\rangle$; $\langle -2, 1 \rangle$

c. $\left\langle \frac{8}{(2t+1)^3}, -\frac{2}{(t+1)^3} \right\rangle$

d. $\left\langle \frac{1}{2} \ln|2t+1|, t - \ln|t+1| \right\rangle + \mathbf{C}$

19. a. $\langle 0, 3, 0 \rangle$; does not exist

b. $\langle 2 \cos 2t, -12 \sin 4t, 1 \rangle$; $\langle 2, 0, 1 \rangle$ c. $\langle -4 \sin 2t, -48 \cos 4t, 0 \rangle$

d. $\left\langle -\frac{1}{2} \cos 2t, \frac{3}{4} \sin 4t, \frac{1}{2} t^2 \right\rangle + \mathbf{C}$ 21. $2\mathbf{j} + \pi\mathbf{k}$

23. $23\mathbf{i} - 41\mathbf{k}$ 25. $\mathbf{r}(t) = \left\langle t + 2, -\frac{1}{2} \cos 2t + \frac{5}{2}, \tan t + 2 \right\rangle$

27. $\mathbf{r}(t) = \langle 4 \tan^{-1} t - \pi, t^2 + t - 2, t^3 - 1 \rangle$

29. $\mathbf{T}(t) = \left\langle \frac{2e^t}{2e^{2t} + 1}, \frac{2e^{2t}}{2e^{2t} + 1}, \frac{1}{2e^{2t} + 1} \right\rangle$; $\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle$

31. a. $\langle 4e^{4t}, 4e^{4t}, 2e^{4t} \rangle$; $6e^{4t}$ b. $\langle 16e^{4t}, 16e^{4t}, 8e^{4t} \rangle$

33. $\mathbf{v}(t) = \langle 2 + \sin t, 3 - 2 \cos t \rangle$;

$\mathbf{r}(t) = \langle 2t + 2 - \cos t, 3t + 2 - 2 \sin t \rangle$

35. a. $\mathbf{v}(t) = \langle 40, -32t + 40\sqrt{3} \rangle$;

$\mathbf{r}(t) = \langle 40t, -16t^2 + 40\sqrt{3}t + 3 \rangle$

b. Approx. 4.37 s; approx. 174.9 ft c. 78 ft

37. a. $\mathbf{v}(t) = \langle 4t + 40, 20, 40 - 32t \rangle$;

$\mathbf{r}(t) = \langle 2t^2 + 40t, 20t, -16t^2 + 40t + 2 \rangle$ b. 2.549 s

c. 126 ft 39. a. $(116, 30)$ b. 39.1 ft c. 2.315 s

d. $\int_0^{2.315} \sqrt{50^2 + (-32t + 50)^2} dt$ e. 129 ft f. 41.4° to 79.4°

41. $(1.47, 3.15, 4.4)$ 43. 12 45. Approx. 6.42

47. a. $\mathbf{v}(t) = \mathbf{i} + t\sqrt{2}\mathbf{j} + t^2\mathbf{k}$ b. 12

49. $\mathbf{r}(s) = \left\langle (\sqrt{1+s} - 1)^2, \frac{4\sqrt{2}}{3}(\sqrt{1+s} - 1)^{3/2}, \right.$

$\left. 2(\sqrt{1+s} - 1) \right\rangle$, for $s \geq 0$ 51. a. $\mathbf{v} = \langle -6 \sin t, 3 \cos t \rangle$,

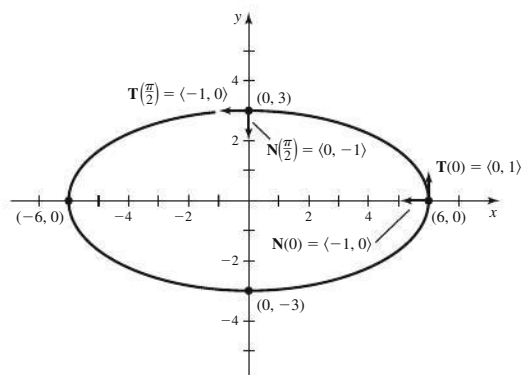
$\mathbf{T} = \frac{\langle -2 \sin t, \cos t \rangle}{\sqrt{1 + 3 \sin^2 t}}$ b. $\kappa(t) = \frac{2}{3(1 + 3 \sin^2 t)^{3/2}}$

c. $\mathbf{N} = \left\langle -\frac{\cos t}{\sqrt{1 + 3 \sin^2 t}}, -\frac{2 \sin t}{\sqrt{1 + 3 \sin^2 t}} \right\rangle$

d. $|\mathbf{N}| = \sqrt{\frac{\cos^2 t + 4 \sin^2 t}{1 + 3 \sin^2 t}} = \sqrt{\frac{(\cos^2 t + \sin^2 t) + 3 \sin^2 t}{1 + 3 \sin^2 t}} = 1$;

$\mathbf{T} \cdot \mathbf{N} = \frac{2 \sin t \cos t - 2 \sin t \cos t}{1 + 3 \sin^2 t} = 0$

e.



53. a. $\mathbf{v}(t) = \langle -\sin t, -2 \sin t, \sqrt{5} \cos t \rangle$,

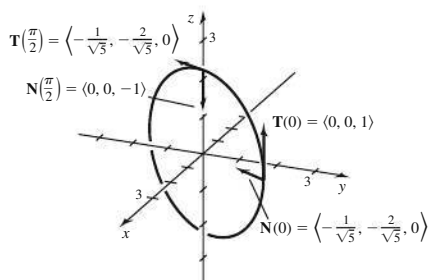
$\mathbf{T}(t) = \left\langle -\frac{1}{\sqrt{5}} \sin t, -\frac{2}{\sqrt{5}} \sin t, \cos t \right\rangle$ b. $\kappa(t) = \frac{1}{\sqrt{5}}$

c. $\mathbf{N}(t) = \left\langle -\frac{1}{\sqrt{5}} \cos t, -\frac{2}{\sqrt{5}} \cos t, -\sin t \right\rangle$

d. $|\mathbf{N}(t)| = \sqrt{\frac{1}{5} \cos^2 t + \frac{4}{5} \cos^2 t + \sin^2 t} = 1$;

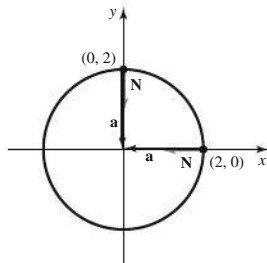
$\mathbf{T} \cdot \mathbf{N} = \left(\frac{1}{5} \cos t \sin t + \frac{4}{5} \cos t \sin t \right) - \sin t \cos t = 0$

e.



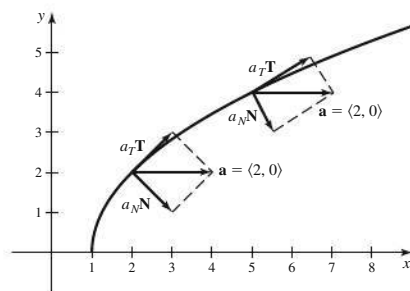
55. a. $\mathbf{a}(t) = 2\mathbf{N} + 0\mathbf{T} = 2\langle -\cos t, -\sin t \rangle$

b.



57. a. $a_T = \frac{2t}{\sqrt{t^2 + 1}}$ and $a_N = \frac{2}{\sqrt{t^2 + 1}}$

b.

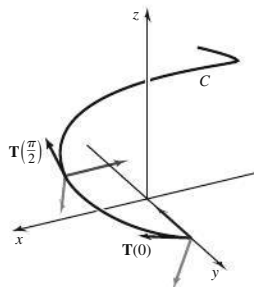


59. $\mathbf{B}(1) = \frac{\langle 3, -3, 1 \rangle}{\sqrt{19}}$; $\tau = \frac{3}{19}$

61. a. $\mathbf{T}(t) = \frac{1}{5} \langle 3 \cos t, -3 \sin t, 4 \rangle$

b. $\mathbf{N}(t) = \langle -\sin t, -\cos t, 0 \rangle$; $\kappa = \frac{3}{25}$

c.



d. Yes

e. $\mathbf{B}(t) = \frac{1}{5} \langle 4 \cos t, -4 \sin t, -3 \rangle$

f. See graph in part (c).

g. Check that $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$ have unit length and are mutually orthogonal.

h. $\tau = -\frac{4}{25}$

63. a. Consider first the case where $a_3 = b_3 = c_3 = 0$, and show that for all $s \neq t$ in I , $\mathbf{r}(t) \times \mathbf{r}(s)$ is a multiple of the constant vector $\langle b_1c_2 - b_2c_1, a_2c_1 - a_1c_2, a_1b_2 - a_2b_1 \rangle$, which implies $\mathbf{r}(t) \times \mathbf{r}(s)$ is always orthogonal to the same vector, and therefore the vectors $\mathbf{r}(t)$ must all lie in the same plane. When a_3 , b_3 , and c_3 are not necessarily 0, the curve still lies in a plane because these constants represent a simple translation of the curve to a different location in $\mathbb{R}^3$.

b. Because the curve lies in a plane, $\mathbf{B}$ is always normal to the plane

and has length 1. Therefore, $\frac{d\mathbf{B}}{ds} = \mathbf{0}$ and $\tau = 0$.

CHAPTER 15

Section 15.1 Exercises, pp. 927–930

1. Independent: x and y ; dependent: z

3. $D = \{(x, y): x \neq 0 \text{ and } y \neq 0\}$ 5. Three 7. 3; 4

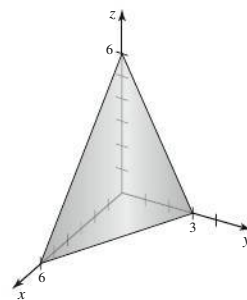
9. a. 1300 ft b. Katie; Katie is 100 ft higher than Zeke. 11. Circles

13. $n = 6$ 15. $D = \mathbb{R}^2$ 17. $D = \{(x, y): x^2 + y^2 \leq 25\}$

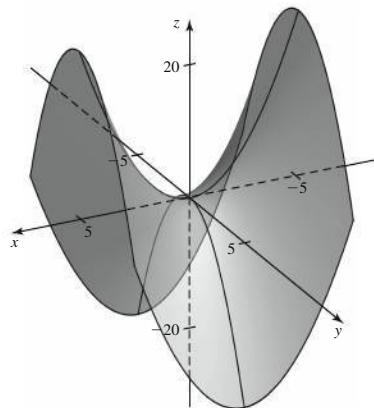
19. $D = \{(x, y): y \neq 0\}$ 21. $D = \{(x, y): y < x^2\}$

23. $D = \{(x, y): xy \geq 0, (x, y) \neq (0, 0)\}$

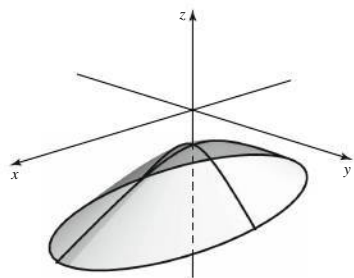
25. Plane; $D = \mathbb{R}^2, R = \mathbb{R}$



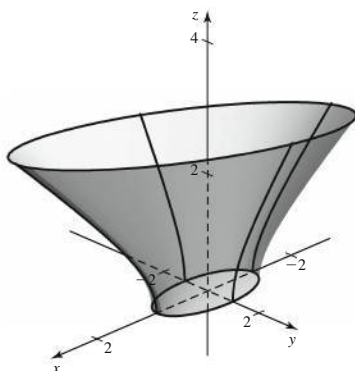
27. Hyperbolic paraboloid; $D = \mathbb{R}^2, R = \mathbb{R}$



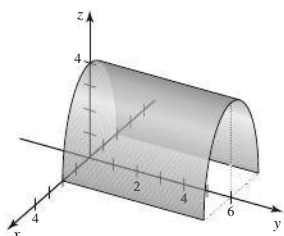
29. Lower part of a hyperboloid of two sheets;
 $D = \mathbb{R}^2, R = (-\infty, -1]$



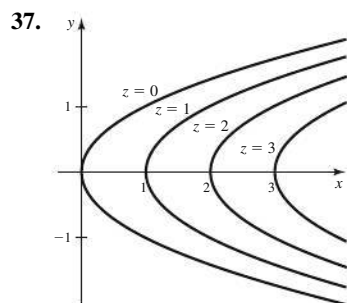
31. Upper half of a hyperboloid of one sheet;
 $D = \{(x, y): x^2 + y^2 \geq 1\}, R = [0, \infty)$



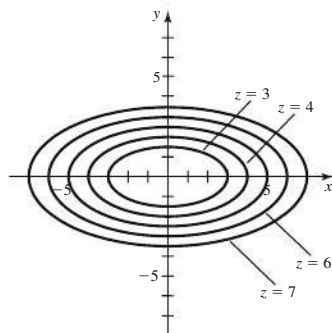
33. Upper half of an elliptical cylinder;
 $D = \{(x, y): -2 \leq x \leq 2, -\infty < y < \infty\}, R = [0, 4]$



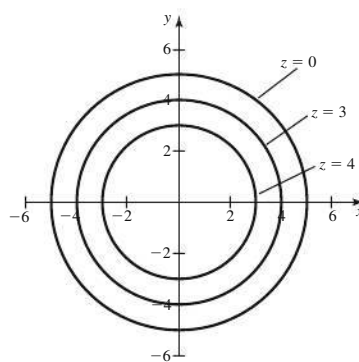
35. a. A b. D c. B d. C



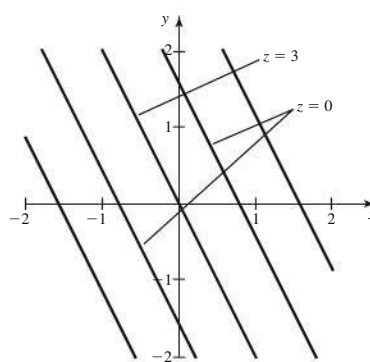
39.



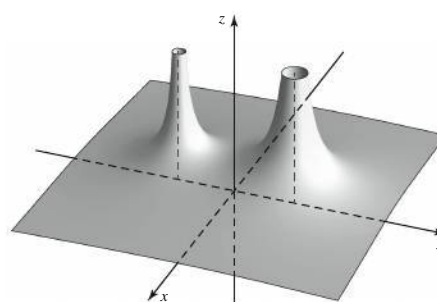
41.



43.

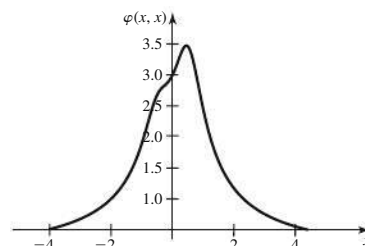


45. a.

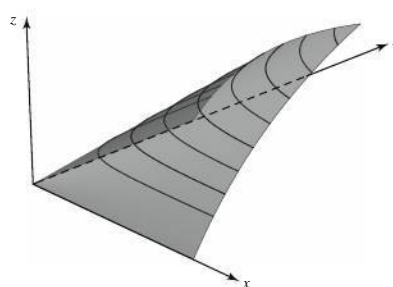


b. $\mathbb{R}^2$ excluding the points $(0, 1)$ and $(0, -1)$

c. $\varphi(2, 3)$ is greater. d.

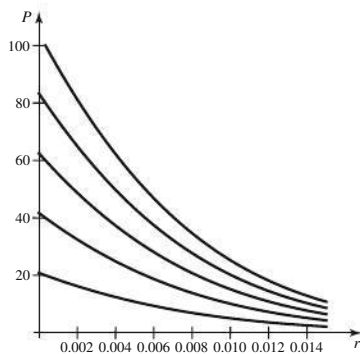


47. a.



b. $R(10, 10) = 5$
c. $R(x, y) = R(y, x)$

49. a. $P = \frac{20,000r}{(1+r)^{240} - 1}$ b. $P = \frac{Br}{(1+r)^{240} - 1}$, with $B = 5000, 10,000, 15,000, 25,000$

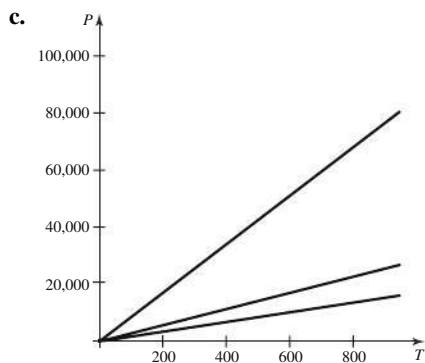
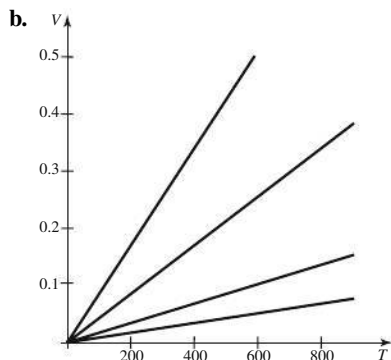
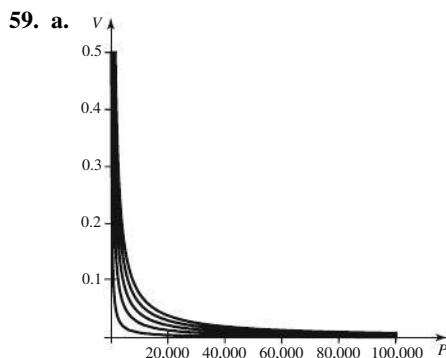


51. $D = \{(x, y, z): x \neq z\}$; all points not on the plane $x = z$

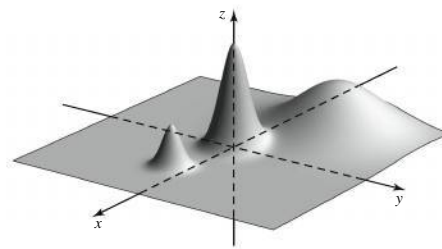
53. $D = \{(x, y, z): y \geq z\}$; all points on or below the plane $y = z$

55. $D = \{(x, y, z): x^2 \leq y\}$; all points on the side of the vertical cylinder $y = x^2$ that contains the positive y -axis

57. a. False b. False c. True



61. a.

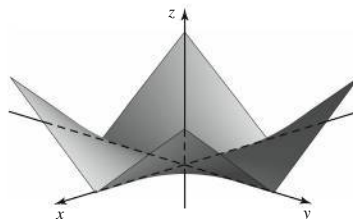


b. $(0, 0), (-5, 3), (4, -1)$

c. $f(0, 0) = 10.17, f(-5, 3) = 5.00, f(4, -1) = 4.00$

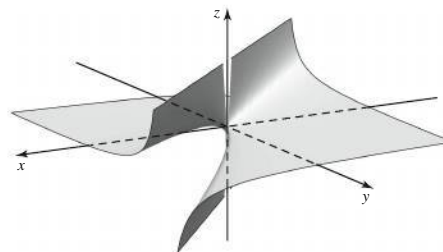
63. a. $D = \mathbb{R}^2, R = [0, \infty)$

b.

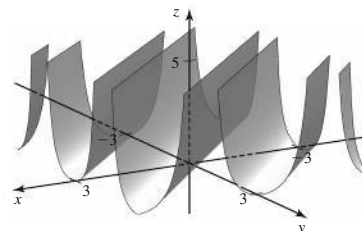


65. a. $D = \{(x, y): x \neq y\}, R = \mathbb{R}$

b.



67. a. $D = \{(x, y): y \neq x + \pi/2 + n\pi \text{ for any integer } n\}, R = [0, \infty)$ b.



69. Peak at the origin 71. Depression at $(1, 0)$ 73. The level curves are $ax + by = d - cz_0$, where z_0 is a constant, which are lines with slope $-a/b$ if $b \neq 0$ or vertical lines if $b = 0$.

75. $z = x^2 + y^2 - C$; paraboloids with vertices at $(0, 0, -C)$

77. $x^2 + 2z^2 = C$; elliptic cylinders parallel to the y -axis

79. $D = \{(x, y): x - 1 \leq y \leq x + 1\}$

81. $D = \{(x, y, z): (x \leq z \text{ and } y \geq -z) \text{ or } (x \geq z \text{ and } y \leq -z)\}$

Section 15.2 Exercises, pp. 937–939

1. The values of $f(x, y)$ are arbitrarily close to L for all (x, y) sufficiently close to (a, b) .
3. Because polynomials of n variables are continuous on all of $\mathbb{R}^n$, limits of polynomials can be evaluated with direct substitution.
5. If the function approaches different values along different paths, the limit does not exist.
7. f must be defined, the limit must exist, and the limit must equal the function value.
9. At any point where the denominator is nonzero
11. 10
13. 101
15. 27
17. $1/(2\pi)$
19. 2
21. 6
23. -1
25. 2
27. $1/(2\sqrt{2}) = \sqrt{2}/4$
29. $L = 1$ along $y = 0$, and $L = -1$ along $x = 0$
31. $L = 1$ along $x = 0$, and $L = -2$ along $y = 0$
33. $L = 2$ along $y = x$, and $L = 0$ along $y = -x$
35. $\mathbb{R}^2$
37. All points except $(0, 0)$
39. $\{(x, y): x \neq 0\}$
41. All points except $(0, 0)$
43. $\mathbb{R}^2$
45. $\mathbb{R}^2$
47. $\mathbb{R}^2$
49. All points except $(0, 0)$
51. $\mathbb{R}^2$
53. $\mathbb{R}^2$
55. 6
57. -1
59. 2
61. a. False

b. False c. True d. False 63. $\frac{1}{2}$ 65. 0 67. Does not exist
69. $\frac{1}{4}$ 71. 1 73. 1 75. 5 77. $b = 1$ 79. 0 81. 1 85. 0

Section 15.3 Exercises, pp. 948–951

1. $f_x(a, b)$ is the slope of the surface in the direction parallel to the positive x -axis, $f_y(a, b)$ is the slope of the surface in the direction parallel to the positive y -axis, both taken at (a, b) . 3. a. Negative

b. Negative c. Negative d. Positive 5. $f_x(x, y) = 6xy$;
 $f_y(x, y) = 3x^2$ 7. $f_{xy} = 0 = f_{yx}$ 9. $f_x(x, y, z) = y + z$;
 $f_y(x, y, z) = x + z$; $f_z(x, y, z) = x + y$

11. $f_x(x, y) = 5y$; $f_y(x, y) = 5x$ 13. $f_x(x, y) = \frac{1}{y}$; $f_y(x, y) = -\frac{x}{y^2}$

15. $f_x(x, y) = e^y$; $f_y(x, y) = xe^y$

17. $f_x(x, y) = 2xye^{x^2y}$; $f_y(x, y) = x^2e^{x^2y}$

19. $f_w(w, z) = \frac{z^2 - w^2}{(w^2 + z^2)^2}$; $f_z(w, z) = -\frac{2wz}{(w^2 + z^2)^2}$

21. $f_x(x, y) = \cos xy - xy \sin xy$; $f_y(x, y) = -x^2 \sin xy$

23. $s_y(y, z) = z^3 \sec^2 yz$; $s_z(y, z) = 2z \tan yz + yz^2 \sec^2 yz$

25. $G_s(s, t) = \frac{\sqrt{st}(t-s)}{2s(s+t)^2}$; $G_t(s, t) = \frac{\sqrt{st}(s-t)}{2t(s+t)^2}$

27. $f_x(x, y) = 2yx^{2y-1}$; $f_y(x, y) = 2x^{2y} \ln x$

29. $h_x(x, y) = \frac{\sqrt{x^2 - 4y} - x}{\sqrt{x^2 - 4y}}$; $h_y(x, y) = \frac{2}{\sqrt{x^2 - 4y}}$

31. $f_x(x, y) = -e^{x^2}$; $f_y(x, y) = 3y^2e^{y^3}$

33. $f_x(x, y) = -\frac{2x}{1 + (x^2 + y^2)^2}$; $f_y(x, y) = -\frac{2y}{1 + (x^2 + y^2)^2}$

35. $h_x(x, y) = (1 + 2y)^x \ln(1 + 2y)$; $h_y(x, y) = 2x(1 + 2y)^{x-1}$

37. $f_x(x, y) = -h(x)$; $f_y(x, y) = h(y)$

39. $h_{xx}(x, y) = 6x$; $h_{xy}(x, y) = 2y = h_{yx}(x, y)$; $h_{yy}(x, y) = 2x$

41. $f_{xx}(x, y) = -16y^3 \sin 4x$; $f_{xy}(x, y) = 12y^2 \cos 4x = f_{yx}(x, y)$;
 $f_{yy}(x, y) = 6y \sin 4x$

43. $p_{uu}(u, v) = \frac{-2u^2 + 2v^2 + 8}{(u^2 + v^2 + 4)^2}$;

$$p_{uv}(u, v) = -\frac{4uv}{(u^2 + v^2 + 4)^2} = p_{vu}(u, v);$$

$$p_{vv}(u, v) = \frac{2u^2 - 2v^2 + 8}{(u^2 + v^2 + 4)^2}$$

45. $F_{rr}(r, s) = 0$; $F_{rs}(r, s) = e^s = F_{sr}(r, s)$; $F_{ss}(r, s) = re^s$

47. $f_{xx}(x, y) = \frac{6xy^2(1 - 2x^6y^4)}{(1 + x^6y^4)^2}$;

$$f_{xy}(x, y) = \frac{6x^2y(1 - x^6y^4)}{(1 + x^6y^4)^2} = f_{yx}(x, y);$$

$$f_{yy}(x, y) = \frac{2x^3(1 - 3x^6y^4)}{(1 + x^6y^4)^2}$$

49. $f_{xy} = e^{x+y} = f_{yx}$ 51. $f_{xy} = -(xy \cos xy + \sin xy) = f_{yx}$

53. $f_{xy}(x, y) = -72y^2(2x - y^3)^2 = f_{yx}(x, y)$

55. $h_x(x, y, z) = h_y(x, y, z) = h_z(x, y, z) = -\sin(x + y + z)$

57. $F_u(u, v, w) = \frac{1}{v + w}$; $F_v(u, v, w) = F_w(u, v, w) = -\frac{u}{(v + w)^2}$

59. $G_r(r, s, t) = \frac{s^3t^5}{2\sqrt{rs^3t^5}}$;

$$G_s(r, s, t) = \frac{3rs^2t^5}{2\sqrt{rs^3t^5}}$$

$$G_t(r, s, t) = \frac{5rs^3t^4}{2\sqrt{rs^3t^5}}$$

61. $h_w(w, x, y, z) = \frac{z}{xy}$; $h_x(w, x, y, z) = -\frac{wz}{x^2y}$;

$$h_y(w, x, y, z) = -\frac{wz}{xy^2}$$
; $h_z(w, x, y, z) = \frac{w}{xy}$

63. b. $g_x(x, y, z) = -\frac{8z}{3(2x - y + z)^2}$;

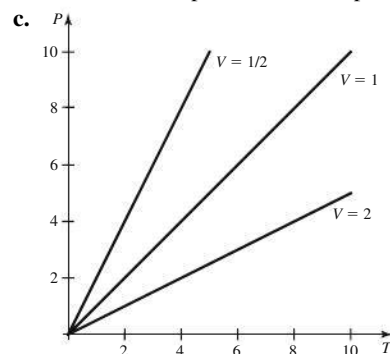
$$g_y(x, y, z) = \frac{4z}{3(2x - y + z)^2}$$

$$g_z(x, y, z) = \frac{4(2x - y)}{3(2x - y + z)^2}$$

65. 1.41 67. 1.55 (answer will vary) 69. a. $\frac{\partial V}{\partial P} = -\frac{kT}{P^2}$; volume

decreases with pressure at fixed temperature. b. $\frac{\partial V}{\partial T} = \frac{k}{P}$; volume

increases with temperature at fixed pressure.



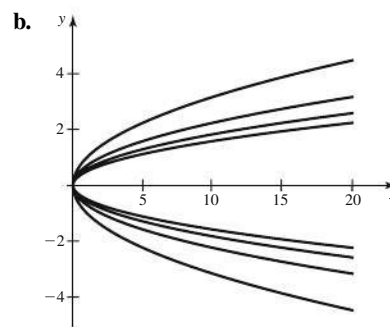
71. a. $\frac{\partial R}{\partial R_1} = \frac{R_2^2}{(R_1 + R_2)^2}$; $\frac{\partial R}{\partial R_2} = \frac{R_1^2}{(R_1 + R_2)^2}$

b. $\frac{\partial R}{\partial R_1} = \frac{R^2}{R_1^2}$; $\frac{\partial R}{\partial R_2} = \frac{R^2}{R_2^2}$ c. Increase d. Decrease

73. $u_t = -16e^{-4t} \cos 2x = u_{xx}$ 75. $u_t = -a^2 A e^{-a^2 t} \cos ax = u_{xx}$

77. a. No b. No c. $f_x(0, 0) = f_y(0, 0) = 0$ d. f_x and f_y are not continuous at $(0, 0)$. 79. a. False b. False c. True

81. a. $z_x(x, y) = \frac{1}{y^2}$; $z_y(x, y) = -\frac{2x}{y^3}$



c. z increases as x increases.

d. z increases as y increases when $y < 0$, z is undefined for $y = 0$, and z decreases as y increases for $y > 0$.

83. a. $\varphi_x(x, y) = -\frac{2x}{(x^2 + (y - 1)^2)^{3/2}} - \frac{x}{(x^2 + (y + 1)^2)^{3/2}}$;

$$\varphi_y(x, y) = -\frac{2(y - 1)}{(x^2 + (y - 1)^2)^{3/2}} - \frac{y + 1}{(x^2 + (y + 1)^2)^{3/2}}$$

b. They both approach zero. c. $\varphi_x(0, y) = 0$

d. $\varphi_y(x, 0) = \frac{1}{(x^2 + 1)^{3/2}}$

87. $\frac{\partial^2 u}{\partial t^2} = -4c^2 \cos(2(x + ct)) = c^2 \frac{\partial^2 u}{\partial x^2}$

89. $\frac{\partial^2 u}{\partial t^2} = c^2 A f''(x + ct) + c^2 B g''(x - ct) = c^2 \frac{\partial^2 u}{\partial x^2}$

91. $u_{xx} = 6x$; $u_{yy} = -6x$

93. $u_{xx} = \frac{2(x-1)y}{((x-1)^2 + y^2)^2} - \frac{2(x+1)y}{((x+1)^2 + y^2)^2}$;

$u_{yy} = -\frac{2(x-1)y}{((x-1)^2 + y^2)^2} + \frac{2(x+1)y}{((x+1)^2 + y^2)^2}$

95. $\varepsilon_1 = \Delta y$, $\varepsilon_2 = 0$ or $\varepsilon_1 = 0$, $\varepsilon_2 = \Delta x$ 97. a. f is continuous at $(0, 0)$. b. f is not differentiable at $(0, 0)$.

c. $f_x(0, 0) = f_y(0, 0) = 0$ d. f_x and f_y are not continuous at $(0, 0)$.

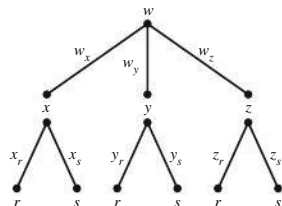
e. Theorem 15.5 does not apply because f_x and f_y are not continuous at $(0, 0)$; Theorem 15.6 does not apply because f is not differentiable at $(0, 0)$. 99. $f_x(x, y) = yh(xy)$; $f_y(x, y) = xh(xy)$

Section 15.4 Exercises, pp. 957–961

1. One dependent, two intermediate, and one independent variable

3. Multiply each of the partial derivatives of w by the t -derivative of the corresponding function and add these expressions.

5.



7. $4t^3 + 3t^2$

9. $z'(t) = 2t \sin 4t^3 + 12t^4 \cos 4t^3$

11. $w'(t) = -\sin t \sin 3t^4 + 12t^3 \cos t \cos 3t^4$

13. $z'(t) = 20(\sin^2 t + 2(3t + 4)^5)^9 (\sin t \cos t + 15(3t + 4)^4)$

15. $w'(t) = 20t^4 \sin(t + 1) + 4t^5 \cos(t + 1)$

17. $V'(t) = e^t((2t + 5) \sin t + (2t + 3) \cos t)$

19. $z_s = 2(s - t) \sin t^2$; $z_t = 2(s - t)(t(s - t) \cos t^2 - \sin t^2)$

21. $z_s = 2s - 3s^2 - 2st + t^2$; $z_t = -s^2 - 2t + 2st + 3t^2$

23. $z_s = (t + 1)e^{st+s+t}$; $z_t = (s + 1)e^{st+s+t}$

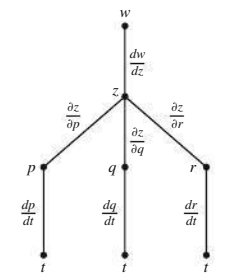
25. $w_s = -\frac{2t(t + 1)}{(st + s - t)^2}$; $w_t = \frac{2s}{(st + s - t)^2}$

27. a. $V'(t) = 2\pi r(t)h(t)r'(t) + \pi r(t)^2 h'(t)$ b. $V'(t) = 0$

c. The volume remains constant.

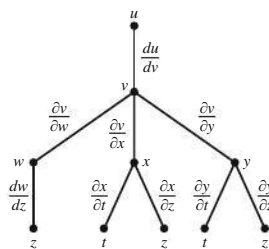
29. $z'(t) = -\frac{2t + 2}{(t + 2t)} - \frac{3t^2}{(t^3 - 2)}$

31.



$\frac{dw}{dt} = \frac{\partial w}{\partial p} \frac{dp}{dt} + \frac{\partial w}{\partial q} \frac{dq}{dt} + \frac{\partial w}{\partial r} \frac{dr}{dt}$

33.



$\frac{\partial u}{\partial z} = \frac{du}{dv} \left(\frac{\partial v}{\partial w} \frac{dw}{dz} + \frac{\partial v}{\partial x} \frac{dx}{dz} + \frac{\partial v}{\partial y} \frac{dy}{dz} \right)$

35. $\frac{dy}{dx} = \frac{x}{2y}$ 37. $\frac{dy}{dx} = -\frac{y}{x}$ 39. $\frac{dy}{dx} = -\frac{x + y}{2y^3 + x}$

41. $\frac{\partial s}{\partial x} = \frac{2x}{\sqrt{x^2 + y^2}}$; $\frac{\partial s}{\partial y} = \frac{2y}{\sqrt{x^2 + y^2}}$

43. $f_{ss} = 2(3s + t)$; $f_{st} = 2(s - t)$; $f_{tt} = -2(s + 3t)$

45. $f_{ss} = \frac{4t^2(-3s^2 + t^2)}{(s^2 + t^2)^3}$; $f_{st} = \frac{8st(s^2 - t^2)}{(s^2 + t^2)^3}$; $f_{tt} = -\frac{4(s^4 - 3s^2t^2)}{(s^2 + t^2)^3}$

47. $f''(s) = 4\left(\frac{6}{s^4} - \frac{2}{s^3} - 1 - 9s + 9s^2\right)$ 49. a. False b. False

51. $w'(t) = 0$ 53. $\frac{\partial z}{\partial x} = -\frac{z^2}{x^2}$ 55. a. $w'(t) = af_x + bf_y + cf_z$

b. $w'(t) = ayz + bxz + cxy = 3abct^2$

c. $w'(t) = \sqrt{a^2 + b^2 + c^2} \frac{t}{|t|}$

d. $w''(t) = a^2 f_{xx} + b^2 f_{yy} + c^2 f_{zz} + 2abf_{xy} + 2acf_{xz} + 2bcf_{yz}$

57. $\frac{\partial z}{\partial x} = -\frac{y + z}{x + y}$; $\frac{\partial z}{\partial y} = -\frac{x + z}{x + y}$ 59. $\frac{\partial z}{\partial x} = -\frac{yz + 1}{xy - 1}$;

$\frac{\partial z}{\partial y} = -\frac{xz + 1}{xy - 1}$ 61. a. $z'(t) = -2x \sin t + 8y \cos t = 3 \sin 2t$

b. $0 < t < \pi/2$ and $\pi < t < 3\pi/2$

63. a. $z'(t) = \frac{(x + y)e^{-t}}{\sqrt{1 - x^2 - y^2}} = \frac{2e^{-2t}}{\sqrt{1 - 2e^{-2t}}}$ b. All $t \geq \frac{1}{2} \ln 2$

65. $E'(t) = mx'x'' + my'y'' + mgy' = 0$

67. a. The volume increases. b. The volume decreases.

69. a. $\frac{\partial P}{\partial V} = -\frac{P}{V}$; $\frac{\partial T}{\partial P} = \frac{V}{k}$; $\frac{\partial V}{\partial T} = \frac{k}{P}$ b. Follows directly from part (a)

71. a. $w'(t) = \frac{2t(t^2 + 1) \cos 2t - (t^2 - 1) \sin 2t}{2(t^2 + 1)^2}$

b. Max value of $t \approx 0.838$, $(x, y, z) \approx (0.669, 0.743, 0.838)$

73. a. $z_x = \frac{x}{r} z_r - \frac{y}{r^2} z_\theta$; $z_y = \frac{y}{r} z_r + \frac{x}{r^2} z_\theta$

b. $z_{xx} = \frac{x^2}{r^2} z_{rr} + \frac{y^2}{r^4} z_{\theta\theta} - \frac{2xy}{r^3} z_{r\theta} + \frac{y^2}{r^3} z_r + \frac{2xy}{r^4} z_\theta$

c. $z_{yy} = \frac{y^2}{r^2} z_{rr} + \frac{x^2}{r^4} z_{\theta\theta} + \frac{2xy}{r^3} z_{r\theta} + \frac{x^2}{r^3} z_r - \frac{2xy}{r^4} z_\theta$

75. a. $\left(\frac{\partial z}{\partial x}\right)_y = -\frac{F_x}{F_z}$ b. $\left(\frac{\partial y}{\partial z}\right)_x = -\frac{F_z}{F_y}$; $\left(\frac{\partial x}{\partial y}\right)_z = -\frac{F_y}{F_x}$

d. $\left(\frac{\partial w}{\partial x}\right)_{y,z} \left(\frac{\partial z}{\partial w}\right)_{x,y} \left(\frac{\partial y}{\partial z}\right)_{x,w} \left(\frac{\partial x}{\partial y}\right)_{z,w} = 1$

77. a. $\left(\frac{\partial w}{\partial x}\right)_y = f_x + f_z \frac{dz}{dx} = 18$ b. $\left(\frac{\partial w}{\partial x}\right)_z = f_x + f_y \frac{dy}{dx} = 8$

d. $\left(\frac{\partial w}{\partial y}\right)_x = -5$; $\left(\frac{\partial w}{\partial y}\right)_z = 4$; $\left(\frac{\partial w}{\partial z}\right)_x = \frac{5}{2}$; $\left(\frac{\partial w}{\partial z}\right)_y = \frac{9}{2}$

Section 15.5 Exercises, pp. 970–973

1. Form the dot product between the unit direction vector $\mathbf{u}$ and the gradient of the function. 3. Direction of steepest ascent
5. The gradient is orthogonal to the level curves of f .
7. -2 9. -7; 0

11. a.

	$(a, b) = (2, 0)$	$(a, b) = (0, 2)$	$(a, b) = (1, 1)$
$\mathbf{u} = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$	$-\sqrt{2}$	$-2\sqrt{2}$	$-3\sqrt{2}/2$
$\mathbf{v} = \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$	$\sqrt{2}$	$-2\sqrt{2}$	$-\sqrt{2}/2$
$\mathbf{w} = \langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$	$\sqrt{2}$	$2\sqrt{2}$	$3\sqrt{2}/2$

b. The function is decreasing at $(2, 0)$ in the direction of $\mathbf{u}$ and increasing at $(2, 0)$ in the directions of $\mathbf{v}$ and $\mathbf{w}$.

13. $\nabla f(x, y) = \langle 6x, -10y \rangle$, $\nabla f(2, -1) = \langle 12, 10 \rangle$

15. $\nabla g(x, y) = \langle 2(x - 4xy - 4y^2), -4x(x + 4y) \rangle$,
 $\nabla g(-1, 2) = \langle -18, 28 \rangle$ 17. $\nabla f(x, y) = e^{2xy} \langle 1 + 2xy, 2x^2 \rangle$,

$\nabla f(1, 0) = \langle 1, 2 \rangle$ 19. $\nabla F(x, y) = -2e^{-x^2-2y^2} \langle x, 2y \rangle$,

$\nabla F(-1, 2) = 2e^{-9} \langle 1, -4 \rangle$ 21. -6 23. $\frac{27}{2} - 6\sqrt{3}$

25. $-\frac{2}{\sqrt{5}}$ 27. -2 29. 0 31. a. Direction of steepest ascent:

$\frac{1}{\sqrt{65}} \langle 1, 8 \rangle$; direction of steepest descent: $-\frac{1}{\sqrt{65}} \langle 1, 8 \rangle$

b. $\langle -8, 1 \rangle$ 33. a. Direction of steepest ascent: $\frac{1}{\sqrt{5}} \langle -2, 1 \rangle$;

direction of steepest descent: $\frac{1}{\sqrt{5}} \langle 2, -1 \rangle$ b. $\langle 1, 2 \rangle$

35. a. Direction of steepest ascent: $\frac{1}{\sqrt{2}} \langle 1, -1 \rangle$;

direction of steepest descent: $\frac{1}{\sqrt{2}} \langle -1, 1 \rangle$ b. $\langle 1, 1 \rangle$

37. a. $\nabla f(3, 2) = -12\mathbf{i} - 12\mathbf{j}$

b. Direction of max increase: $\theta = \frac{5\pi}{4}$; direction of max decrease:

$\theta = \frac{\pi}{4}$; directions of no change: $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

c. $g(\theta) = -12 \cos \theta - 12 \sin \theta$ d. $\theta = \frac{5\pi}{4}$, $g(\frac{5\pi}{4}) = 12\sqrt{2}$

e. $\nabla f(3, 2) = 12\sqrt{2} \langle \cos \frac{5\pi}{4}, \sin \frac{5\pi}{4} \rangle$, $|\nabla f(3, 2)| = 12\sqrt{2}$

39. a. $\nabla f(\sqrt{3}, 1) = \frac{\sqrt{6}}{6} \langle \sqrt{3}, 1 \rangle$ b. Direction of max increase:

$\theta = \frac{\pi}{6}$; direction of max decrease: $\theta = \frac{7\pi}{6}$; directions of no change:

$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$ c. $g(\theta) = \frac{\sqrt{2}}{2} \cos \theta + \frac{\sqrt{6}}{6} \sin \theta$ d. $\theta = \frac{\pi}{6}$, $g(\frac{\pi}{6}) = \frac{\sqrt{6}}{3}$

e. $\nabla f(\sqrt{3}, 1) = \frac{\sqrt{6}}{3} \langle \cos \frac{\pi}{6}, \sin \frac{\pi}{6} \rangle$, $|\nabla f(\sqrt{3}, 1)| = \frac{\sqrt{6}}{3}$

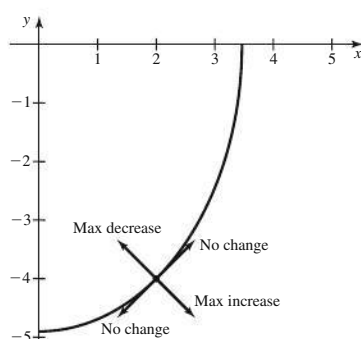
41. a. $\nabla F(-1, 0) = \frac{2}{e} \mathbf{i}$ b. Direction of max increase: $\theta = 0$; direction

of max decrease: $\theta = \pi$; directions of no change: $\theta = \pm \frac{\pi}{2}$

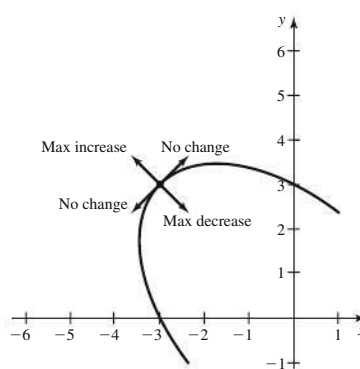
c. $g(\theta) = \frac{2}{e} \cos \theta$ d. $\theta = 0$, $g(0) = \frac{2}{e}$

e. $\nabla F(-1, 0) = \frac{2}{e} \langle \cos 0, \sin 0 \rangle$, $|\nabla F(-1, 0)| = \frac{2}{e}$

43.



45.



47. $y' = 0$

49. Vertical tangent

51. $y' = -2/\sqrt{3}$

53. Vertical tangent

55. a. $\nabla f = \langle 1, 0 \rangle$ b. $x = 4 - t$, $y = 4$, $t \geq 0$

c. $\mathbf{r}(t) = \langle 4 - t, 4, 8 - t \rangle$, for $t \geq 0$

57. a. $\nabla f = \langle -2x, -4y \rangle$ b. $y = x^2$, $x \geq 1$

c. $\mathbf{r}(t) = \langle t, t^2, 4 - t^2 - 2t^4 \rangle$, for $t \geq 1$

59. a. $\nabla f(x, y, z) = 2x\mathbf{i} + 4y\mathbf{j} + 8z\mathbf{k}$, $\nabla f(1, 0, 4) = 2\mathbf{i} + 32\mathbf{k}$

b. $\frac{1}{\sqrt{257}}(\mathbf{i} + 16\mathbf{k})$ c. $2\sqrt{257}$ d. $17\sqrt{2}$

61. a. $\nabla f(x, y, z) = 4yz\mathbf{i} + 4xz\mathbf{j} + 4xy\mathbf{k}$,

$\nabla f(1, -1, -1) = 4\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$ b. $\frac{1}{\sqrt{3}}(\mathbf{i} - \mathbf{j} - \mathbf{k})$ c. $4\sqrt{3}$

d. $\frac{4}{\sqrt{3}}$ 63. a. $\nabla f(x, y, z) = \cos(x + 2y - z)(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$,

$\nabla f(\frac{\pi}{6}, \frac{\pi}{6}, -\frac{\pi}{6}) = -\frac{1}{2}\mathbf{i} - \mathbf{j} + \frac{1}{2}\mathbf{k}$ b. $\frac{1}{\sqrt{6}}(-\mathbf{i} - 2\mathbf{j} + \mathbf{k})$

c. $\sqrt{6}/2$ d. $-\frac{1}{2}$

65. a. $\nabla f(x, y, z) = \frac{2}{1 + x^2 + y^2 + z^2}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$,

$\nabla f(1, 1, -1) = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} - \frac{1}{2}\mathbf{k}$ b. $\frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} - \mathbf{k})$

c. $\frac{\sqrt{3}}{2}$ d. $\frac{5}{6}$ 67. a. False b. False c. False d. True

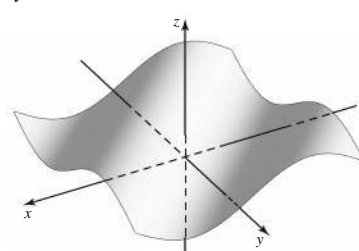
69. $\pm \frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$ 71. $\pm \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$

73. $x = x_0 + at$, $y = y_0 + bt$ 75. a. $\nabla f(x, y, z) = \langle 2x, 2y, 2z \rangle$,
 $\nabla f(1, 1, 1) = \langle 2, 2, 2 \rangle$ b. $x + y + z = 3$

77. a. $\nabla f(x, y, z) = e^{x+y-z} \langle 1, 1, -1 \rangle$, $\nabla f(1, 1, 2) = \langle 1, 1, -1 \rangle$

b. $x + y - z = 0$

79. a.



b. $\mathbf{v} = \pm \langle 1, 1 \rangle$

c. $\mathbf{v} = \pm \langle 1, -1 \rangle$

83. $\langle \mathbf{u}, \mathbf{v} \rangle = \langle \pi \cos \pi x \sin 2\pi y, 2\pi \sin \pi x \cos 2\pi y \rangle$

87. $\nabla f(x, y) = \frac{1}{(x^2 + y^2)^2} \langle y^2 - x^2 - 2xy, x^2 - y^2 - 2xy \rangle$

89. $\nabla f(x, y, z) = -\frac{1}{\sqrt{25 - x^2 - y^2 - z^2}} \langle x, y, z \rangle$

91. $\nabla f(x, y, z) = \frac{(y + xz) \langle 1, z, y \rangle - (x + yz) \langle z, 1, x \rangle}{(y + xz)^2}$
 $= \frac{1}{(y + xz)^2} \langle y(1 - z^2), x(z^2 - 1), y^2 - x^2 \rangle$

Section 15.6 Exercises, pp. 980–983

1. The gradient of f is a multiple of $\mathbf{n}$.
 3. $F_x(a, b, c)(x - a) + F_y(a, b, c)(y - b) + F_z(a, b, c)(z - c) = 0$
 5. Multiply the change in x by $f_x(a, b)$ and the change in y by $f_y(a, b)$, and add both terms to f . 7. $dz = f_x(x, y) dx + f_y(x, y) dy$
 9. $z = 5x - 3y + 5$ 11. $3x - y + 6z = 4$ 13. $2x + y + z = 4$; $4x + y + z = 7$ 15. $x + y + z = 6$; $3x + 4y + z = 12$
 17. $z = -8x - 4y + 16$ and $z = 4x + 2y + 7$ 19. $z = y + 1$ and $z = x + 1$ 21. $x + \frac{1}{2}y + \sqrt{3}z = 2 + \frac{\sqrt{3}\pi}{6}$ and $\frac{1}{2}x + y + \sqrt{3}z = \frac{5\sqrt{3}\pi}{6} - 2$ 23. $\frac{1}{2}x + \frac{2}{3}y + 2\sqrt{3}z = -2$ and $x - 2y + 2\sqrt{14}z = 2$ 25. $z = 8x - 4y - 4$ and $z = -x - y - 1$
 27. $z = \frac{7}{25}x - \frac{1}{25}y - \frac{2}{5}$ and $z = -\frac{7}{25}x + \frac{1}{25}y + \frac{6}{5}$
 29. $z = \frac{1}{2}x + \frac{1}{2}y + \frac{\pi}{4} - 1$
 31. $\frac{1}{6}(x - \pi) + \frac{\pi}{6}(y - 1) + \pi\left(z - \frac{1}{6}\right) = 0$
 33. a. $L(x, y) = 4x + y - 6$ b. $L(2.1, 2.99) = 5.39$
 35. a. $L(x, y) = -6x - 4y + 7$ b. $L(3.1, -1.04) = -7.44$
 37. a. $L(x, y, z) = x + y + 2z$ b. $L(0.1, -0.2, 0.2) = 0.3$
 39. $dz = -6dx - 5dy = -0.1$ 41. $dz = dx + dy = 0.05$
 43. a. The surface area decreases. b. Impossible to say
 c. $\Delta S \approx 53.3$ d. $\Delta S \approx 33.95$ e. $R dR = r dr$ 45. $\frac{\Delta A}{A} \approx 3.5\%$
 47. $dw = (y^2 + 2xz) dx + (2xy + z^2) dy + (x^2 + 2yz) dz$
 49. $dw = \frac{dx}{y + z} - \frac{u + x}{(y + z)^2} dy - \frac{u + x}{(y + z)^2} dz + \frac{du}{y + z}$
 51. a. $\Delta c \approx 0.035$ b. When $\theta = \frac{\pi}{20}$ 53. a. True b. True
 c. False 55. $(1, -1, 1)$ and $(1, -1, -1)$
 57. Points with $x = 0, \pm\frac{\pi}{2}, \pm\pi$ and $y = \pm\frac{\pi}{2}$, or points with $x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$ and $y = 0, \pm\pi$
 59. a. $\Delta S \approx 0.749$ b. More sensitive to changes in r
 61. a. $\Delta A \approx \frac{2}{1225} = 0.00163$ b. No. The batting average increases more if the batter gets a hit than it decreases if he fails to get a hit.
 c. Yes. The answer depends on whether A is less than 0.500 or greater than 0.500. 63. a. $\Delta V \approx \frac{21}{5000} = 0.0042$ b. $\frac{\Delta V}{V} \approx -4\%$ c. $2p$
 65. a. $f_r = n(1 - r)^{n-1}$, $f_n = -(1 - r)^n \ln(1 - r)$
 b. $\Delta P \approx 0.027$ c. $\Delta P \approx 2 \times 10^{-20}$ 67. $\Delta R \approx 7/540 \approx 0.013\Omega$
 69. a. Apply the Chain Rule. b. Follows directly from (a)
 c. $d(\ln(xy)) = \frac{dx}{x} + \frac{dy}{y}$ d. $d(\ln(x/y)) = \frac{dx}{x} - \frac{dy}{y}$
 e. $\frac{df}{f} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2} + \cdots + \frac{dx_n}{x_n}$

Section 15.7 Exercises, pp. 993–996

1. The local maximum occurs at the highest point on the surface; you cannot get to a higher point in any direction. 3. The partial derivatives are both zero or do not exist. 5. The discriminant is a determinant; it is defined as $D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b)$.
 7. f has an absolute minimum value on R at (a, b) if $f(a, b) \leq f(x, y)$ for all (x, y) in R . 9. Saddle point 11. Local min 13. $(0, 0)$
 15. $(0, 1)$, $(0, -1)$ 17. $(0, 0)$, $(2, 2)$, and $(-2, -2)$
 19. $(0, 2)$, $(\pm 1, 2)$ 21. $(3, 0)$, $(-15, 6)$ 23. Saddle point at $(0, 0)$

25. Local min at $(0, 0)$ 27. Saddle point at $(0, 0)$; local min at $(1, 1)$ and $(-1, -1)$ 29. $(0, 0)$; Second Derivative Test is inconclusive; absolute min of 4 at $(0, 0)$ 31. Local min at $(2, 0)$

33. Saddle point at $(0, 0)$; local max at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$; local min at $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
 35. Local min at $(-1, 0)$; local max at $(1, 0)$ 37. Saddle point at $(0, 1)$; local min at $(\pm 2, 0)$ 39. Saddle point at $(0, 0)$ 41. Saddle point 43. Height = 32 in, base is 16 in $\times$ 16 in; volume is 8192 in³
 45. $2\text{ m} \times 2\text{ m} \times 1\text{ m}$ 47. Absolute min: $0 = f(0, 1)$; absolute max: $9 = f(0, -2)$ 49. Absolute min: $4 = f(0, 0)$; absolute max: $7 = f(\pm 1, \pm 1)$ 51. Absolute min: $0 = f(1, 0)$; absolute max: $3 = f(1, 1) = f(1, -1)$ 53. Absolute min: $1 = f(1, -2) = f(1, 0)$; absolute max: $4 = f(1, -1)$
 55. Absolute min: $0 = f(0, 0)$; absolute max: $\frac{7}{8} = f\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$

57. a. 1.83; 82.6° b. 14% 59. Absolute min: $-4 = f(0, 0)$; no absolute max on R 61. Absolute max: $2 = f(0, 0)$; no absolute min on R 63. $P\left(\frac{4}{3}, \frac{2}{3}, \frac{4}{3}\right)$ 65. $(3, 4, 5)$, $(3, 4, -5)$

67. a. True b. False c. True d. True
 69. Local min at $(0.3, -0.3)$; saddle point at $(0, 0)$

71. a.–d. $x = y = z = \frac{200}{3}$

73. a. $P\left(1, \frac{1}{3}\right)$ b. $P\left(\frac{1}{3}(x_1 + x_2 + x_3), \frac{1}{3}(y_1 + y_2 + y_3)\right)$

- c. $P(\bar{x}, \bar{y})$, where $\bar{x} = \frac{1}{n} \sum_{k=1}^n x_k$ and $\bar{y} = \frac{1}{n} \sum_{k=1}^n y_k$

- d. $d(x, y) = \sqrt{x^2 + y^2} + \sqrt{(x - 2)^2 + y^2} + \sqrt{(x - 1)^2 + (y - 1)^2}$. Absolute max: $1 + \sqrt{3} = f\left(1, \frac{1}{\sqrt{3}}\right)$

77. $y = \frac{22}{13}x + \frac{46}{13}$ 79. $a = b = c = 3$

81. a. $\nabla d_1(x, y) = \frac{x - x_1}{d_1(x, y)} \mathbf{i} + \frac{y - y_1}{d_1(x, y)} \mathbf{j}$

- b. $\nabla d_2(x, y) = \frac{x - x_2}{d_2(x, y)} \mathbf{i} + \frac{y - y_2}{d_2(x, y)} \mathbf{j}$

- $\nabla d_3(x, y) = \frac{x - x_3}{d_3(x, y)} \mathbf{i} + \frac{y - y_3}{d_3(x, y)} \mathbf{j}$

- c. Follows from $\nabla f = \nabla d_1 + \nabla d_2 + \nabla d_3$ d. Three unit vectors add to zero. e. P is the vertex at the large angle.

- f. $P(0.255457, 0.304504)$ 83. a. Local max at $(1, 0)$, $(-1, 0)$

- b. Local max at $(1, 0)$ and $(-1, 0)$ 85. $\frac{abc\sqrt{3}}{2}$

Section 15.8 Exercises, pp. 1002–1004

1. The level curve of f is tangent to the curve $g = 0$ at the optimal point; therefore, the gradients are parallel.

3. $1 = 2\lambda x$, $4 = 2\lambda y$, $x^2 + y^2 - 1 = 0$

5. Abs. min: 1; abs. max: 8 7. Abs. min: $-2\sqrt{5}$ at $\left(-\frac{2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$; abs. max: $2\sqrt{5}$ at $\left(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$

9. Abs. min: -2 at $(-1, -1)$; abs. max: 2 at $(1, 1)$

11. Abs. min: -3 at $(-\sqrt{3}, \sqrt{3})$ and $(\sqrt{3}, -\sqrt{3})$; abs. max: 9 at $(3, 3)$ and $(-3, -3)$

13. Abs. min: e^{-9} at $(-3, 3)$ and $(3, -3)$; abs. max: e^3 at $(\sqrt{3}, \sqrt{3})$ and $(-\sqrt{3}, -\sqrt{3})$ 15. Abs. min: 9 at $(0, 3)$; abs. max: 34 at $(-\sqrt{15}, -2)$ and $(\sqrt{15}, -2)$ 17. Abs. min: $-2\sqrt{11}$ at $\left(-\frac{2}{\sqrt{11}}, -\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}\right)$; abs. max: $2\sqrt{11}$ at $\left(\frac{2}{\sqrt{11}}, \frac{6}{\sqrt{11}}, -\frac{2}{\sqrt{11}}\right)$

19. Abs. min: $-\frac{\sqrt{5}}{2}$ at $\left(-\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right)$; abs. max: $\frac{\sqrt{5}}{2}$ at $\left(\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right)$
 21. Abs. min: -5 at $(-2, -2, -1)$; abs. max: 5 at $(2, 2, 1)$

23. Abs. min: -10 at $(-5, 0, 0)$; abs. max: $\frac{29}{2}$ at $\left(2, 0, \pm\sqrt{\frac{21}{2}}\right)$

25. Abs. min: $-\sqrt{3}$ at $\left(0, -\frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right)$; abs. max: $\frac{7}{4}$ at $\left(\frac{1}{2}, \frac{1}{2}, 1\right)$
 and $\left(-\frac{1}{2}, \frac{1}{2}, 1\right)$ 27. $18 \text{ in} \times 18 \text{ in} \times 36 \text{ in}$ 29. Abs. min: 0.6731 ;

abs. max: 1.1230 31. 2×1 33. $\left(-\frac{3}{17}, \frac{29}{17}, -3\right)$

35. Abs. min: $\sqrt{38 - 6\sqrt{29}}$ (or $\sqrt{29} - 3$); abs. max: $\sqrt{38 + 6\sqrt{29}}$ (or $\sqrt{29} + 3$) 37. $\ell = 3$ and $g = \frac{3}{2}$; $U = 15\sqrt{2}$

39. $\ell = \frac{16}{5}$ and $g = 1$; $U = 20.287$ 41. a. True b. False

43. $\frac{\sqrt{6}}{3} \text{ m} \times \frac{\sqrt{6}}{3} \text{ m} \times \frac{\sqrt{6}}{6} \text{ m}$ 45. $2 \times 1 \times \frac{2}{3}$ 47. $P\left(\frac{4}{3}, \frac{2}{3}, \frac{4}{3}\right)$

49. Abs. min: 1 ; abs. max: 9 51. Abs. min: 0 ; abs. max: 3

53. $K = 7.5$ and $L = 5$ 57. Abs. max: 8 59. Abs. max:

$\sqrt{a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2}$ 61. a. Gradients are perpendicular to level surfaces. b. If the gradient were not in the plane spanned by ∇g and ∇h , f could be increased (decreased) by moving the point slightly. c. ∇f is a linear combination of ∇g and ∇h , since it belongs to the plane spanned by these two vectors. d. The gradient condition from part (c), as well as the constraints, must be satisfied.

63. Abs. min: $2 - 4\sqrt{2}$; abs. max: $2 + 4\sqrt{2}$

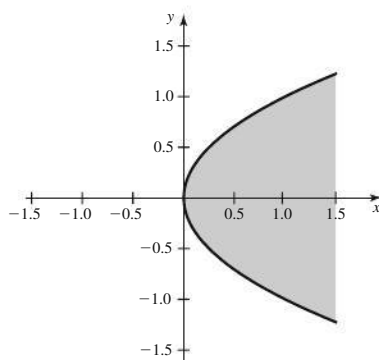
65. a. $y + 1 = \lambda y$, $x + 1 = \lambda x$, $xy = 4$ c. Abs. min of 108 over the curve C_1 d. Abs. max of 100 over the curve C_2

e. The constraint curve is unbounded, so there is no guarantee that an abs. min or max occurs over the curve $xy = 4$.

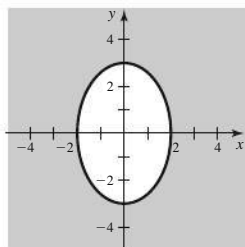
Chapter 15 Review Exercises, pp. 1005–1007

1. a. True b. False c. False d. False

3. $D = \{(x, y): x \geq y^2\}$

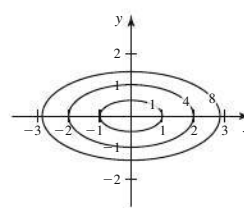


5. $D = \left\{(x, y): \frac{x^2}{4} + \frac{y^2}{9} \geq 1\right\}$



7. $D = \{(x, y): x^2 + y^2 \geq 1\}$; $R = (-\infty, 0]$; lower half of the hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$

9.



11. 2 13. Does not exist

15. $\frac{2}{3}$ 17. 4

19. $\{(x, y): y > x^2 + 1\}$

21. $f_x = 6xy^5$; $f_y = 15x^2y^4$

23. $f_x = \frac{2xy^2}{(x^2 + y^2)^2}$; $f_y = -\frac{2x^2y}{(x^2 + y^2)^2}$

25. $f_x = y(1 + xy)e^{xy}$; $f_y = x(1 + xy)e^{xy}$ 27. $f_{xx} = 4y^2e^{2xy}$;
 $f_{xy} = 2e^{2xy}(2xy + 1) = f_{yx}$; $f_{yy} = 4x^2e^{2xy}$

29. $\frac{\partial^2 u}{\partial x^2} = 6y = -\frac{\partial^2 u}{\partial y^2}$ 31. a. V increases with R if r is fixed,

$V_R > 0$; V decreases if r increases and R is fixed, $V_r < 0$.

b. $V_r = -4\pi r^2$; $V_R = 4\pi R^2$ c. The volume increases more if R is increased. 33. $4t + 2 \ln 5$

35. $w_r = \frac{3r + s}{r(r + s)}$; $w_s = \frac{r + 3s}{s(r + s)}$; $w_t = \frac{1}{t}$

37. $\frac{dy}{dx} = -\frac{2xy}{2y^2 + (x^2 + y^2) \ln(x^2 + y^2)}$

39. a. $z'(t) = -24 \sin t \cos t = -12 \sin 2t$

b. $z'(t) > 0$ for $\frac{\pi}{2} < t < \pi$ and $\frac{3\pi}{2} < t < 2\pi$

41. a.

	$(a, b) = (0, 0)$	$(a, b) = (2, 0)$	$(a, b) = (1, 1)$
$\mathbf{u} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$	0	$4\sqrt{2}$	$-2\sqrt{2}$
$\mathbf{v} = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$	0	$-4\sqrt{2}$	$-6\sqrt{2}$
$\mathbf{w} = \left\langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$	0	$-4\sqrt{2}$	$2\sqrt{2}$

b. The function is increasing at $(2, 0)$ in the direction of $\mathbf{u}$ and decreasing at $(2, 0)$ in the directions of $\mathbf{v}$ and $\mathbf{w}$.

43. $\nabla g = \langle 2xy^3, 3x^2y^2 \rangle$; $\nabla g(-1, 1) = \langle -2, 3 \rangle$; $D_{\mathbf{u}}g(-1, 1) = 2$

45. $\nabla h = \left\langle \frac{x}{\sqrt{2 + x^2 + 2y^2}}, \frac{2y}{\sqrt{2 + x^2 + 2y^2}} \right\rangle$;

$\nabla h(2, 1) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$; $D_{\mathbf{u}}h(2, 1) = \frac{7\sqrt{2}}{10}$

47. $\nabla f = \langle y \cos xy, x \cos xy, -\sin z \rangle$; $\nabla f(1, \pi, 0) = \langle -\pi, -1, 0 \rangle$;

$D_{\mathbf{u}}f(1, \pi, 0) = -\frac{1}{7}(3 + 2\pi)$

49. a. Direction of steepest ascent: $\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$; direction of

steepest descent: $\mathbf{u} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

b. No change: $\mathbf{u} = \pm \left(\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} \right)$

51. Tangent line is vertical; $\nabla f(2, 0) = -8\mathbf{i}$

53. $E = \frac{kx}{x^2 + y^2}\mathbf{i} + \frac{ky}{x^2 + y^2}\mathbf{j}$

55. $y = 2$ and $12x + 3y - 2z = 12$

57. $x + 2y + 3z = 6$ and $x - 2y + 3z = 6$

59. $x + y - z = 0$ and $x + y - z = 0$

61. a. $L(x, y) = x + 5y$ b. $L(1.95, 0.05) = 2.2$ 63. Approx. -4%

65. a. $\Delta V \approx -0.1\pi \text{ m}^3$ b. $\Delta S \approx -0.05\pi \text{ m}^2$

67. Saddle point at $(0, 0)$; local min at $(2, -2)$

69. Saddle points at $(0, 0)$ and $(-2, 2)$; local max at $(0, 2)$; local min at $(-2, 0)$ 71. Abs. min: $-1 = f(1, 1) = f(-1, -1)$; abs. max: $49 = f(2, -2) = f(-2, 2)$

73. Abs. min: $-\frac{1}{2} = f\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; abs. max: $\frac{1}{2} = f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

75. Abs. min: $\frac{23}{2} = f\left(\frac{1}{3}, \frac{5}{6}\right)$ abs. max: $\frac{29}{2} = f\left(\frac{5}{3}, \frac{7}{6}\right)$;

77. Abs. min: $-\sqrt{6} = f\left(-\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}\right)$;

abs. max: $\sqrt{6} = f\left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6}\right)$

79. $\frac{2a^2}{\sqrt{a^2 + b^2}}$ by $\frac{2b^2}{\sqrt{a^2 + b^2}}$

81. $x = \frac{1}{2} + \frac{\sqrt{10}}{20}$, $y = \frac{3}{2} + \frac{3\sqrt{10}}{20} = 3x$, $z = \frac{1}{2} + \frac{\sqrt{10}}{2} = \sqrt{10}x$

83. $(1, 2, 5)$

CHAPTER 16

Section 16.1 Exercises, pp. 1015–1017

1. $\int_0^2 \int_1^3 xy \, dy \, dx$ or $\int_1^3 \int_0^2 xy \, dx \, dy$ 3. $\int_{-2}^4 \int_1^5 f(x, y) \, dy \, dx$ or

$\int_1^5 \int_{-2}^4 f(x, y) \, dx \, dy$ 5. 48 7. 4 9. $\frac{32}{3}$ 11. 4 13. $\frac{224}{9}$

15. $10 - 2e$ 17. $\frac{1}{2}$ 19. $e^2 + 3$ 21. $\frac{1}{2}$ 23. $10\sqrt{5} - 4\sqrt{2} - 14$

25. $\frac{117}{2}$ 27. $\frac{\pi^2}{4} + 1$ 29. $\frac{4}{3}$ 31. $\frac{9 - e^2}{2}$ 33. $\frac{4}{11}$ 35. $\frac{1}{4}$

37. 136 39. 3 41. $e^2 - 3$ 43. $e^{16} - 17$ 45. $\ln \frac{5}{3}$ 47. $\frac{1}{2 \ln 2}$

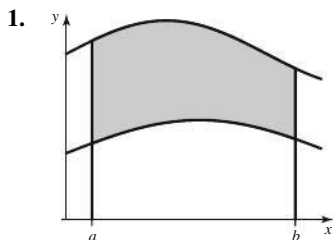
49. $\frac{8}{3}$ 51. a. True b. False c. True 53. a. 1475 b. The sum of products of population densities and areas is a Riemann sum.

55. $\int_c^d \int_a^b f(x) \, dy \, dx = (c - d) \int_a^b f(x) \, dx$. The integral is the area of the cross section of S . 57. $a = \pi/6, 5\pi/6$ 59. $a = \sqrt{6}$

61. a. $\frac{1}{2}\pi^2 + \pi$ b. $\frac{1}{2}\pi^2 + \pi$ c. $\frac{1}{2}\pi^2 + 2$

63. $f(a, b) - f(a, 0) - f(0, b) + f(0, 0)$

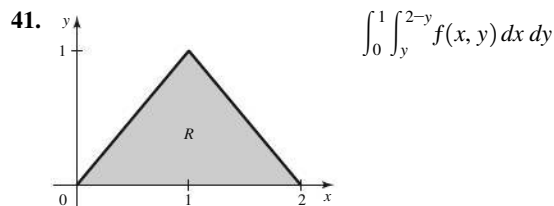
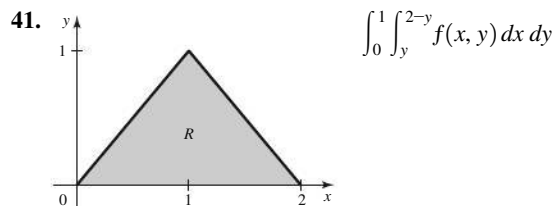
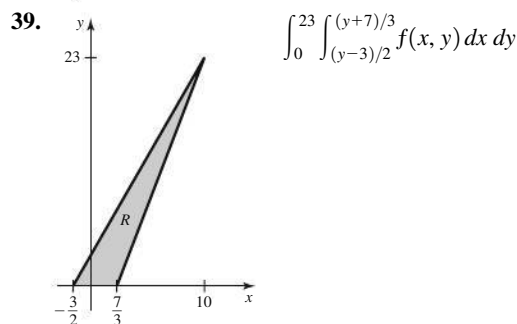
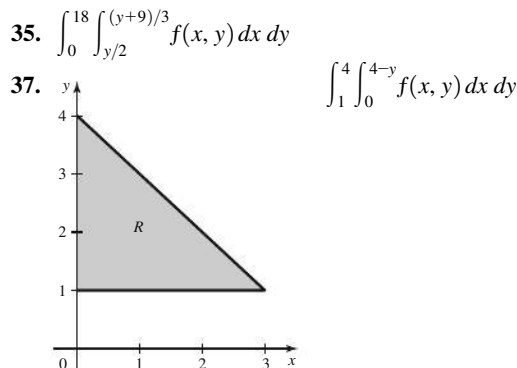
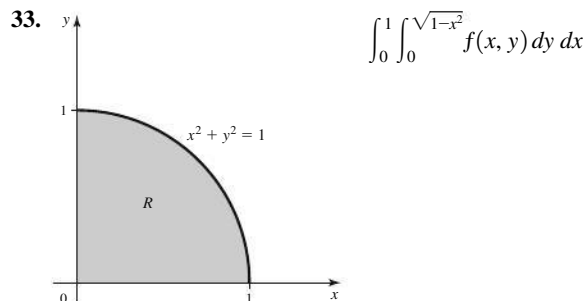
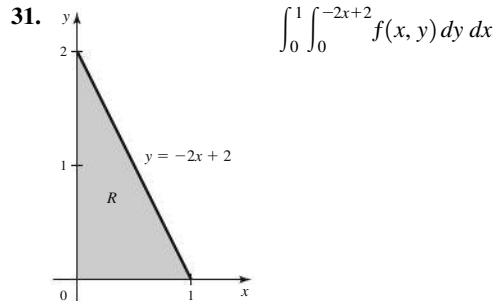
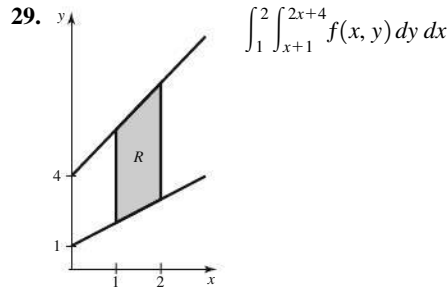
Section 16.2 Exercises, pp. 1024–1027



3. $dx \, dy$ 5. $\int_0^1 \int_{x^2}^{\sqrt{x}} f(x, y) \, dy \, dx$ 7. 4 9. $\int_0^2 \int_{x^3}^{4x} f(x, y) \, dy \, dx$

11. 2 13. $\frac{8}{3}$ 15. 0 17. $e - 1$ 19. $\frac{\ln^3 2}{6}$

21. 2 23. $\frac{\pi}{2} - 1$ 25. 0 27. $\pi - 1$

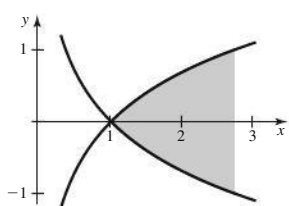


43. 2 45. 12 47. 5 49. 14 51. 32 53. $\frac{9}{8}$ 55. $\frac{1}{4} \ln 2$

57. $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x, y) dx dy$ 59. $\int_0^{\ln 2} \int_{1/2}^{e^{-x}} f(x, y) dy dx$

61. $\int_0^{\pi/2} \int_0^{\cos x} f(x, y) dy dx$ 63. $\frac{1}{2}(e-1)$ 65. 0 67. $\frac{2}{3}$

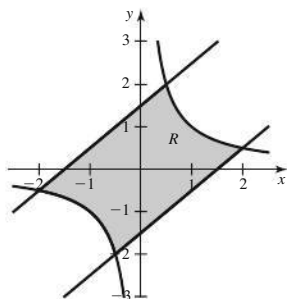
69. $\int_1^e \int_{-\ln x}^{\ln x} f(x, y) dy dx$



71. $\frac{11}{12}$ 73. $\frac{32}{3}$ 75. 12π 77. $\frac{43}{6}$ 79. $\frac{2}{3}$ 81. 16 83. $4a\pi$

85. $\frac{32}{3}$ 87. 1 89. $\frac{140}{3}$ 91. a. False b. False c. False

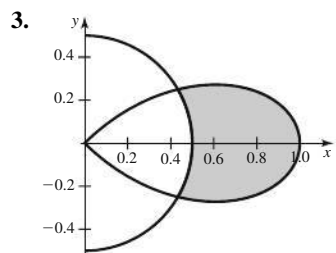
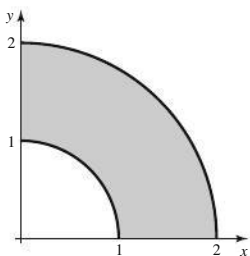
93. 30 95. $\frac{a}{3}$ 97. a.



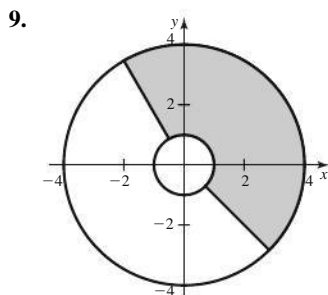
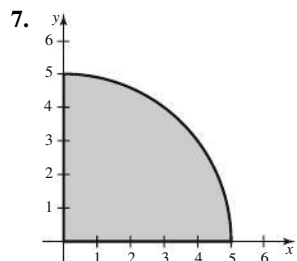
b. $\frac{15}{4} + 4 \ln 2$ c. $2 \ln 2 - \frac{5}{64}$ 99. $\frac{3}{8e^2}$ 101. 1

Section 16.3 Exercises, pp. 1033–1036

1. It is called a polar rectangle because r and θ vary between two constants.



5. Evaluate the integral $\int_{\alpha}^{\beta} \int_{g(\theta)}^{h(\theta)} r dr d\theta$.

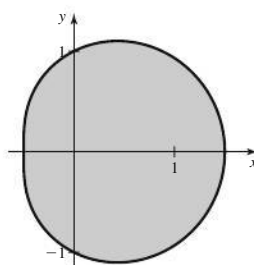


11. $\frac{64\pi}{3}$ 13. $(8 - 24e^{-2})\pi$ 15. $\frac{7\pi}{2}$ 17. $\frac{9\pi}{2}$ 19. $\frac{37\pi}{3}$

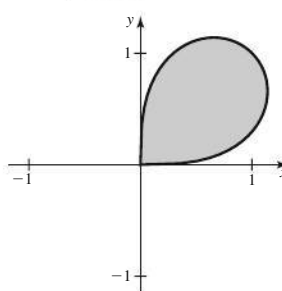
21. 128π 23. 0 25. $(2 - \sqrt{3})\pi$ 27. $2\pi/5$ 29. $\frac{14\pi}{3}$

31. $\frac{81\pi}{2}$ 33. π 35. 8π 37. 81π 39. $\frac{2\pi}{3}(7\sqrt{7} - 15)$

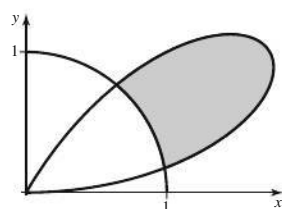
41. $\int_0^{2\pi} \int_0^{1+\frac{1}{2}\cos\theta} g(r, \theta) r dr d\theta$



43. $\int_0^{\pi/2} \int_0^{\sqrt{2}\sin 2\theta} g(r, \theta) r dr d\theta$



45. $\int_{\pi/18}^{5\pi/18} \int_1^{2\sin 3\theta} g(r, \theta) r dr d\theta$



47. $3\pi/2$ 49. π 51. $\frac{3\pi}{2} - 2\sqrt{2}$ 53. $2a/3$

55. a. False b. True c. True 57. The hyperboloid ($V = \frac{112\pi}{3}$)

59. a. $R = \{(r, \theta) : -\pi/4 \leq \theta \leq \pi/4 \text{ or } 3\pi/4 \leq \theta \leq 5\pi/4\}$

b. $\frac{a^4}{4}$ 61. $\frac{32}{9}$ 63. $2\pi(1 - 2\ln \frac{3}{2})$ 65. 1

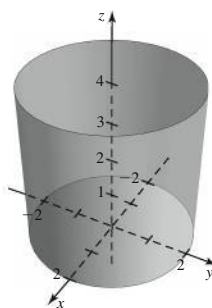
67. $\pi/4$ 69. a. $\frac{16\pi}{3}$ b. 2.78 71. $30\pi + 42$

73. c. $\sqrt{\pi}/2, 1/2$, and $\sqrt{\pi}/4$ 75. a. $I = \frac{\sqrt{2}}{2} \tan^{-1} \frac{\sqrt{2}}{2}$

b. $I = \frac{\sqrt{2}}{4} \tan^{-1} \frac{\sqrt{2}a}{2} + \frac{a}{2\sqrt{a^2+1}} \tan^{-1} \frac{1}{\sqrt{a^2+1}}$ c. $\frac{\sqrt{2}\pi}{8}$

Section 16.4 Exercises, pp. 1043–1047

1.



3. $\int_{-9}^9 \int_{-\sqrt{81-x^2}}^{\sqrt{81-x^2}} \int_{-\sqrt{81-x^2-y^2}}^{\sqrt{81-x^2-y^2}} f(x, y, z) dz dy dx$

5. $\int_0^1 \int_0^{\sqrt{1-z^2}} \int_0^{\sqrt{1-z^2-x^2}} f(x, y, z) dy dx dz$ 7. 24 9. 8 11. $\frac{2}{\pi}$

13. 0 15. 8 17. $\frac{16}{3}$ 19. $1 - \ln 2$

21. $\frac{2\pi(1 + 19\sqrt{19} - 20\sqrt{10})}{3}$ 23. $\frac{27\pi}{2}$ 25. 12π

27. $\frac{5}{12}$ 29. 8 31. $\int_0^1 \int_y^1 \int_0^{2\sqrt{1-x^2}} f(x, y, z) dz dx dy$

33. $\int_0^1 \int_0^{2\sqrt{1-x^2}} \int_0^x f(x, y, z) dy dz dx$

35. $\int_0^1 \int_0^{2\sqrt{1-y^2}} \int_{y^2}^{\frac{1}{2}\sqrt{4-z^2}} f(x, y, z) dx dz dy$

37. $\int_0^1 \int_0^2 \int_0^{1-y} dz dx dy$, $\int_0^2 \int_0^1 \int_0^{1-z} dy dz dx$, $\int_0^1 \int_0^2 \int_0^{1-z} dy dx dz$,
 $\int_0^1 \int_0^{1-y} \int_0^2 dx dz dy$, $\int_0^1 \int_0^{1-z} \int_0^2 dx dy dz$ 39. $\frac{256}{9}$ 41. $\frac{2}{3}$

43. $(10\sqrt{10} - 1)\frac{\pi}{6}$ 45. $\frac{3 \ln 2}{2} + \frac{e}{16} - 1$

47. $\int_0^4 \int_{y/4-1}^0 \int_0^5 dz dx dy = 10$ 49. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2}} dz dy dx = \frac{2}{3}$

51. $\frac{8}{\pi}$ 53. $\frac{10}{3}$ 55. a. False b. False c. False 57. 2

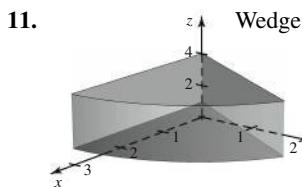
59. 1 61. $\frac{16}{3}$ 63. $\frac{16}{3}$ 65. $a = 2\sqrt{2}$ 67. $V = \frac{\pi r^2 h}{3}$

69. $V = \frac{\pi h^2}{3}(3R - h)$ 71. $V = \frac{4\pi abc}{3}$ 73. $\frac{1}{24}$

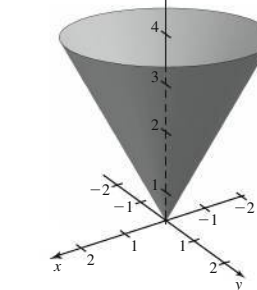
Section 16.5 Exercises, pp. 1059–1063

1. r measures the distance from the point to the z -axis, θ is the angle that the segment from the point to the z -axis makes with the positive xz -plane, and z is the directed distance from the point to the xy -plane.
 3. A cone 5. It approximates the volume of the cylindrical wedge formed by the changes Δr , $\Delta \theta$, and Δz .

7. $\int_{\alpha}^{\beta} \int_{g(\theta)}^{h(\theta)} \int_{G(r, \theta)}^{H(r, \theta)} w(r, \theta, z) r dz dr d\theta$ 9. Cylindrical coordinates



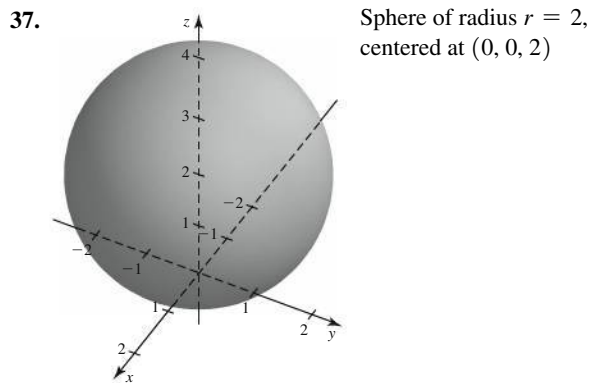
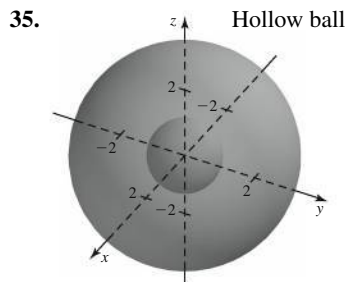
13.



15. 2π 17. $4\pi/5$ 19. $\pi(1 - e^{-1})/2$ 21. $9\pi/4$

23. 560π 25. 396π 27. The paraboloid ($V = 44\pi/3$)

29. $\frac{20\pi}{3}$ 31. $\frac{(16 + 17\sqrt{29})\pi}{3}$ 33. $\frac{1}{3}$



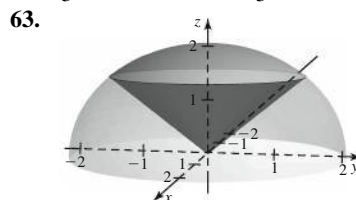
39. a. $(3960, 0.74, -2.13)$, $(-1426.85, -2257.05, 2924.28)$
 b. $(3960, 0.84, 0.22)$, $(2877.61, 637.95, 2644.62)$ c. 5666 mi

41. $\frac{\pi}{2}$ 43. $4\pi \ln 2$ 45. $\pi\left(\frac{188}{9} - \frac{32\sqrt{3}}{3}\right)$ 47. $\frac{32\pi\sqrt{3}}{9}$

49. $\frac{5\pi}{12}$ 51. $\frac{8\pi}{3}$ 53. $\frac{8\pi}{3}(9\sqrt{3} - 11)$ 55. a. True b. True

57. $z = \sqrt{x^2 + y^2} - 1$; upper half of a hyperboloid of one sheet

59. $\frac{8\pi}{3}(1 - e^{-512}) \approx \frac{8\pi}{3}$ 61. 32π



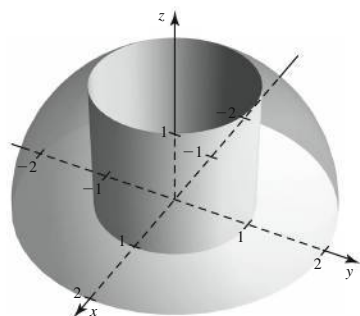
$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} g(r, \theta, z) r dz dr d\theta,$

$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_0^z g(r, \theta, z) r dr dz d\theta$

$+ \int_0^{2\pi} \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-z^2}} g(r, \theta, z) r dr dz d\theta,$

$\int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} \int_0^{2\pi} g(r, \theta, z) r d\theta dz dr$

65.



$$\int_{\pi/6}^{\pi/2} \int_0^{2\pi} \int_{\csc \varphi}^2 g(\rho, \varphi, \theta) \rho^2 \sin \varphi \, d\rho \, d\theta \, d\varphi,$$

$$\int_{\pi/6}^{\pi/2} \int_{\csc \varphi}^2 \int_0^{2\pi} g(\rho, \varphi, \theta) \rho^2 \sin \varphi \, d\theta \, d\rho \, d\varphi$$

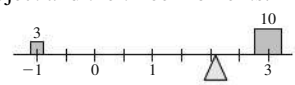
67. $32\sqrt{3}\pi/9$ 69. $2\sqrt{2}/3$ 71. $7\pi/2$ 73. 95.6036

77. $V = \frac{\pi r^2 h}{3}$ 79. $V = \frac{\pi}{3} (R^2 + rR + r^2)h$

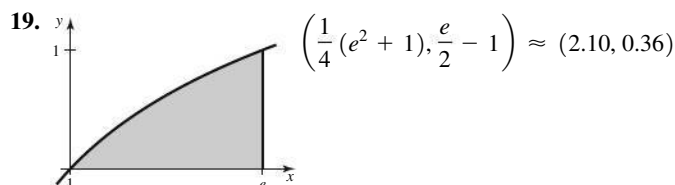
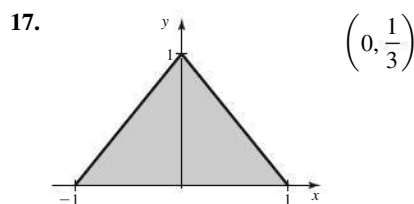
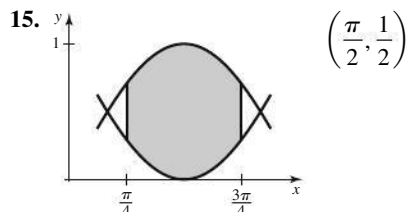
81. $V = \frac{\pi R^3 (8r - 3R)}{12r}$

Section 16.6 Exercises, pp. 1070–1072

1. The pivot should be located at the center of mass of the system.
 3. Use a double integral. Integrate the density function over the region occupied by the plate. 5. Use a triple integral to find the mass of the object and the three moments.

7.  $\frac{27}{13}$ 9. Mass is $2 + \pi$; $\bar{x} = \frac{\pi}{2}$

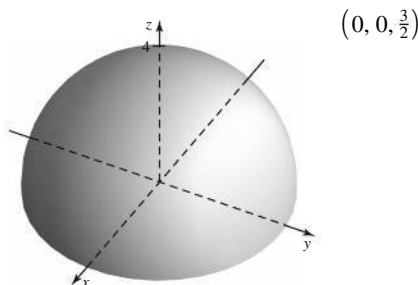
11. Mass is $\frac{20}{3}$; $\bar{x} = \frac{9}{5}$ 13. Mass is 10; $\bar{x} = \frac{8}{3}$



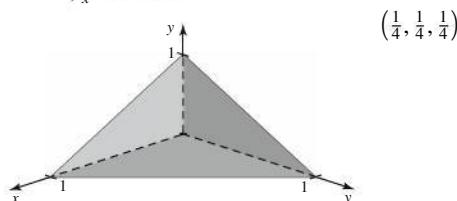
21. $\left(\frac{7}{3}, 1\right)$; density increases to the right. 23. $\left(\frac{16}{11}, \frac{16}{11}\right)$; density increases toward the hypotenuse of the triangle.

25. $\left(0, \frac{16 + 3\pi}{16 + 12\pi}\right) \approx (0, 0.4735)$; density increases away from the x-axis.

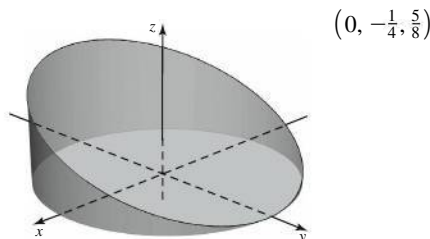
27.



29.



31.



33. $\left(\frac{7}{3}, \frac{1}{2}, \frac{1}{2}\right)$ 35. $(0, 0, \frac{198}{85})$ 37. $\left(\frac{2}{3}, \frac{7}{3}, \frac{1}{3}\right)$ 39. a. False

b. True c. False d. False 41. $\bar{x} = \frac{\ln(1 + L^2)}{2 \tan^{-1} L}$, $\lim_{L \rightarrow \infty} \bar{x} = \infty$

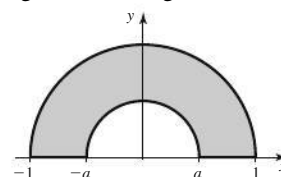
43. $\left(0, \frac{8}{9}\right)$ 45. $\left(0, \frac{8}{3\pi}\right)$ 47. $\left(\frac{5}{6}, 0\right)$ 49. $\left(\frac{128}{105\pi}, \frac{128}{105\pi}\right)$

51. On the line of symmetry, $2a/\pi$ units above the diameter

53. $\left(\frac{2a}{3(4 - \pi)}, \frac{2a}{3(4 - \pi)}\right)$ 55. $h/4$ units

57. $h/3$ units, where h is the height of the triangle 59. $3a/8$ units

61. a. $\left(0, \frac{4(1 + a + a^2)}{3(1 + a)\pi}\right)$



b. $a = \frac{1}{2} \left(-1 + \sqrt{1 + \frac{16}{3\pi - 4}}\right) \approx 0.4937$

63. Depth = $\frac{40\sqrt{10} - 4}{333}$ cm ≈ 0.3678 cm

65. a. $(\bar{x}, \bar{y}) = \left(\frac{-r^2}{R + r}, 0\right)$ (origin at center of large circle);

$(\bar{x}, \bar{y}) = \left(\frac{R^2 + Rr + r^2}{R + r}, 0\right)$ (origin at common point of the circles)

b. Hint: Solve $\bar{x} = R - 2r$.

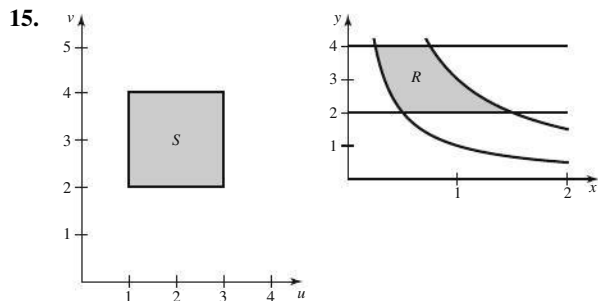
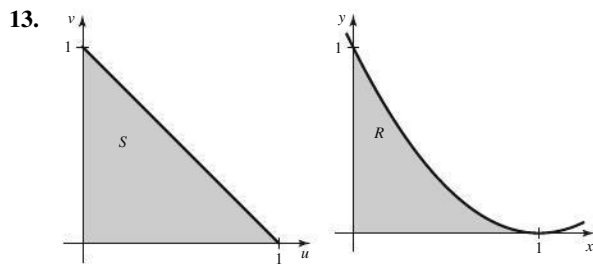
Section 16.7 Exercises, pp. 1082–1084

1. The image of S is the 2×2 square with vertices at $(0, 0)$, $(2, 0)$, $(2, 2)$, and $(0, 2)$. 3. $\int_0^1 \int_0^1 f(u + v, u - v) 2 \, du \, dv$

5. The rectangle with vertices at $(0, 0)$, $(2, 0)$, $\left(2, \frac{1}{2}\right)$, and $\left(0, \frac{1}{2}\right)$

7. The square with vertices at $(0, 0)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$, $(1, 0)$, and $\left(\frac{1}{2}, -\frac{1}{2}\right)$

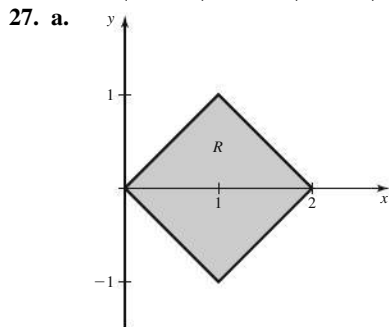
9. The region above the x-axis and bounded by the curves $y^2 = 4 \pm 4x$ 11. The upper half of the unit circle



17. -9 19. $-4(u^2 + v^2)$ 21. -1

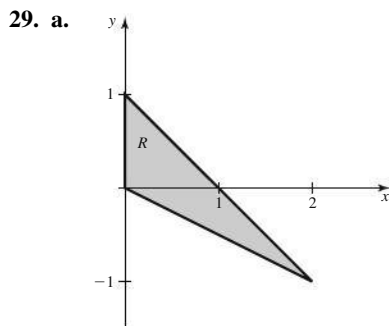
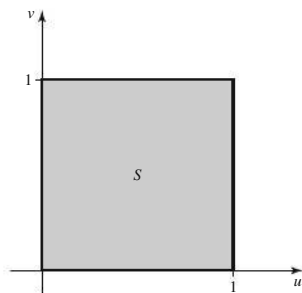
23. $x = (u + v)/3$, $y = (2u - v)/3$; $-1/3$

25. $x = -(u + 3v)$, $y = -(u + 2v)$; -1

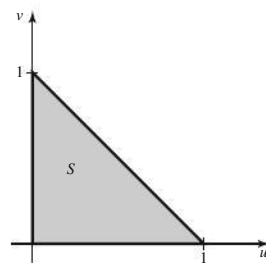


b. $0 \leq u \leq 1$, $0 \leq v \leq 1$

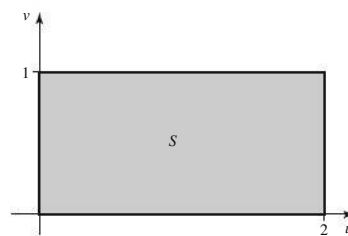
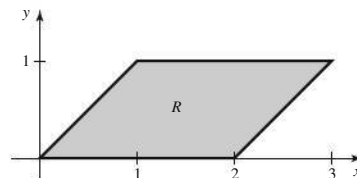
c. $J(u, v) = -2$ d. 0



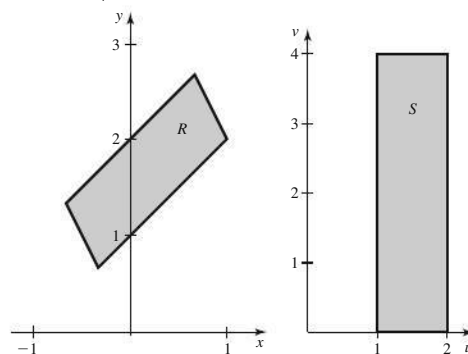
b. $0 \leq u \leq 1$, $0 \leq v \leq 1 - u$ c. $J(u, v) = 2$ d. $256\sqrt{2}/945$



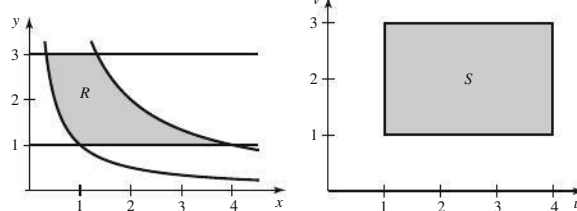
31. $4\sqrt{2}/3$



33. $3844/5625$



35. $\frac{15 \ln 3}{2}$



37. 2 39. $2w(u^2 - v^2)$ 41. 5 43. $1024\pi/3$

45. a. True b. True c. True

47. Hint: $J(\rho, \varphi, \theta) = \begin{vmatrix} \sin \varphi \cos \theta & \rho \cos \varphi \cos \theta & -\rho \sin \varphi \sin \theta \\ \sin \varphi \sin \theta & \rho \cos \varphi \sin \theta & \rho \sin \varphi \cos \theta \\ \cos \varphi & -\rho \sin \varphi & 0 \end{vmatrix}$

49. $a^2b^2/2$ 51. $(a^2 + b^2)/4$ 53. $4\pi abc/3$

55. $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{3c}{8}\right)$ 57. a. $x = a^2 - \frac{y^2}{4a^2}$

b. $x = \frac{y^2}{4b^2} - b^2$ c. $J(u, v) = 4(u^2 + v^2)$ d. $\frac{80}{3}$ e. 160

f. Vertical lines become parabolas opening downward with vertices on the positive y -axis, and horizontal lines become parabolas opening upward with vertices on the negative y -axis. 59. **a.** S is stretched in the positive u - and v -directions but not in the w -direction. The amount of stretching increases with u and v . **b.** $J(u, v, w) = ad$

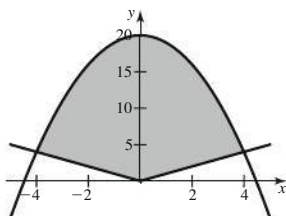
c. Volume = ad **d.** $\left(\frac{a+b+c}{2}, \frac{d+e}{2}, \frac{1}{2}\right)$

Chapter 16 Review Exercises, pp. 1084–1088

1. **a.** False **b.** True **c.** False **d.** False 3. $\frac{26}{3}$

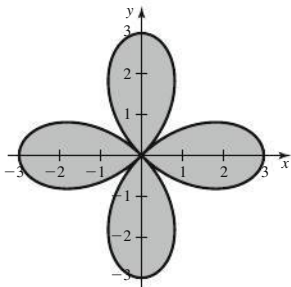
5. $\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y) dx dy$ 7. $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x, y) dy dx$

9. $\frac{304}{3}$ 11. $\frac{\sqrt{17} - \sqrt{2}}{2}$

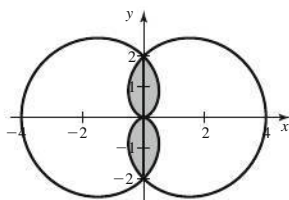


13. 8π 15. $\frac{2}{7\pi^2}$ 17. $\frac{1}{5}$

19. $\frac{9\pi}{2}$



21. $6\pi - 16$



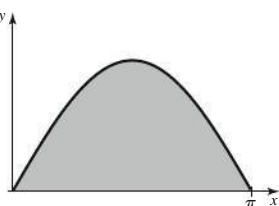
23. 2 25. $\int_0^1 \int_{2y}^2 \int_0^{\sqrt{z^2-4y^2/2}} f(x, y, z) dx dz dy$ 27. $\pi - \frac{4}{3}$

29. $8 \sin^2 2 = 4(1 - \cos 4)$ 31. $\frac{848}{9}$ 33. $\frac{8}{15}$ 35. $\frac{16}{3}$

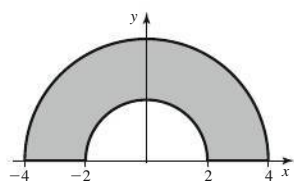
37. $\frac{128}{3}$ 39. $\frac{\pi}{6} - \frac{\sqrt{3}}{2} + \frac{1}{2}$ 41. $\frac{1}{3}$ 43. $\frac{1}{3}$ 45. π

47. 4π 49. $\frac{28\pi}{3}$ 51. $\frac{2048\pi}{105}$

53. $(\bar{x}, \bar{y}) = \left(\frac{\pi}{2}, \frac{\pi}{8}\right)$



55. $(\bar{x}, \bar{y}) = \left(0, \frac{56}{9\pi}\right)$



57. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 24)$ 59. $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{63}{10}\right)$

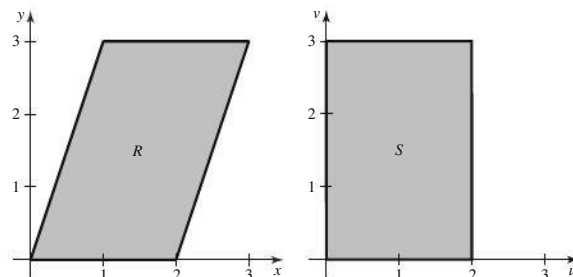
61. $\frac{h}{3}$ 63. $\left(\frac{4\sqrt{2}a}{3\pi}, \frac{4(2-\sqrt{2})a}{3\pi}\right)$ 65. **a.** $\frac{4\pi}{3}$ **b.** $\frac{16Q}{3}$

67. $R = \{(x, y): 0 \leq x \leq 1, 0 \leq y \leq 1\}$

69. The parallelogram with vertices $(0, 0)$, $(3, 1)$, $(4, 4)$, and $(1, 3)$

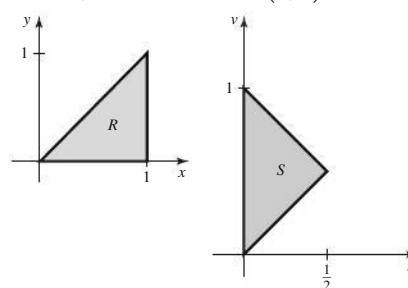
71. 10 73. 6

75. **a.**



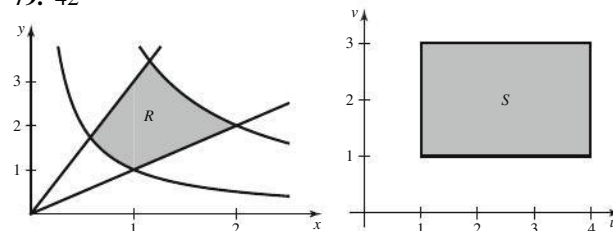
b. $0 \leq u \leq 2, 0 \leq v \leq 3$ **c.** $J(u, v) = 1$ **d.** $\frac{63}{2}$

77. **a.**



b. $u \leq v \leq 1 - u, 0 \leq u \leq \frac{1}{2}$ **c.** $J(u, v) = 2$ **d.** $\frac{1}{60}$

79. 42



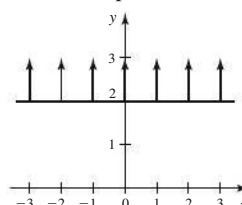
81. $-\frac{7}{16}$

CHAPTER 17

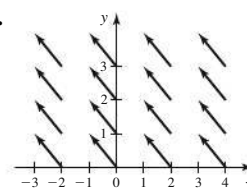
Section 17.1 Exercises, pp. 1096–1098

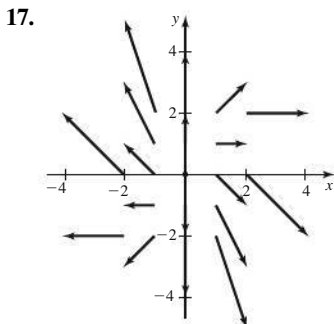
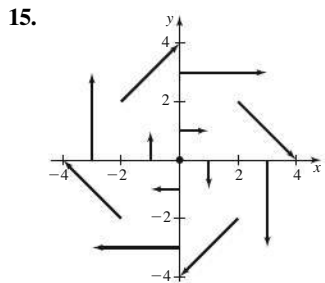
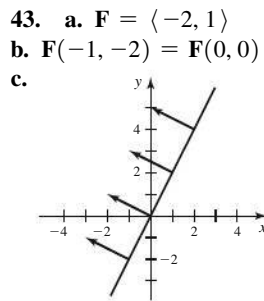
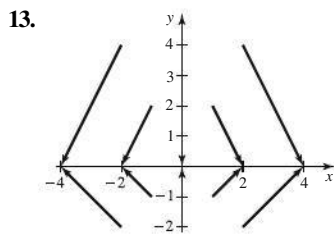
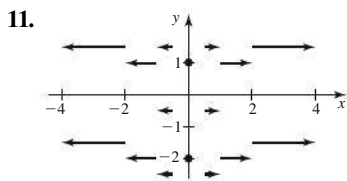
1. $\mathbf{F} = \langle f, g, h \rangle$ evaluated at (x, y, z) is the velocity vector of an air particle at (x, y, z) at a fixed point in time. 3. At selected points (a, b) , plot the vector $\langle f(a, b), g(a, b) \rangle$. 5. It shows the direction in which the temperature increases the fastest and the amount of increase.

7.

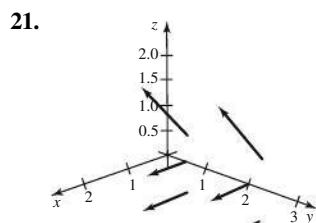
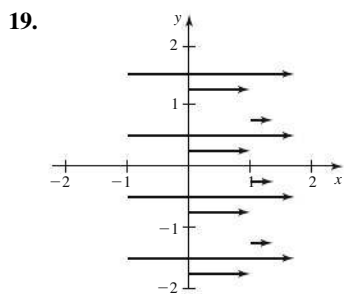
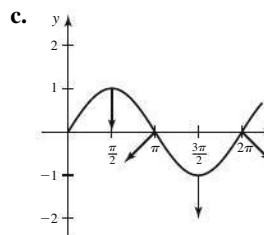


9.

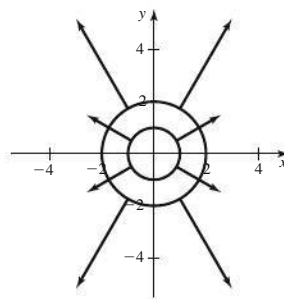




45. a. $\mathbf{F} = \langle \cos x, -1 \rangle$
 b. $\mathbf{F}\left(\frac{\pi}{2}, 1\right) = \langle 0, -1 \rangle$; $\mathbf{F}(\pi, 0) = \langle -1, -1 \rangle$;
 $\mathbf{F}\left(\frac{3\pi}{2}, -1\right) = \langle 0, -1 \rangle$; $\mathbf{F}(2\pi, 0) = \langle 1, -1 \rangle$



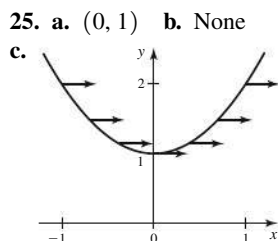
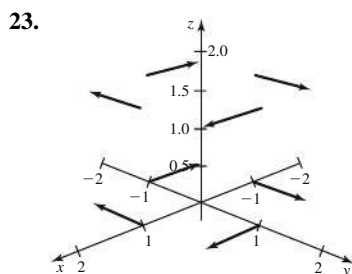
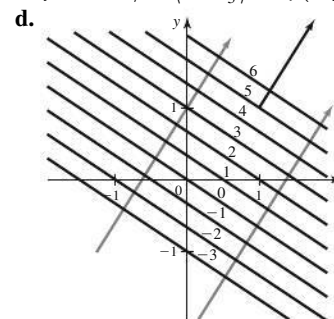
47. $\nabla\varphi(x, y) = 2\langle x, y \rangle$



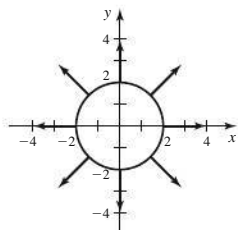
49. a. $\nabla\varphi(x, y) = \langle 2, 3 \rangle$

b. $y' = -2/3$, $\langle 1, -2/3 \rangle \cdot \nabla\varphi(1, 1) = 0$

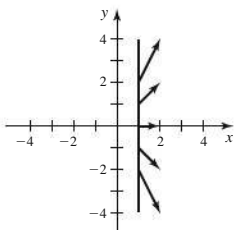
c. $y' = -2/3$, $\langle 1, -2/3 \rangle \cdot \nabla\varphi(x, y) = 0$



27. a. None
 b. At all points on C
 c.



29. a. None
 b. $(1, 0)$
 c.



31. $\mathbf{F} = \langle -y, x \rangle$ or $\mathbf{F} = \langle -1, 1 \rangle$

33. $\mathbf{F}(x, y) = \frac{\langle x, y \rangle}{\sqrt{x^2 + y^2}} = \frac{\mathbf{r}}{|\mathbf{r}|}$, $\mathbf{F}(0, 0) = \mathbf{0}$

35. $\nabla\varphi(x, y) = \langle 2xy - y^2, x^2 - 2xy \rangle$

37. $\nabla\varphi(x, y) = \langle 1/y, -x/y^2 \rangle$ 39. $\nabla\varphi(x, y, z) = \langle x, y, z \rangle = \mathbf{r}$

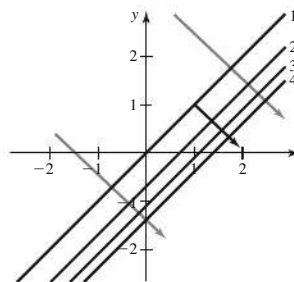
41. $\nabla\varphi(x, y, z) = -(x^2 + y^2 + z^2)^{-3/2} \langle x, y, z \rangle = -\frac{\mathbf{r}}{|\mathbf{r}|^3}$

51. a. $\nabla\varphi(x, y) = \langle e^{x-y}, -e^{x-y} \rangle = e^{x-y} \langle 1, -1 \rangle$

b. $y' = 1$, $\langle 1, 1 \rangle \cdot \nabla\varphi(1, 1) = 0$

c. $y' = 1$, $\langle 1, 1 \rangle \cdot \nabla\varphi(x, y) = 0$

d.

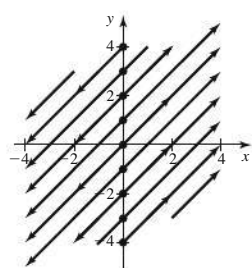


53. a. True b. False c. True 55. a. $\mathbf{E} = \frac{c}{x^2 + y^2} \langle x, y \rangle$

b. $|\mathbf{E}| = \left| \frac{c}{|\mathbf{r}|^2} \mathbf{r} \right| = \frac{c}{r}$ c. Hint: The equipotential curves are circles

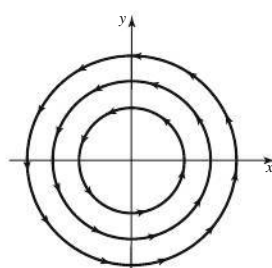
centered at the origin. 57. The slope of the streamline at (x, y) is $y'(x)$, which equals the slope of the vector $\mathbf{F}(x, y)$, which is g/f . Therefore, $y'(x) = g/f$.

59.



$$y = x + C$$

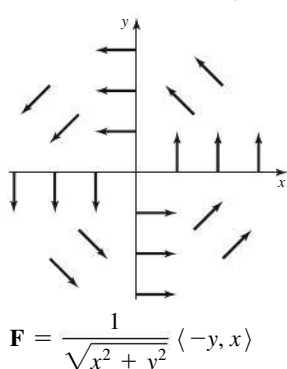
61.



$$x^2 + y^2 = C$$

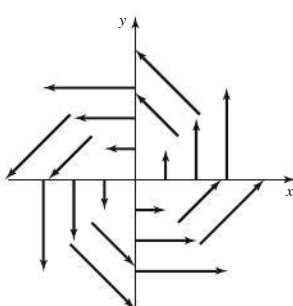
63. For $\theta = 0$: $\mathbf{u}_r = \mathbf{i}$ and $\mathbf{u}_\theta = \mathbf{j}$
 for $\theta = \frac{\pi}{2}$: $\mathbf{u}_r = \mathbf{j}$ and $\mathbf{u}_\theta = -\mathbf{i}$
 for $\theta = \pi$: $\mathbf{u}_r = -\mathbf{i}$ and $\mathbf{u}_\theta = -\mathbf{j}$
 for $\theta = \frac{3\pi}{2}$: $\mathbf{u}_r = -\mathbf{j}$ and $\mathbf{u}_\theta = \mathbf{i}$

65.



$$\mathbf{F} = \frac{1}{\sqrt{x^2 + y^2}} \langle -y, x \rangle$$

67.



$$\mathbf{F} = r \mathbf{u}_\theta$$

Section 17.2 Exercises, pp. 1110–1114

1. A line integral is taken along a curve; an ordinary single-variable integral is taken along an interval. 3. $\int_{\pi/2}^{\pi} \frac{1}{t} \cos t \sqrt{\sin^2 t + 1} dt$
 5. $\mathbf{r}(t) = \langle 1 + 4t, 2 + 2t, 3 - 3t \rangle$, for $0 \leq t \leq 1$
 7. $\mathbf{r}(t) = \langle t^2 + 1, t \rangle$, for $2 \leq t \leq 4$
 9. a. $\int_0^2 (t + 6t^5) dt$ b. 66 11. $\int_C \mathbf{F} \cdot d\mathbf{r}$ and $\int_C f dx + g dy + h dz$
 13. 7 15. Take the line integral of $\mathbf{F} \cdot \mathbf{T}$ along the curve with arc length as the parameter. 17. 0 19. 100 21. 8 23. $-40\pi^2$
 25. 128π 27. $\frac{\sqrt{2}}{2} \ln 10$ 29. $\frac{112}{9}$ 31. 8 33. 414 35. 409.5
 37. $\frac{15}{2}$ 39. $\sqrt{101}$ 41. $\frac{17}{2}$ 43. 49 45. $\frac{3}{4\sqrt{10}}$ 47. a. Negative
 b. Positive 49. 0 51. 16 53. 0 55. $\frac{3\sqrt{3}}{10}$ 57. b. 0
 59. a. Negative b. -4π 61. a. True b. True c. True d. True
 63. a. Both paths require the same work: $W = 28,200$.
 b. Both paths require the same work: $W = 28,200$.
 65. a. $\frac{5\sqrt{5} - 1}{12}$ b. $\frac{5\sqrt{5} - 1}{12}$ c. The results are identical.
 67. The work equals zero for all three paths.
 69. $8\pi(48 + 7\pi - 128\pi^2) \approx -29,991.4$ 71. 2π 73. a. 4 b. -4
 e. 0 75. Hint: Show that $\int_C \mathbf{F} \cdot \mathbf{T} ds = \pi r^2(c - b)$.
 77. Hint: Show that $\int_C \mathbf{F} \cdot \mathbf{n} ds = \pi r^2(a + d)$. 79. a. $\ln a$ b. No
 c. $\frac{1}{6} \left(1 - \frac{1}{a^2} \right)$ d. Yes e. $W = \frac{3^{1-p/2}}{2-p} (a^{2-p} - 1)$, for $p \neq 2$;
 otherwise, $W = \ln a$ f. $p > 2$ 81. ab

Section 17.3 Exercises, pp. 1121–1123

1. A simple curve has no self-intersections; the initial and terminal points of a closed curve are identical. 3. Test for equality of partial derivatives as given in Theorem 17.3. 5. Integrate f with respect to x and make the constant of integration a function of y to obtain $\varphi = \int f dx + h(y)$; finally, set $\frac{\partial \varphi}{\partial y} = g$ to determine h . 7. 0
 9. Conservative 11. Not conservative 13. Conservative
 15. Conservative 17. $\varphi(x, y) = \frac{1}{2}(x^2 + y^2)$ 19. Not conservative
 21. $\varphi(x, y) = \sqrt{x^2 + y^2}$ 23. $\varphi(x, y, z) = xz + y$
 25. Not conservative 27. $\varphi(x, y, z) = xy + yz + zx$
 29. $\varphi(x, y) = \sqrt{x^2 + y^2 + z^2}$ 31. a, b. 0 33. a, b. 2 35. 3
 37. -10 39. 24 41. $-\frac{1}{2}$ 43. $-\pi^2$ 45. 0 47. 0 49. 0
 51. -5 53. 0 55. 1 57. a. False b. True c. True d. True
 e. True 59. 10 61. 25 63. a. Negative b. Positive c. No
 67. a. Compare partial derivatives.

$$\text{b. } \varphi(x, y, z) = \frac{GMm}{\sqrt{x^2 + y^2 + z^2}} = \frac{GMm}{|\mathbf{r}|}$$

$$\text{c. } \varphi(B) - \varphi(A) = GMm \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \quad \text{d. No}$$

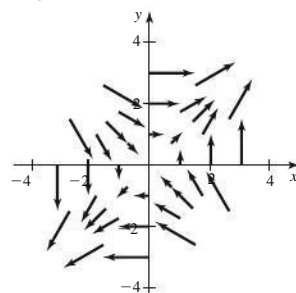
$$\text{69. a. } \frac{\partial}{\partial y} \left(\frac{-y}{(x^2 + y^2)^{p/2}} \right) = \frac{-x^2 + (p-1)y^2}{(x^2 + y^2)^{1+p/2}} \text{ and}$$

$$\frac{\partial}{\partial x} \left(\frac{x}{(x^2 + y^2)^{p/2}} \right) = \frac{(1-p)x^2 + y^2}{(x^2 + y^2)^{1+p/2}}$$

- b. The two partial derivatives in (a) are equal if $p = 2$.
 c. $\varphi(x, y) = \tan^{-1}(y/x)$ 73. $\varphi(x, y) = \frac{1}{2}(x^2 + y^2)$
 75. $\varphi(x, y) = \frac{1}{2}(x^4 + x^2y^2 + y^4)$

Section 17.4 Exercises, pp. 1133–1136

1. In both forms, the integral of a derivative is computed from boundary data. 3. Area $= \frac{1}{2} \oint_C x dy - y dx$, where C encloses the region. 5. The integral in the flux form of Green's Theorem vanishes.
 7. $\mathbf{F} = \langle y, x \rangle$



9. a. 0 b. 2 c. Yes d. No 11. a. -4 b. 0 c. No d. Yes
 13. a. y^2 b. $12x^2y + 2xy$ c. No d. No 15. a. 1; no
 b. $\mathbf{r}_1(t) = \langle t, t^2 \rangle$, for $0 \leq t \leq 1$, and $\mathbf{r}_2(t) = \langle 1 - t, 1 - t \rangle$, for
 $0 \leq t \leq 1$ (answers may vary) c. Both integrals equal $\frac{1}{6}$ d. 0
 17. a. -4 b. Both integrals equal -8 . 19. a. $4x$ b. $\frac{16}{3}$
 21. 25π 23. 16π 25. 32 27. a. 2 b. Both integrals equal 8π .
 29. a. $2y$ b. $\frac{16}{15}$ 31. 104 33. $\frac{31 - 3e^4}{6}$ 35. 6 37. $\frac{8}{3}$
 39. $8 - \frac{\pi}{2}$ 41. a. 0 b. 3π 43. a. 0 b. $-\frac{15\pi}{2}$ 45. a. 0
 b. 2π 47. a. $\frac{16}{3}$ b. 0 49. a. True b. False c. True

51. Note: $\frac{\partial f}{\partial y} = 0 = \frac{\partial g}{\partial x}$ 53. The integral becomes $\iint_R 2 \, dA$.

55. a. $f_x = g_y = 0$ b. $\psi(x, y) = -2x + 4y$

57. a. $f_x = e^{-x} \sin y = -g_y$ b. $\psi(x, y) = e^{-x} \cos y$

59. a. Hint: $f_x = e^x \cos y$, $f_y = -e^x \sin y$,

$g_x = -e^x \sin y$, $g_y = -e^x \cos y$

b. $\varphi(x, y) = e^x \cos y$, $\psi(x, y) = e^x \sin y$

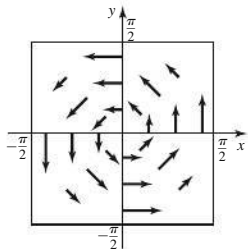
61. a. Hint: $f_x = -\frac{y}{x^2 + y^2}$, $f_y = \frac{x}{x^2 + y^2}$,

$g_x = \frac{x}{x^2 + y^2}$, $g_y = \frac{y}{x^2 + y^2}$

b. $\varphi(x, y) = x \tan^{-1} \frac{y}{x} + \frac{y}{2} \ln(x^2 + y^2) - y$,

$\psi(x, y) = y \tan^{-1} \frac{y}{x} - \frac{x}{2} \ln(x^2 + y^2) + x$

63. a.

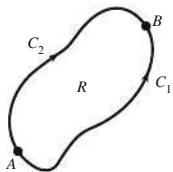


$\mathbf{F} = \langle -4 \cos x \sin y, 4 \sin x \cos y \rangle$ b. Yes, the divergence equals zero.

c. No, the two-dimensional curl equals $8 \cos x \cos y$. d. 0 e. 32

67. c. The vector field is undefined at the origin.

69.



Basic ideas: Let C_1 and C_2 be two smooth simple curves from A to B.

$$\int_{C_1} \mathbf{F} \cdot \mathbf{n} \, ds - \int_{C_2} \mathbf{F} \cdot \mathbf{n} \, ds = \oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_R (f_x + g_y) \, dA = 0$$

$$\text{and } \int_{C_1} \mathbf{F} \cdot \mathbf{n} \, ds = \int_{C_1} \psi_x \, dx + \psi_y \, dy = \int_{C_1} d\psi = \psi(B) - \psi(A)$$

71. Use $\nabla \varphi \cdot \nabla \psi = \langle f, g \rangle \cdot \langle -g, f \rangle = 0$.

Section 17.5 Exercises, pp. 1143–1146

1. Compute $f_x + g_y + h_z$. 3. There is no source or sink.

5. It indicates the axis and the angular speed of the circulation at a point. 7. 0 9. 3 11. 0 13. $2(x + y + z)$

15. $\frac{x^2 + y^2 + 3}{(1 + x^2 + y^2)^2}$ 17. $\frac{1}{|\mathbf{r}|^2}$ 19. $-\frac{1}{|\mathbf{r}|^4}$ 21. a. Positive for

both points b. $\text{div } \mathbf{F} = 2$ c. Outward everywhere d. Positive

23. a. $\text{curl } \mathbf{F} = 2\mathbf{i}$ b. $|\text{curl } \mathbf{F}| = 2$

25. a. $\text{curl } \mathbf{F} = 2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ b. $|\text{curl } \mathbf{F}| = 2\sqrt{3}$ 27. $3y\mathbf{k}$

29. $-4z\mathbf{j}$ 31. 0 33. 0 35. Follows from partial differentiation of

$$\frac{1}{(x^2 + y^2 + z^2)^{3/2}} \quad 37. \text{ Combine Exercise 36 with Theorem 17.10.}$$

39. a. False b. False c. False d. False e. False 41. a. No

b. No c. Yes, scalar function d. No e. No f. No g. Yes, vector field h. No i. Yes, vector field 43. a. At $(0, 1, 1)$, $\mathbf{F}$ points in the positive x -direction; at $(1, 1, 0)$, $\mathbf{F}$ points in the negative z -direction; at $(0, 1, -1)$, $\mathbf{F}$ points in the negative x -direction; and at $(-1, 1, 0)$, $\mathbf{F}$ points in the positive z -direction. These vectors

circle the y -axis in the counterclockwise direction looking along $\mathbf{a}$ from head to tail. b. The argument in part (a) can be repeated in any plane perpendicular to the y -axis to show that the vectors of $\mathbf{F}$ circle the y -axis in the counterclockwise direction looking along $\mathbf{a}$ from head to tail. Alternatively, computing the cross product, we find that $\mathbf{F} = \mathbf{a} \times \mathbf{r} = \langle z, 0, -x \rangle$, which is a rotation field in any plane perpendicular to $\mathbf{a}$. 45. Compute an explicit expression for $\mathbf{a} \times \mathbf{r}$ and then take the required partial derivatives. 47. $\text{div } \mathbf{F}$ has a maximum value of 6 at $(1, 1, 1)$, $(1, -1, -1)$, $(-1, 1, 1)$, and $(-1, -1, -1)$. 49. $\mathbf{n} = \langle a, b, 2a + b \rangle$, where a and b are real numbers 51. $\mathbf{F} = \frac{1}{2}(y^2 + z^2)\mathbf{i}$; no 53. a. The wheel does not spin.

b. Clockwise, looking in the positive y -direction c. The wheel does

not spin. 55. $\omega = \frac{10}{\sqrt{3}}$, or $\frac{5}{\sqrt{3}\pi} \approx 0.9189$ revolution per unit time

57. $\mathbf{F} = -200ke^{-x^2+y^2+z^2}(-x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$

$$\nabla \cdot \mathbf{F} = -200k(1 + 2(x^2 + y^2 + z^2))e^{-x^2+y^2+z^2}$$

59. a. $\mathbf{F} = -\frac{GMm\mathbf{r}}{|\mathbf{r}|^3}$ b. See Theorem 17.11.

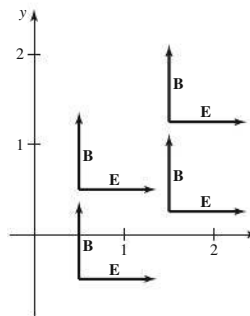
$$61. \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

63. a. Use $\nabla \times \mathbf{B} = -Ak \cos(kz - \omega t)\mathbf{i}$ and

$$\frac{\partial \mathbf{E}}{\partial t} = -A\omega \cos(kz - \omega t)\mathbf{i} \quad \text{b.}$$



Section 17.6 Exercises, pp. 1159–1161

1. $\mathbf{r}(u, v) = \langle a \cos u, a \sin u, v \rangle$, $0 \leq u \leq 2\pi$, $0 \leq v \leq h$

3. $\mathbf{r}(u, v) = \langle a \sin u \cos v, a \sin u \sin v, a \cos u \rangle$, $0 \leq u \leq \pi$, $0 \leq v \leq 2\pi$ 5. Use the parameterization from Exercise 3 and

compute $\int_0^\pi \int_0^{2\pi} f(a \sin u \cos v, a \sin u \sin v, a \cos u) a^2 \sin u \, dv \, du$.

7. The normal vectors point outward. 9. $\langle u, v, \frac{1}{3}(16 - 2u + 4v) \rangle$, $|u| < \infty$, $|v| < \infty$ 11. $\langle v \cos u, v \sin u, v \rangle$, $0 \leq u \leq 2\pi$,

$2 \leq v \leq 8$ 13. $\langle 3 \cos u, 3 \sin u, v \rangle$, $0 \leq u \leq \frac{\pi}{2}$, $0 \leq v \leq 3$

15. The plane $z = 2x + 3y - 1$ 17. Part of the upper half of the cone $z^2 = 16x^2 + 16y^2$ of height 12 and radius 3 (with $y \geq 0$)

19. 28π 21. $16\sqrt{3}$ 23. $\pi r \sqrt{r^2 + h^2}$ 25. 1728π 27. 0

29. 12 31. $4\pi\sqrt{5}$ 33. $\frac{(65\sqrt{65} - 1)\pi}{24}$ 35. $\frac{2\sqrt{3}}{3}$ 37. $\frac{1250\pi}{3}$

39. $e - 1$ 41. $\frac{1}{4\pi}$ 43. -8 45. 0 47. 4π 49. a. True

b. False c. True d. True 51. $8\pi(4\sqrt{17} + \ln(\sqrt{17} + 4))$

53. $8\pi a$ 55. 8 57. a. 0 b. 0; the flow is tangent to the surface

(radial flow). 59. $2\pi ah$ 61. $-400\left(e - \frac{1}{e}\right)^2$ 63. $8\pi a$

65. a. $4\pi(b^3 - a^3)$ b. The net flux is zero. 67. $(0, 0, \frac{2}{3}h)$

69. $(0, 0, \frac{7}{6})$ 73. $\text{Flux} = \iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \iint_R dA$

Section 17.7 Exercises, pp. 1169–1171

1. The integral measures the circulation along the closed curve C .
 3. Under certain conditions, the accumulated rotation of the vector field over the surface S equals the net circulation on the boundary of S .
 5. Both integrals equal -2π . 7. Both integrals equal zero. 9. Both integrals equal -18π . 11. -24π 13. $-\frac{128}{3}$ 15. 15π 17. 0
 19. 0 21. -2π 23. -4π 25. $\nabla \times \mathbf{v} = \langle 1, 0, 0 \rangle$; a paddle wheel with its axis aligned with the x -axis will spin with maximum angular speed counterclockwise (looking in the negative x -direction) at all points. 27. $\nabla \times \mathbf{v} = \langle 0, -2, 0 \rangle$; a paddle wheel with its axis aligned with the y -axis will spin with maximum angular speed clockwise (looking in the negative y -direction) at all points. 29. a. False b. False
 c. True d. True 31. 0 33. 0 35. 2π 37. $\pi(\cos \varphi - \sin \varphi)$; maximum for $\varphi = 0$ 39. The circulation is 48π ; it depends on the radius of the circle but not on the center. 41. a. The normal vectors point toward the z -axis on the curved surface of S and in the direction of $\langle 0, 1, 0 \rangle$ on the flat surface of S . b. 2π c. 2π 43. The integral is π for all a . 45. a. 0 b. 0 47. b. 2π for any circle of radius r centered at the origin c. $\mathbf{F}$ is not differentiable along the z -axis.

49. Apply the Chain Rule. 51. $\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z} \right) dA$

Section 17.8 Exercises, pp. 1179–1182

1. The surface integral measures the flow across the boundary.
 3. The flux across the boundary equals the cumulative expansion or contraction of the vector field inside the region. 5. 32π
 7. The outward fluxes are equal. 9. Both integrals equal 96π .
 11. Both integrals equal zero. 13. 0 15. 0 17. $16\sqrt{6}\pi$ 19. $\frac{2}{3}$
 21. $-\frac{128}{3}\pi$ 23. 24π 25. -224π 27. 12π 29. 20
 31. a. False b. False c. True 33. 0 35. $\frac{3}{2}$ 37. b. The net flux between the two spheres is $4\pi(a^2 - \varepsilon^2)$. 39. b. Use $\nabla \cdot \mathbf{E} = 0$.

- c. The flux across S is the sum of the contributions from the individual charges. d. For an arbitrary volume, we find

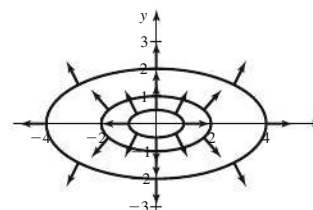
$$\frac{1}{\varepsilon_0} \iiint_D q(x, y, z) \, dV = \iint_S \mathbf{E} \cdot \mathbf{n} \, dS = \iiint_D \nabla \cdot \mathbf{E} \, dV.$$

e. Use $\nabla^2 \varphi = \nabla \cdot \nabla \varphi$. 41. 0 43. $e^{-1} - 1$ 45. $800\pi a^3 e^{-a^2}$

Chapter 17 Review Exercises, pp. 1182–1184

1. a. False b. True c. False d. False e. True

3. $\nabla \varphi = \langle 2x, 8y \rangle$



5. $-\frac{\mathbf{r}}{|\mathbf{r}|^3}$ 7. $\mathbf{n} = \frac{1}{2} \langle x, y \rangle$ 9. $\frac{7}{8}(e^{48} - 1)$ 11. Both integrals

equal zero. 13. 0 15. The circulation is -4π ; the outward flux is zero. 17. The circulation is zero; the outward flux is 2π .

19. $\frac{4v_0 L^3}{3}$ 21. $\varphi(x, y, z) = xy + yz^2$ 23. $\varphi(x, y, z) = xye^z$

25. 0 for both methods 27. a. $-\pi$ b. $\mathbf{F}$ is not conservative.

29. 0 31. $\frac{20}{3}$ 33. 8π 35. The circulation is zero; the outward flux equals 2π . 37. a. $b = c$ b. $a = -d$ c. $a = -d$ and $b = c$

39. $\nabla \cdot \mathbf{F} = 4\sqrt{x^2 + y^2 + z^2} = 4|\mathbf{r}|$, $\nabla \times \mathbf{F} = \mathbf{0}$, $\nabla \cdot \mathbf{F} \neq 0$; irrotational but not source free 41. $\nabla \cdot \mathbf{F} = 2y + 12xz^2$, $\nabla \times \mathbf{F} = \mathbf{0}$, $\nabla \cdot \mathbf{F} \neq 0$; irrotational but not source free

43. a. -1 b. 0 c. $\mathbf{n} = \frac{1}{\sqrt{3}} \langle -1, 1, 1 \rangle$ 45. 18π 47. $4\sqrt{3}$

49. $\frac{8\sqrt{3}}{3}$ 51. 8π 53. $4\pi a^2$ 55. a. Use $x = y = 0$ to confirm the highest point; use $z = 0$ to confirm the base. b. The hemisphere

S has the greater surface area— $2\pi a^2$ for S versus $\frac{5\sqrt{5} - 1}{6} \pi a^2$ for T .

57. 0 59. 99π 61. 0 63. $\frac{972}{5}\pi$ 65. $\frac{124}{5}\pi$ 67. $\frac{32}{3}$

Answers

CHAPTER D2

Section D2.1, pp. D2-10–D2-13

1. The order of a differential equation is the order of the highest order derivative in the equation. 3. A differential equation of the form $y''(t) + p(t)y'(t) + q(t)y(t) = f(t)$ is homogeneous if $f(t) = 0$ on the interval of interest. 5. If one function is a nonzero constant multiple of the other on the interval, then the two functions are linearly dependent. 7. Find two linearly independent solutions, y_1 and y_2 , of the corresponding homogeneous differential equation. Find any particular solution, y_p , of the original differential equation. The general solution is then $y = c_1y_1 + c_2y_2 + y_p$, where c_1 and c_2 are arbitrary constants.

9. order 2; linear; nonhomogeneous 11. order 2; nonlinear; nonhomogeneous

$$13. y'' - 4y = (3e^{2t} - 5e^{-2t})'' - 4(3e^{2t} - 5e^{-2t}) \\ = 12e^{2t} - 20e^{-2t} - 12e^{2t} + 20e^{-2t} \\ = 0$$

$$15. y'' - 9y = (4e^{3t} + 3e^{-3t} - 2t)'' - 9(4e^{3t} + 3e^{-3t} - 2t) \\ = 36e^{3t} + 27e^{-3t} - 36e^{3t} - 27e^{-3t} + 18t \\ = 18t$$

$$17. y'' - y' - 2y = (c_1e^{-t} + c_2e^{2t})'' - (c_1e^{-t} + c_2e^{2t})' \\ - 2(c_1e^{-t} + c_2e^{2t}) \\ = c_1e^{-t} + 4c_2e^{2t} - (-c_1e^{-t} + 2c_2e^{2t}) \\ - 2c_1e^{-t} - 2c_2e^{2t} \\ = 0$$

$$19. y'' + 6y' + 25y \\ = (c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t)'' \\ + 6(c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t)' \\ + 25(c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t) \\ = (-7c_1e^{-3t}\sin 4t - 24c_1e^{-3t}\cos 4t + 24c_2e^{-3t}\sin 4t - 7c_2e^{-3t}\cos 4t) \\ + 6(-3c_1e^{-3t}\sin 4t + 4c_1e^{-3t}\cos 4t - 4c_2e^{-3t}\sin 4t - 3c_2e^{-3t}\cos 4t) + 25(c_1e^{-3t}\sin 4t + c_2e^{-3t}\cos 4t) \\ = (-7 - 18 + 25)c_1e^{-3t}\sin 4t + (-24 + 24)c_1e^{-3t}\cos 4t \\ + (24 - 24)c_2e^{-3t}\sin 4t + (-7 - 18 + 25)c_2e^{-3t}\cos 4t \\ = 0$$

$$21. ty'' - (t+1)y' + y = t[c_1e^t + c_2(t+1)]'' - (t+1) \cdot [c_1e^t + c_2(t+1)]' + [c_1e^t + c_2(t+1)] \\ = t(c_1e^t)' - (t+1)(c_1e^t + c_2)' + [c_1e^t + c_2(t+1)] \\ = tc_1e^t - tc_1e^t - c_1e^t - (t+1)c_2 + c_1e^t + c_2(t+1) \\ = 0$$

$$23. y = c_1e^{-6t} + c_2e^{6t}$$

$$25. y = c_1e^{-t} + c_2te^{-t}$$

$$27. y'' - y = (e^{-3t})'' - e^{-3t} = 9e^{-3t} - e^{-3t} = 8e^{-3t}$$

$$29. y'' - 4y' + 4y = (t^2e^{2t})'' - 4(t^2e^{2t})' + 4t^2e^{2t} \\ = (4t^2e^{2t} + 8te^{2t} + 2e^{2t}) - 4(2te^{2t} + 2te^{2t}) + 4t^2e^{2t} \\ = (4 - 8 + 4)t^2e^{2t} + (8 - 8)te^{2t} + 2e^{2t} \\ = 2e^{2t}$$

$$31. \text{ Let } y_1 = \frac{e^{-t}}{2}, y_2 = \frac{e^{-t}}{2} + 3e^{7t}, \text{ and } y_3 = y_2 - y_1 = 3e^{7t}. \text{ Then}$$

$$y_1'' - 49y_1 = \left(\frac{e^{-t}}{2}\right)'' - 49\left(\frac{e^{-t}}{2}\right) = \frac{e^{-t}}{2} - \frac{49e^{-t}}{2} = -24e^{-t}$$

$$y_2'' - 49y_2 = \left(\frac{e^{-t}}{2} + 3e^{7t}\right)'' - 49\left(\frac{e^{-t}}{2} + 3e^{7t}\right) \\ = \frac{e^{-t}}{2} + 147e^{7t} - \frac{49e^{-t}}{2} - 147e^{7t} = -24e^{-t}$$

$$y_3'' - 49y_3 = (3e^{7t})'' - 49(3e^{7t}) = 147e^{7t} - 147e^{7t} = 0$$

$$33. \text{ Let } y_1 = -e^t, y_2 = 6e^{4t} - e^t, \text{ and } y_3 = y_2 - y_1 = 6e^{4t} - e^t - (-e^t) = 6e^{4t}. \text{ Then}$$

$$y_1'' - y_1' - 12y_1 = (-e^t)'' - (-e^t)' - 12(-e^t) \\ = -e^t + e^t + 12e^t = 12e^t$$

$$y_2'' - y_2' - 12y_2 \\ = (6e^{4t} - e^t)'' - (6e^{4t} - e^t)' - 12(6e^{4t} - e^t) \\ = (96e^{4t} - e^t) - (24e^{4t} - e^t) - 72e^{4t} + 12e^t = 12e^t \\ y_3'' - y_3' - 12y_3 = (6e^{4t})'' - (6e^{4t})' - 12(6e^{4t}) \\ = 96e^{4t} - 24e^{4t} - 72e^{4t} = 0$$

$$35. \text{ homogeneous solutions: } \sin \sqrt{2}t, \cos \sqrt{2}t \\ \text{general solution: } y = c_1 \sin \sqrt{2}t + c_2 \cos \sqrt{2}t + e^t$$

$$37. \text{ homogeneous solutions: } e^{\frac{3}{2}t} \sin 4t, e^{\frac{3}{2}t} \cos 4t \\ \text{general solution: } y = c_1e^{\frac{3}{2}t} \sin 4t + c_2e^{\frac{3}{2}t} \cos 4t + 100t + 48$$

$$39. y = 4 \cos 3t \quad 41. y = -e^{5t} - 2e^{-4t}$$

$$43. y = \frac{1}{16}e^{4t} + \frac{1}{16}e^{-4t} - t^2 - \frac{1}{8} \quad 45. y = \frac{3}{4}t^{-2} + \frac{1}{4}t^2$$

$$47. \text{ a. False b. True c. False d. False e. False}$$

$$49. y'' - 12y' + 36y = (c_1e^{6t} + c_2te^{6t} + t^2e^{6t})'' \\ - 12(c_1e^{6t} + c_2te^{6t} + t^2e^{6t})' + 36(c_1e^{6t} + c_2te^{6t} + t^2e^{6t}) \\ = (36c_1e^{6t} + 12c_2e^{6t} + 36c_2te^{6t} + 2e^{6t} + 24te^{6t} + 36t^2e^{6t}) \\ - 12(6c_1e^{6t} + c_2e^{6t} + 6c_2te^{6t} + 2te^{6t} + 6t^2e^{6t}) \\ + 36(c_1e^{6t} + c_2te^{6t} + t^2e^{6t}) \\ = (36 - 72 + 36)c_1e^{6t} + (12 - 12)c_2e^{6t} + (36 - 72 + 36)c_2te^{6t} \\ + 2e^{6t} + (24 - 24)te^{6t} + (36 - 72 + 36)t^2e^{6t} \\ = 2e^{6t}$$

$$51. t^2y'' - 3ty' + 4y \\ = t^2(c_1t^2 + c_2t^2 \ln t)'' - 3t(c_1t^2 + c_2t^2 \ln t)' + 4(c_1t^2 + c_2t^2 \ln t) \\ = t^2(2c_1 + 3c_2 + 2c_2 \ln t) - 3t(2c_1t + c_2t + 2c_2t \ln t) \\ + 4(c_1t^2 + c_2t^2 \ln t) = (2c_1 + 3c_2 - 6c_1 - 3c_2 + 4c_1)t^2 \\ + (2c_2 - 6c_2 + 4c_2)t^2 \ln t \\ = 0$$

$$53. t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right)y \\ = t^2(c_1t^{-1/2} \cos t + c_2t^{-1/2} \sin t)'' + t(c_1t^{-1/2} \cos t + c_2t^{-1/2} \sin t)' \\ + \left(t^2 - \frac{1}{4}\right)(c_1t^{-1/2} \cos t + c_2t^{-1/2} \sin t) \\ = t^2\left(\frac{3}{4}c_1t^{-5/2} \cos t + \frac{3}{4}c_2t^{-5/2} \sin t - c_2t^{-3/2} \cos t + c_1t^{-3/2} \sin t \right. \\ \left. - c_1t^{-1/2} \cos t - c_2t^{-1/2} \sin t\right) \\ + t\left(-c_1t^{-1/2} \sin t + c_2t^{-1/2} \cos t - \frac{1}{2}c_1t^{-3/2} \cos t - \frac{1}{2}c_2t^{-3/2} \sin t\right) \\ + \left(t^2 - \frac{1}{4}\right)(c_1t^{-1/2} \cos t + c_2t^{-1/2} \sin t)$$

$$= \left(\frac{3}{4} - \frac{1}{2} - \frac{1}{4}\right)c_1 t^{-1/2} \cos t + \left(\frac{3}{4} - \frac{1}{2} - \frac{1}{4}\right)c_2 t^{-1/2} \sin t \\ + (-1 + 1)c_2 t^{1/2} \cos t + (1 - 1)c_1 t^{1/2} \sin t \\ + (-1 + 1)c_1 t^{3/2} \cos t + (-1 + 1)c_2 t^{3/2} \sin t \\ = 0$$

$$55. \mathbf{a.} \quad y'' - y = (e^t)'' - e^t = e^t - e^t = 0$$

$$y'' - y = (e^{-t})'' - e^{-t} = e^{-t} - e^{-t} = 0$$

The functions are not multiples of each other and so are a linearly independent set of solutions. **b.** $y = \sinh t$ and $y = \cosh t$ are linear combinations of the solutions e^{-t} and e^t and so must be solutions themselves by the Superposition Principle. They are independent since

$$\text{one is not a multiple of the other. } \mathbf{c.} \quad (\sinh t)' = \frac{e^t + e^{-t}}{2} = \cosh t$$

$$\text{and } (\cosh t)' = \frac{e^t - e^{-t}}{2} = \sinh t \text{ so } (\sinh t)'' - \sinh t =$$

$$(\cosh t)' - \sinh t = \sinh t - \sinh t = 0$$

$$(\cosh t)'' - \cosh t = (\sinh t)' - \cosh t = \cosh t - \cosh t = 0$$

$$\mathbf{d.} \quad y = c_1 e^{-t} + c_2 e^t, y = c_3 \sinh t + c_4 \cosh t \quad \mathbf{e.} \quad \text{For } y = e^{kt},$$

$$y'' - k^2 y = (e^{kt})'' - k^2 e^{kt} = (k e^{kt})' - k^2 e^{kt} = k^2 e^{kt} - k^2 e^{kt} = 0.$$

$$\text{For } y = e^{-kt}, y'' - k^2 y = (e^{-kt})'' - k^2 e^{-kt} = (-k e^{-kt})' - k^2 e^{-kt} =$$

$$k^2 e^{-kt} - k^2 e^{-kt} = 0. \quad \mathbf{f.} \quad y = c_1 e^{-kt} + c_2 e^{kt}, y = c_3 \sinh kt + c_4 \cosh kt$$

$$57. \quad y^{(4)} - 16y$$

$$= (c_1 e^{-2t} + c_2 e^{2t} + c_3 \sin 2t + c_4 \cos 2t)^{(4)}$$

$$= 16(c_1 e^{-2t} + c_2 e^{2t} + c_3 \sin 2t + c_4 \cos 2t)$$

$$= (16c_1 e^{-2t} + 16c_2 e^{2t} + 16c_3 \sin 2t + 16c_4 \cos 2t)$$

$$= 16(c_1 e^{-2t} + c_2 e^{2t} + c_3 \sin 2t + c_4 \cos 2t)$$

$$= 0$$

$$59. \mathbf{a.} \quad \text{By the chain rule } \frac{d}{dt}(y'(t)^2) = 2y'(t) \frac{d}{dt}y'(t) = 2y'(t)y''(t)$$

$$\mathbf{b.} \quad y''y' = 1 \Rightarrow 2y''y' = 2 \Rightarrow (y'(t)^2)' = 2 \text{ using part (a).}$$

$$\mathbf{c.} \quad (y'(t)^2)' = 2 \Rightarrow y'(t)^2 = \int 2 dt = 2t + c_1 \text{ so}$$

$$y'(t) = \pm \sqrt{2t + c_1} \quad \mathbf{d.} \quad \text{From part (c), } y(t) = \int \pm \sqrt{2t + c_1} dt =$$

$$\pm \left(\frac{1}{2}\right)^{2/3} (2t + c_1)^{3/2} + c_2 = c_2 \pm \frac{1}{3} (2t + c_1)^{3/2}$$

$$61. \quad y = c_1 e^{3t} + c_2 - \frac{4}{3}t \quad 63. \quad y = -\frac{1}{\sqrt{c_1}} \tan^{-1} \frac{t}{\sqrt{c_1}} + c_2 \text{ if } c_1 > 0;$$

$$y = -\frac{1}{2\sqrt{|c_1|}} \ln \left| \frac{t - \sqrt{|c_1|}}{t + \sqrt{|c_1|}} \right| + c_2 \text{ if } c_1 < 0$$

$$65. \mathbf{a.} \quad y'' + 3y' + \frac{25}{4}y$$

$$= [e^{-3t/2}(c_1 \sin 2t + c_2 \cos 2t)]'' + 3[e^{-3t/2}(c_1 \sin 2t + c_2 \cos 2t)]' \\ + \frac{25}{4}[e^{-3t/2}(c_1 \sin 2t + c_2 \cos 2t)]$$

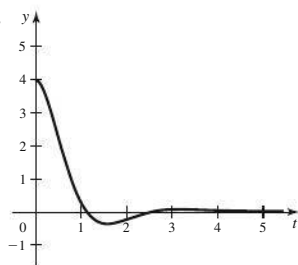
$$= \left(6c_2 - \frac{7}{4}c_1\right)e^{-3t/2} \sin 2t + \left(-6c_1 - \frac{7}{4}c_2\right)e^{-3t/2} \cos 2t$$

$$+ \left(-6c_2 - \frac{9}{2}c_1\right)e^{-3t/2} \sin 2t + \left(6c_1 - \frac{9}{2}c_2\right)e^{-3t/2} \cos 2t$$

$$+ \frac{25}{4}c_1 e^{-3t/2} \sin 2t + \frac{25}{4}c_2 e^{-3t/2} \cos 2t$$

$$= 0$$

$$\mathbf{b.} \quad y = e^{-3t/2}(3 \sin 2t + 4 \cos 2t) \quad \mathbf{c.}$$

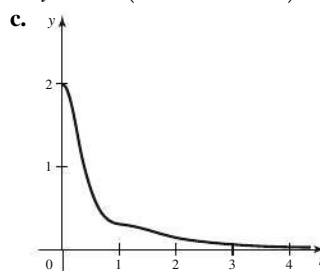


$$67. \mathbf{a.} \quad y'' + 6y' + 25y$$

$$= [e^{-3t}(c_1 \sin 4t + c_2 \cos 4t)]'' + 6[e^{-3t}(c_1 \sin 4t + c_2 \cos 4t)]' \\ + 25[e^{-3t}(c_1 \sin 4t + c_2 \cos 4t)]$$

$$= (24c_2 - 7c_1)e^{-3t} \sin 4t + (-24c_1 - 7c_2)e^{-3t} \cos 4t + e^{-t} \\ + (-24c_2 - 18c_1)e^{-3t} \sin 4t + (24c_1 - 18c_2)e^{-3t} \cos 4t - 6e^{-t} \\ + 25c_1 e^{-3t} \sin 4t + 25c_2 e^{-3t} \cos 4t + 25e^{-t} \\ = 20e^{-t}$$

$$\mathbf{b.} \quad y = e^{-3t}(\sin 4t + \cos 4t) + e^{-t}$$



$$69. \mathbf{a.} \quad \text{By the Chain Rule } \frac{d}{dt}\varphi(x) = \frac{d\varphi}{dx} \frac{dx}{dt} = -F(x)x'(t), \text{ then}$$

$$\frac{d}{dt} \left[\frac{1}{2} m(x'(t))^2 + \varphi(x) \right] = 2 \left(\frac{1}{2} m \right) x'(t) x''(t) - F(x) x'(t) \\ = mx'(t)x''(t) - F(x)x'(t) \\ = x'(t)(mx''(t) - F(x)) \\ = x'(t)(0) = 0$$

$$\text{Since } v = x'(t), \text{ this relation can be written } \frac{d}{dt} \left[\frac{1}{2} mv^2 + \varphi(x) \right] = 0.$$

$$\mathbf{b.} \quad \text{By part a., the derivative of } E(t) \text{ is identically 0, so } E(t) \text{ is constant.}$$

Section D2.2, pp. D2-23–D2-25

$$1. \quad y = e^{rt}$$

$$3. \quad \text{Case 1: real distinct roots}$$

$$\text{Case 2: real repeated roots}$$

$$\text{Case 3: complex roots}$$

$$5. \quad y = c_1 e^{rt} + c_2 t e^{rt} \quad 7. \quad y = c_1 e^{-2t} \sin 3t + c_2 e^{-2t} \cos 3t$$

$$9. \quad y = c_1 e^{-5t} + c_2 e^{5t} \quad 11. \quad y = c_1 e^{3t} + c_2$$

$$13. \quad y = c_1 e^{-5t} + c_2 e^{2t} \quad 15. \quad y = c_1 e^{-6t} + c_2 e^{6t}; y = \frac{3}{2} e^{-6t} + \frac{3}{2} e^{6t}$$

$$17. \quad y = c_1 e^{-3t} + c_2 e^{6t}; y = -\frac{4}{9} e^{-3t} + \frac{4}{9} e^{6t}$$

$$19. \quad y = c_1 e^{-t/2} + c_2 e^{5t/2}; y = \frac{5}{2} e^{-t/2} + \frac{1}{2} e^{5t/2}$$

$$21. \quad y = c_1 e^t + c_2 t e^t; y = 4e^t - 4te^t$$

$$23. \quad y = c_1 e^{t/2} + c_2 t e^{t/2}; y = e^{t/2} + \frac{3}{2} t e^{t/2}$$

$$25. \quad y = c_1 e^{-2t} + c_2 t e^{-2t}; y = e^{-2t} + 2t e^{-2t}$$

$$27. \quad y = c_1 \sin 3t + c_2 \cos 3t; y = -\frac{8}{3} \sin 3t + 8 \cos 3t$$

$$29. \quad y = c_1 e^t \sin 2t + c_2 e^t \cos 2t; y = -e^t \sin 2t + e^t \cos 2t$$

$$31. \quad y = c_1 e^{-3t} \sin t + c_2 e^{-3t} \cos t; y = 6e^{-3t} \sin t$$

$$33. \quad y = t^{-1} + t \quad 35. \quad y = \frac{31}{8} t^{-3} + \frac{17}{8} t^5 \quad 37. \quad y = 4t^{-3} - 4t^{-2}$$

$$39. \mathbf{a.} \quad \text{False} \quad \mathbf{b.} \quad \text{False} \quad \mathbf{c.} \quad \text{False} \quad \mathbf{d.} \quad \text{True} \quad \mathbf{e.} \quad \text{False}$$

$$\begin{aligned}
 41. A \cos(\omega t + \varphi) &= A \cos \omega t \cos \varphi - A \sin \omega t \sin \varphi \\
 &= c_1 \sin \omega t + c_2 \cos \omega t \\
 \Rightarrow c_1 &= -A \sin \varphi, c_2 = A \cos \varphi \\
 \Rightarrow \sqrt{c_1^2 + c_2^2} &= \sqrt{A^2 \sin^2 \varphi + A^2 \cos^2 \varphi} = \sqrt{A^2(1)} = A \\
 \Rightarrow -\frac{c_1}{c_2} &= \frac{A \sin \varphi}{A \cos \varphi} = \tan \varphi
 \end{aligned}$$

$$43. y = 3\sqrt{2} \sin\left(4t + \frac{3\pi}{4}\right) \quad 45. y = 2 \sin\left(2t + \frac{2\pi}{3}\right)$$

$$47. y = c_1 e^{-2t} + c_2 e^{3t} + c_3 \quad 49. y = c_1 e^{2t} + c_2 e^{4t} + c_3$$

$$51. y = c_1 \sin t + c_2 \cos t + c_3 \sin 2t + c_4 \cos 2t$$

$$53. y = c_1 t^{-1} + c_2 t^{-1} \ln t \quad 55. y = c_1 t^{-3} + c_2 t^{-3} \ln t$$

$$57. y = c_1 t^{-3} \sin(4 \ln t) + c_2 t^{-3} \cos(4 \ln t)$$

$$59. y = \sqrt{t} \left(c_1 \sin \frac{\ln t}{2} + c_2 \cos \frac{\ln t}{2} \right)$$

$$61. y = c_1 \sin kt + c_2 \cos kt + c_3 t + c_4$$

63. a. As seen in the text, the associated quadratic equation is $p^2 + (a-1)p + b = 0$. If there is a repeated root, the discriminant in the quadratic formula is 0, and the root is

$$p = \frac{-(a-1) \pm \sqrt{0}}{2} = \frac{1-a}{2}. \quad \text{b. If } y = t^p v(t) \text{ then}$$

$$\begin{aligned}
 t^2 y'' + aty' + by &= t^2(t^p v)'' + at(t^p v)' + b(t^p v) \\
 &= t^2(t^{p+2} v'' + 2pt^{p+1} v' + p(p-1)t^{p-2} v) + at(t^p v' + pt^{p-1} v) + bt^p v \\
 &= t^p [t^2 v'' + (2p+a)tv' + (p^2 + (a-1)p + b)v] \\
 &= t^p [t^2 v'' + (1)tv' + (0)v] \quad (\text{using part (a) and the fact that } p \text{ satisfies } p^2 + (a-1)p + b = 0) \\
 &= t^p (t^2 v'' + tv') = 0
 \end{aligned}$$

$$\text{so } t^2 v'' + tv' = 0.$$

$$\text{c. If } w = v' \text{ then } t^2 v'' + tv' = 0 \Rightarrow t^2 w' + tw = 0$$

$$\Rightarrow t^2 \frac{dw}{dt} + tw = 0 \Rightarrow \frac{dw}{w} = -\frac{dt}{t}. \text{ Integrating both sides gives}$$

$$\ln w = -\ln t + c \Rightarrow e^{\ln w} = w = e^{-\ln t + c} = e^c e^{-\ln t} = c_1 \left(\frac{1}{t} \right)$$

$$\text{so } w = \frac{c_1}{t}. \quad \text{d. Integrating both sides gives: } w = v' = \frac{c_1}{t} \Rightarrow$$

$$v = c_1 \ln t + c_2. \text{ Then } y = t^p v(t) = t^p (c_1 \ln t + c_2) = c_1 t^p \ln t + c_2 t^p \text{ is the general solution. } \quad 65. \text{ a. } u \text{ has the form of the general solution with } c_1 = \frac{1}{r_1 - r_2} \text{ and } c_2 = -\frac{1}{r_1 - r_2}. \quad \text{b. Apply}$$

$$\text{L'Hôpital's rule, treating } r_1 \text{ as a variable: } \lim_{r_1 \rightarrow r_2} \frac{e^{r_1 t} - e^{r_2 t}}{r_1 - r_2} =$$

$$\lim_{r_1 \rightarrow r_2} \frac{te^{r_1 t}}{1} = te^{r_2 t}. \text{ If } r_1 = r_2 \text{ the linearly independent solutions are } e^{r_1 t} \text{ and } te^{r_1 t}.$$

Section D2.3, pp. D2-33–D2-34

1. Find two linearly independent solutions, y_1 and y_2 , of the corresponding homogeneous equation. Find a particular solution, y_p , of the equation. Form the general solution, $y = c_1 y_1 + c_2 y_2 + y_p$.

$$3. y_p = Ae^{-4t} \quad 5. y_p = Ae^{-t} \sin 4t + Be^{-t} \cos 4t$$

$$7. y_p = (At^2 + Bt + C)e^{-t} \quad 9. y_p = -\frac{2}{9}t - \frac{1}{9}$$

$$11. y_p = -\frac{1}{9}t^4 - \frac{2}{27}t^3 - \frac{1}{18}t^2 - \frac{7}{162}t - \frac{13}{972} \quad 13. y_p = \frac{1}{3}e^{-2t}$$

$$15. y_p = -\frac{6}{11}e^{-3t} \quad 17. y_p = -\frac{3}{5} \sin 2t \quad 19. y_p = -\frac{1}{5} \sin t - \frac{2}{5} \cos t$$

$$21. y_p = -\frac{2}{3}e^t + \frac{1}{4} \quad 23. y_p = \left(-\frac{1}{6}t + \frac{1}{72}\right)e^{-t}$$

$$25. y_p = \frac{1}{36} \sin 2t + \frac{1}{12}t \cos 2t$$

$$27. y_p = \left(\frac{1}{5}t + \frac{13}{50}\right) \sin t - \left(\frac{2}{5}t - \frac{9}{50}\right) \cos t \quad 29. y_p = \frac{3}{2}te^t$$

$$31. y_p = -\frac{4}{5}te^{-3t} \quad 33. y_p = 2te^{-2t} \quad 35. \text{ a. False b. False c. True}$$

$$37. y = c_1 \sin t + c_2 \cos t - \frac{4}{3} \sin 2t; y = \frac{8}{3} \sin t + \cos t - \frac{4}{3} \sin 2t$$

$$39. y = c_1 e^{-2t} \sin t + c_2 e^{-2t} \cos t + \frac{12}{5};$$

$$y = -\frac{19}{5}e^{-2t} \sin t - \frac{7}{5}e^{-2t} \cos t + \frac{12}{5}$$

$$41. y = c_1 \sin 3t + c_2 \cos 3t + t \sin 3t; y = t \sin 3t$$

$$43. y_p = -3t^4 - 9t^3 - 63t^2 - 181t - 307$$

$$45. y_p = \left(-\frac{25}{13}t - \frac{1550}{1521}\right)e^{-t} \sin 3t + \left(\frac{50}{39}t - \frac{150}{169}\right)e^{-t} \cos 3t$$

$$47. y_p = -e^t \quad 49. y_p = \left(t - \frac{10}{3}\right)e^{2t}$$

51. Assume $y_{p1}'' + py_{p1}' + qy_{p1} = f(t)$ and $y_{p2}'' + py_{p2}' + qy_{p2} = g(t)$. Then

$$\begin{aligned}
 (y_{p1} + y_{p2})'' + p(y_{p1} + y_{p2})' + q(y_{p1} + y_{p2}) &= (y_{p1}'' + y_{p2}'') + p(y_{p1}' + y_{p2}') + q(y_{p1} + y_{p2}) \\
 &= y_{p1}'' + y_{p2}'' + py_{p1}' + py_{p2}' + qy_{p1} + qy_{p2} \\
 &= (y_{p1}'' + py_{p1}' + qy_{p1}) + (y_{p2}'' + py_{p2}' + qy_{p2}) \\
 &= f(t) + g(t)
 \end{aligned}$$

Section D2.4, pp. D2-47–D2-52

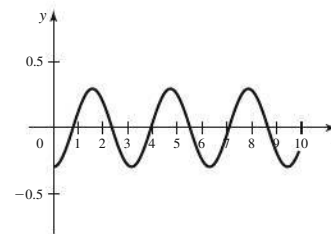
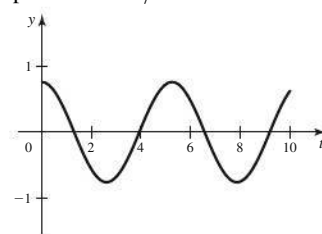
1. A system is damped if the object in motion encounters resistance to the movement; otherwise the motion is undamped. The motion is free if no external forces act on the object; otherwise, the motion is forced.

3. Beats occur when the natural frequency of the oscillator ω_0 is close in value to the forcing frequency ω .

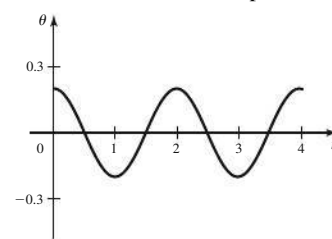
5. Find a solution y_h to the homogeneous equation. Find a particular solution y_p , form the general solution $y_h + y_p$, and evaluate constants using the initial conditions.

$$7. y = 0.75 \cos 1.2t; \text{ period} = 5\pi/3$$

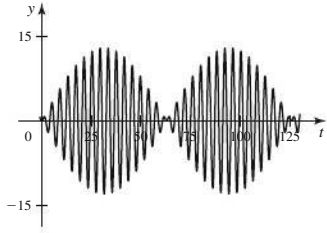
$$9. y = -0.3 \cos 2t; \text{ period} = \pi$$



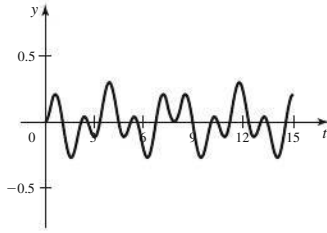
$$11. \theta = 0.2 \cos \sqrt{10}t; \text{ period} = 2\pi/\sqrt{10}$$



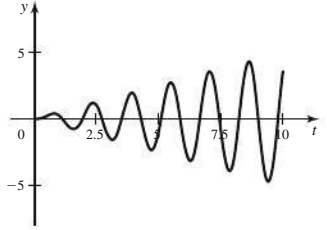
13. a. $\omega = 1.5$: $y = \frac{200}{31} \cos \frac{3}{2}t - \frac{200}{31} \cos \frac{8}{5}t$. This solution has beats.



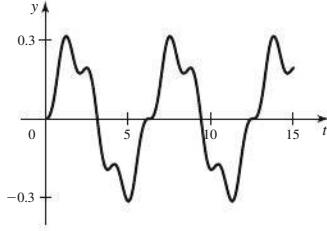
b. $\omega = 4$: $y = \frac{25}{168} \cos \frac{8}{5}t - \frac{25}{168} \cos 4t$



15. a. $\omega = 4$: $y = \frac{1}{8} \sin 4t - \frac{1}{2}t \cos 4t$; resonance case

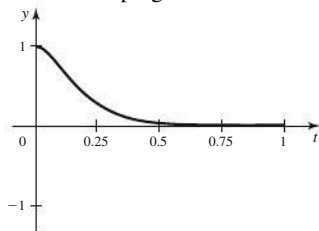
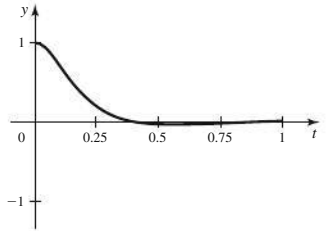


b. $\omega = 1$: $y = \frac{4}{15} \sin t - \frac{1}{15} \sin 4t$

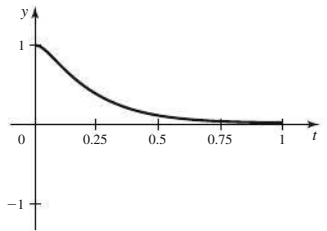


17. a. $y = \frac{4}{3}e^{-8t} \sin 6t + e^{-8t} \cos 6t$; b. $y = e^{-10t} + 10te^{-10t}$; critical damping

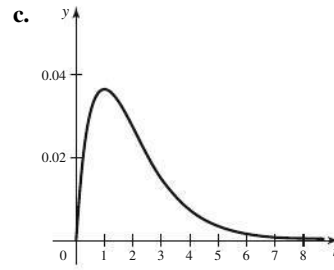
underdamping



c. $y = -\frac{1}{3}e^{-20t} + \frac{4}{3}e^{-5t}$; overdamping

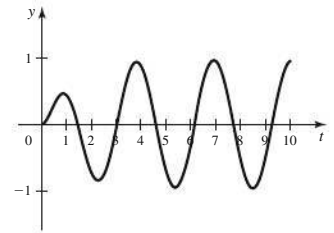


19. a. $k = 250 \text{ N/m}$ b. $y = \frac{1}{10}te^{-t}$



The maximum displacement is $\frac{1}{10}e^{-1} \approx 0.037 \text{ m}$. d. With an increase in k of 50%, the motion becomes underdamped, maximum displacement decreases to 0.034 m. With an increase in k of 100%, the motion becomes underdamped; maximum displacement decreases to 0.032 m. e. With a decrease in k of 50%, the motion becomes overdamped; maximum displacement increases to 0.041 m.

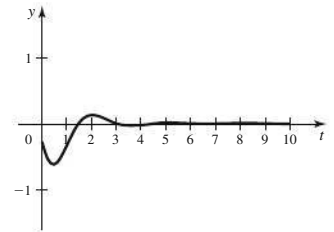
21. a. $y = -\frac{18}{17}e^{-t} \sin 2t - \frac{4}{17}e^{-t} \cos 2t + \frac{16}{17} \sin 2t + \frac{4}{17} \cos 2t$



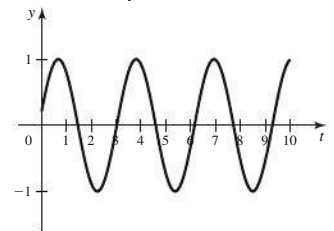
b. transient solution: $y = -\frac{18}{17}e^{-t} \sin 2t - \frac{4}{17}e^{-t} \cos 2t$

steady-state: $y = \frac{16}{17} \sin 2t + \frac{4}{17} \cos 2t$

Transient solution



Steady-state solution



c. After approximately 3 seconds.

23. a. $y = c_1 e^{-t/2} \sin t + c_2 e^{-t/2} \cos t + \frac{32\omega}{16\omega^4 - 24\omega^2 + 25} \sin \omega t - \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25} \cos \omega t$

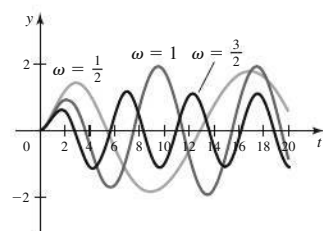
b. $y = -\frac{16\omega^2 + 20}{16\omega^4 - 24\omega^2 + 25} e^{-t/2} \sin t + \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25} e^{-t/2} \cos t + \frac{32\omega}{16\omega^4 - 24\omega^2 + 25} \sin \omega t - \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25} \cos \omega t$

c. transient solution:

$$y = -\frac{16\omega^2 + 20}{16\omega^4 - 24\omega^2 + 25} e^{-t/2} \sin t + \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25} e^{-t/2} \cos t$$

steady-state solution:

$$\frac{32\omega}{16\omega^4 - 24\omega^2 + 25} \sin \omega t - \frac{32\omega^2 - 40}{16\omega^4 - 24\omega^2 + 25} \cos \omega t$$



25. When $L = 0$, the equation $LQ'' + RQ' + \frac{1}{C}Q = E(t)$; becomes

$$RQ' + \frac{1}{C}Q = E(t). \text{ Solution is } Q = -\frac{1}{20}e^{-20t} + \frac{1}{20} \text{ and it}$$

approaches a steady-state value of $\frac{1}{20}$.

27. a. $y = 50e^{-40t} - 50e^{-60t}$; steady-state current: $y = 0$

29. a. $y = \frac{40\sqrt{15}}{3}e^{-80t} \sin 2\sqrt{15}t$ b. transient current:

$$y = \frac{40\sqrt{15}}{3}e^{-80t} \sin 2\sqrt{15}t; \text{ steady-state current: } y = 0$$

31. $Q = \frac{12}{5} - \frac{4}{5}e^{-4t}(3 \cos 3t + 4 \sin 3t); I = 20e^{-4t} \sin 3t$

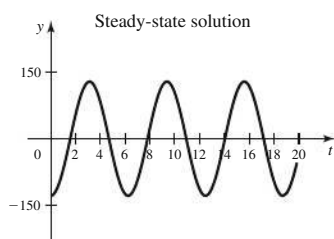
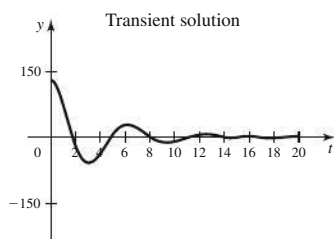
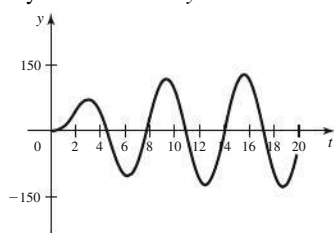
33. a. True b. True c. True d. False e. False f. True

35. a. $y = 16e^{-t/4} \sin t + 128e^{-t/4} \cos t + 16 \sin t - 128 \cos t$

transient solution: $y = 16e^{-t/4} \sin t + 128e^{-t/4} \cos t$

steady-state solution: $y = 16 \sin t - 128 \cos t$

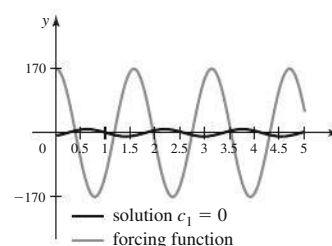
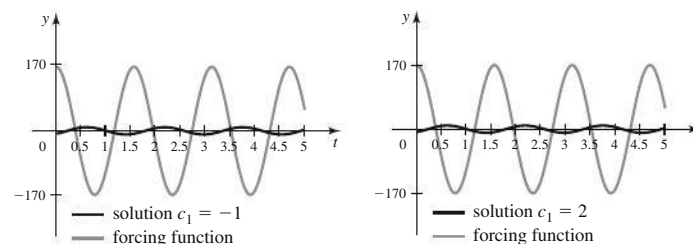
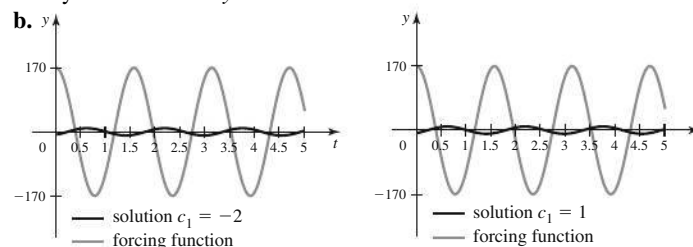
b. c. Around $t = 20$;
 $\lim_{t \rightarrow \infty} (16e^{-t/4} \sin t + 128e^{-t/4} \cos t) = 0$ so the solution approaches the steady-state solution.



37. a. $y = c_1 e^{-2t} + c_2 e^{-t} + 6 \sin 4t - 7 \cos 4t$

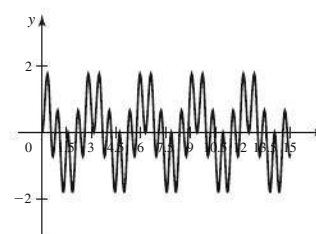
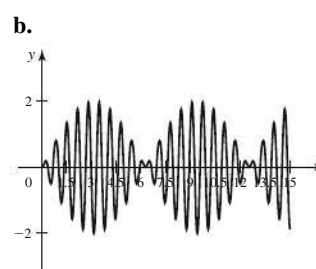
transient solution: $y = c_1 e^{-2t} + c_2 e^{-t}$

steady-state solution: $y = 6 \sin 4t - 7 \cos 4t$



c. In all cases, the solution approaches the steady state solution quickly. The steady-state solution has smaller magnitude than the external force function and is shifted.

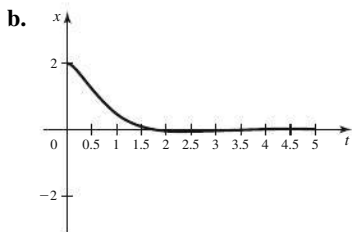
$$39. \text{ a. } \cos \omega t - \cos \omega_0 t = 2 \sin \left(\frac{\omega_0 t - \omega t}{2} \right) \sin \left(\frac{\omega_0 t + \omega t}{2} \right) \\ = 2 \sin \frac{(\omega_0 - \omega)t}{2} \sin \frac{(\omega_0 + \omega)t}{2}$$



$$\begin{aligned}
 41. C^* &= \frac{F_0}{\sqrt{c^2\omega^2 + m^2(\omega_0^2 - \omega^2)^2}} = \frac{\omega E_0}{\sqrt{R^2\omega^2 + L^2\left(\frac{1}{CL} - \omega^2\right)^2}} \\
 &= \frac{\omega E_0}{\sqrt{R^2\omega^2 + \omega^2 \frac{L^2}{\omega^2} \left(\frac{1}{CL} - \omega^2\right)^2}} \\
 &= \frac{\omega E_0}{\sqrt{R^2\omega^2 + \omega^2 \left(\frac{L}{\omega} \left(\frac{1}{CL} - \omega^2\right)\right)^2}} = \frac{\omega E_0}{\omega \sqrt{R^2 + \left(\frac{L}{\omega} \left(\frac{1}{CL} - \omega^2\right)\right)^2}} \\
 &= \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}} \\
 &= \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}
 \end{aligned}$$

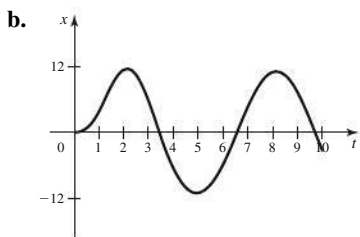
$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$ is smallest when $\omega L = \frac{1}{\omega C}$ or $\omega^2 = \frac{1}{CL}$.

$$43. \text{ a. } x = \frac{4\sqrt{7}}{7} e^{-3t/2} \sin \frac{\sqrt{7}}{2} t + 2e^{-3t/2} \cos \frac{\sqrt{7}}{2} t$$



c. The motion is underdamped.

$$45. \text{ a. } x = -\frac{22}{5} e^{-t/2} \sin 2t + \frac{16}{5} e^{-t/2} \cos 2t + \frac{52}{5} \sin t - \frac{16}{5} \cos t$$



c. The motion approaches a steady-state solution of

$$x = \frac{52}{5} \sin t - \frac{16}{5} \cos t.$$

47. a. There are two forces acting: inertia, given by $ma = m s''(t) = m \ell \theta''(t)$ and weight of the bob. The component of weight in the direction of motion is $mg \sin \theta(t)$. By Newton's law, the sum of the forces is 0 or $m \ell \theta''(t) + mg \sin \theta(t) = 0 \Rightarrow m \ell \theta''(t) = -mg \sin \theta(t)$.

$$\text{b. } m \ell \theta''(t) = -mg \sin \theta(t) \Rightarrow \theta''(t) = -\frac{g}{\ell} \sin \theta(t)$$

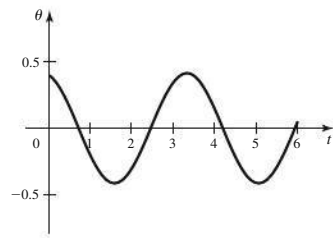
$$= -\omega_0^2 \sin \theta(t) \text{ or } \theta'' + \omega_0^2 \sin \theta = 0$$

c. Using $\sin \theta \approx \theta$ in the equation from (c) gives $\theta'' + \omega_0^2 \theta = 0$.

d. The frequency is given by $\omega_0^2 = \frac{g}{\ell}$ or $\omega_0 = \sqrt{\frac{g}{\ell}}$. The period is

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\frac{g}{\ell}}} = 2\pi \sqrt{\frac{\ell}{g}}. \text{ Doubling the length increases the period}$$

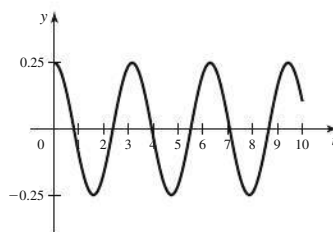
by a factor of $\sqrt{2}$. 49. a. $\theta(t) = -\frac{\sqrt{15}}{35} \sin \frac{7\sqrt{15}}{15} t + \frac{2}{5} \cos \frac{7\sqrt{15}}{15} t$



b. $\theta(t) = \frac{\sqrt{211}}{35} \sin \left(\frac{7\sqrt{15}}{15} t + \varphi \right)$ where $\tan \varphi = -\frac{14\sqrt{15}}{15}$
or $\theta(t) \approx 0.415 \sin (1.807t + 1.841)$.

$$\text{c. } T = 2\pi \sqrt{\frac{\ell}{g}} = 2\pi \sqrt{\frac{3}{9.8}} = 3.476 \text{ seconds.}$$

$$51. \text{ a. } y = 0.25 \cos 2t$$

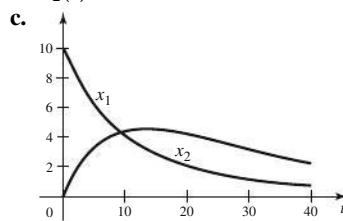


$$\text{b. } y = 0.25 \cos 2t \quad \text{c. } T = \pi$$

53. a. $x_1' = -k_1 x_1 + k_2 x_2 + f(t) \Rightarrow x_1'' = -k_1 x_1' + k_2 x_2' + f'(t)$
 $\Rightarrow k_2 x_2' = x_1'' + k_1 x_1' - f'(t)$. Eliminate x_2 from the given first-order system by multiplying the equation for x_1' by $(k_2 + k_3)$ and the equation for x_2' by k_2 and adding. This results in $(k_2 + k_3)x_1' + k_2 x_2' = -k_1 k_3 x_1 + (k_2 + k_3)f(t)$. Substitute $k_2 x_2' = x_1'' + k_1 x_1' - f'(t)$ and solve for x_1'' to get $x_1'' = -(k_1 + k_2 + k_3)x_1' - k_1 k_3 x_1 + (k_2 + k_3)f(t) + f'(t)$. Replace x_1 with x . b. Because x is another name for x_1 , $x_1(0) = A$ is the same as $x(0) = A$. Using the first DE gives $x'(0) = x_1'(0) = -k_1 x_1(0) + k_2 x_2(0) + f(0) = -k_1 x_1(0) + f(0)$.

$$55. \text{ a. } x_1(t) = 6.59e^{-0.1322t} + 3.41e^{-0.0378t}$$

$$\text{b. } x_2(t) = -10.60e^{-0.1322t} + 10.60e^{-0.0378t}$$



Section D2.5, pp. D2-61–D2-63

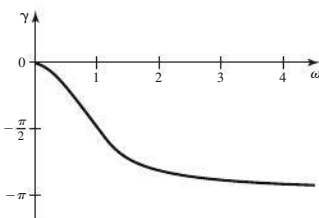
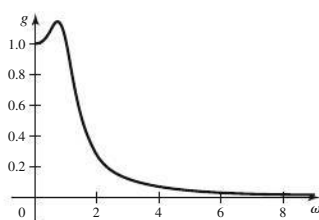
1. $H(\omega) = g(\omega)e^{i\gamma(\omega)}$ 3. The phase lag function gives the phase of the output relative to the input. The output lags the input.

5. a. $g(\omega) = \frac{1}{\sqrt{(1 - \omega^2)^2 + \omega^2}}$;

$\tan \gamma(\omega) = \frac{\omega}{\omega^2 - 1}$

b. Local maximum of the gain function at $\omega = \frac{\sqrt{2}}{2}$.

Weak damping.

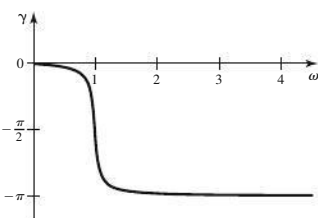
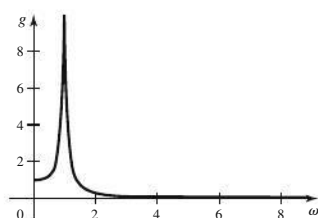


7. a. $g(\omega) = \frac{1}{\sqrt{(1 - \omega^2)^2 + \frac{\omega^2}{100}}}$;

$\tan \gamma(\omega) = \frac{\omega}{10(\omega^2 - 1)}$

b. Local maximum of the gain function at $\omega = \frac{\sqrt{398}}{20}$.

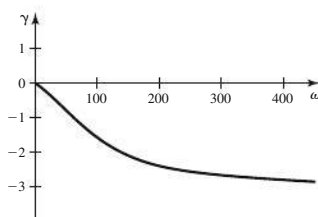
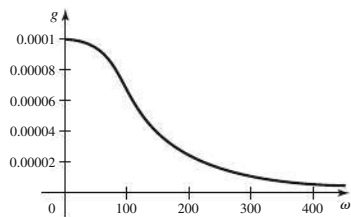
Weak damping.



9. a. $g(\omega) = \frac{1}{\sqrt{(10,000 - \omega^2)^2 + 22,500\omega^2}}$;

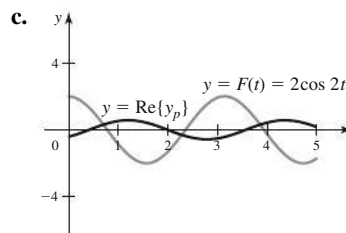
$\tan \gamma(\omega) = \frac{150\omega}{\omega^2 - 10,000}$

b. Local maximum of the gain function at $\omega = 0$. Strong damping.



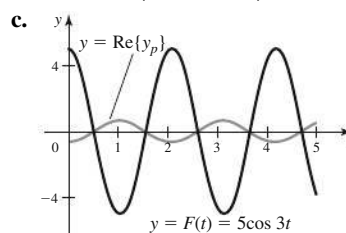
11. a. See Exercise 5a.

b. $\text{Re}\{y_p\} = \frac{2}{\sqrt{13}} \cos\left(2t + \tan^{-1} \frac{2}{3} - \pi\right)$
 $\approx 0.555 \cos(2t - 2.554)$



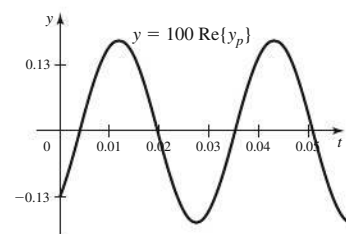
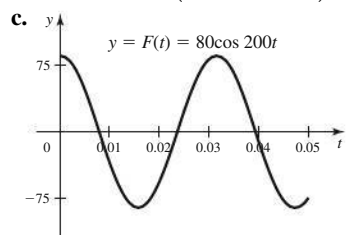
13. a. See Exercise 7a.

b. $\text{Re}\{y_p\} = \frac{50}{\sqrt{6409}} \cos\left(3t + \tan^{-1} \frac{3}{80} - \pi\right)$
 $\approx 0.625 \cos(3t - 3.104)$



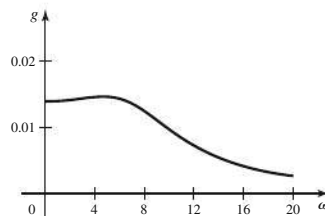
15. a. See Exercise 9a.

b. $\text{Re}\{y_p\} = \frac{\sqrt{2}}{750} \cos(200t + \tan^{-1} 1 - \pi)$
 $\approx 0.00189 \cos(200t - 2.356)$

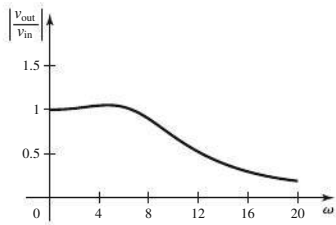


17. a. $g(\omega) = \frac{1}{\sqrt{(72 - \omega^2)^2 + 100\omega^2}}$

b. Local maximum of the gain function at $\omega \approx \sqrt{22}$. c. $(\sqrt{22}, \infty)$

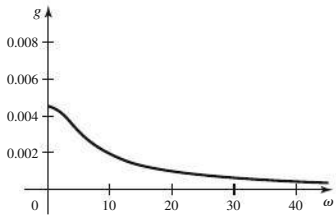


$$d. \left| \frac{v_{out}}{v_{in}} \right| = \frac{72}{\sqrt{(72 - \omega^2)^2 + 100\omega^2}}$$

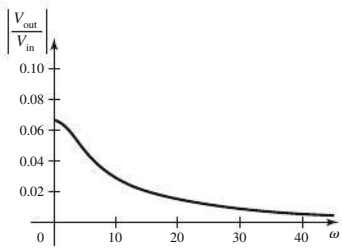


$$19. a. g(\omega) = \frac{1}{\sqrt{(15 - \omega^2)^2 + 2500\omega^2}}$$

b. Local maximum of the gain function at $\omega = 0$. c. $[0, \infty)$



$$d. \left| \frac{v_{out}}{v_{in}} \right| = \frac{15}{\sqrt{(15 - \omega^2)^2 + 2500\omega^2}}$$



21. a. True b. True

23. a. With 1 in the numerator, the smaller the denominator, the larger the value of $g(\omega)$. b. A square root will be smallest when the quantity under the square root is smallest.

$$c. h'(\omega) = \frac{d}{d\omega}[(\omega_0^2 - \omega^2)^2 + b^2\omega^2] = 2(\omega_0^2 - \omega^2)(-2\omega) + 2b\omega$$

$$= 2\omega(-2(\omega_0^2 - \omega^2) + b) = 2\omega(b - 2\omega_0^2 + 2\omega^2)$$

d. From part (c), $h'(\omega) = 2\omega(b^2 - 2\omega_0^2 + 2\omega^2)$. If $b < \sqrt{2}\omega_0$, then $h'(\omega) = 0$ has a real solution: $b^2 - 2\omega_0^2 + 2\omega^2 = 0$

$$\Rightarrow 2\omega^2 = 2\omega_0^2 - b^2 \Rightarrow \omega = \sqrt{\frac{2\omega_0^2 - b^2}{2}} = \sqrt{\omega_0^2 - \frac{b^2}{2}}$$

That this gives a local minimum for h , and hence a local maximum for g , can be verified by the First or Second Derivative Tests. The

maximum value of g is $\frac{\sqrt{2}}{\sqrt{b^2 + b^2(2\omega_0^2 - b^2)}}$.

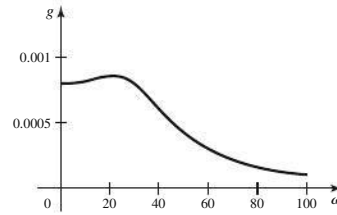
e. If $b \geq \sqrt{2}\omega_0$, then $b^2 \geq 2\omega_0^2$ and $b^2 - 2\omega_0^2 + 2\omega^2 \geq 0$. Hence $h'(\omega) = 2\omega(b^2 - 2\omega_0^2 + 2\omega^2) \geq 0$ for $\omega \geq 0$ and h is an increasing

function. It follows that $g = \frac{1}{\sqrt{h}}$ is decreasing for $\omega \geq 0$ and so

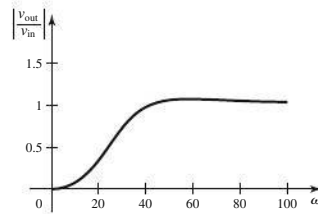
has a local maximum at $\omega = 0$. f. Part (e); the gain function is maximized at 0.

$$25. a. g(\omega) = \frac{1}{\sqrt{(\omega^2 - 1250)^2 + 1600\omega^2}}$$

b. Increases until $\omega = 15\sqrt{2}$ then decreases.

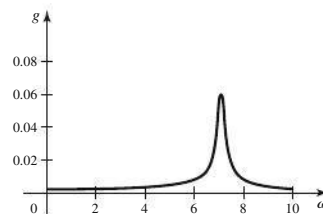


$$c. \left| \frac{v_{out}}{v_{in}} \right| = \frac{\omega^2}{\sqrt{(\omega^2 - 1250)^2 + 1600\omega^2}}$$

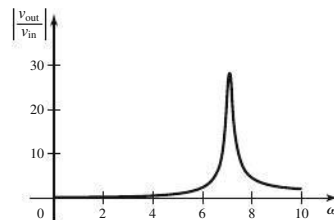


$$27. a. g(\omega) = \frac{4}{\sqrt{16(\omega^2 - 50)^2 + \omega^2}}$$

b. Increases until $\omega = \frac{\sqrt{3198}}{8} \approx 7.069$ then decreases.



$$c. \left| \frac{v_{out}}{v_{in}} \right| = \frac{4\omega^2}{\sqrt{16(\omega^2 - 50)^2 + \omega^2}}$$



Chapter D2 Review Exercises, pp. D2-63–D2-64

1. a. True b. False c. False d. False e. True

$$3. y = c_1 e^{-2t} + c_2 e^{4t} \quad 5. y = c_1 \sin 6t + c_2 \cos 6t$$

$$7. y = c_1 e^{-2t} + c_2 t e^{-2t} \quad 9. y = c_1 e^t \sin 2t + c_2 e^t \cos 2t$$

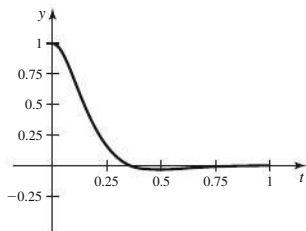
$$11. y_p = \frac{1}{7} \cos 2t \quad 13. y_p = -\frac{1}{6}t - \frac{5}{36} - \frac{1}{5}e^{-t}$$

$$15. y_p = \frac{1}{4}t \sin 4t \quad 17. y = c_1 \sin 2t + c_2 \cos 2t - \frac{3}{5} \sin 3t$$

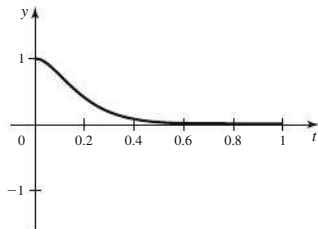
$$19. y = c_1 e^{-2t} \sin t + c_2 e^{-2t} \cos t + \frac{1}{4} \sin t + \frac{1}{4} \cos t$$

21. $y = c_1 e^{-t} + c_2 e^t + t e^t$

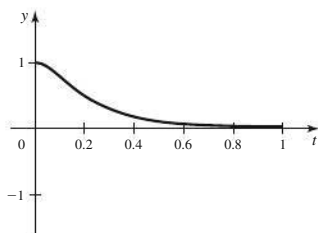
23. a. $y = \frac{3\sqrt{7}}{7} e^{-15t/2} \sin \frac{5\sqrt{7}}{2} t + e^{-15t/2} \cos \frac{5\sqrt{7}}{2} t$; underdamping



b. $y = e^{-10t} + 10te^{-10t}$; critical damping



c. $y = -\frac{1}{3} e^{-20t} + \frac{4}{3} e^{-5t}$; overdamping



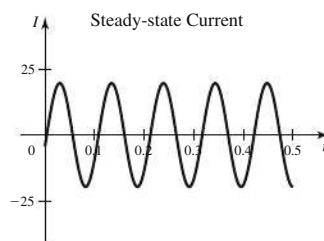
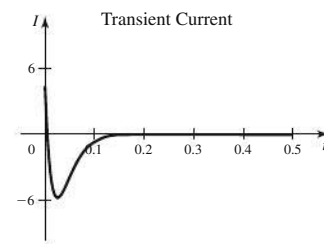
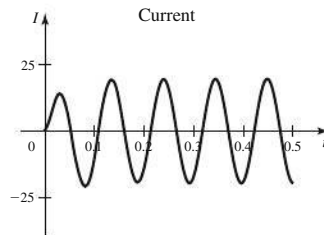
25. a. $I = 50e^{-60t} - \frac{600}{13} e^{-40t} + \frac{250}{13} \sin 60t - \frac{50}{13} \cos 60t$

b. Transient current:

$$I = 50e^{-60t} - \frac{600}{13} e^{-40t}$$

c. Steady-state current:

$$I = \frac{250}{13} \sin 60t - \frac{50}{13} \cos 60t$$



d. The solutions are the same.

Index

Note:

- Italics indicate figures or margin notes.
- “t” indicates a table.
- “e” indicates an exercise.
- GP indicates a Guided Project (located online in MyLab Math).
- AP-, B-, and C- indicate Appendices A, B, and C respectively (Appendices B and C are online at bit.ly/2y3Nck3 and bit.ly/2Rvf08V).

A

Absolute convergence, 692–694, 695e, 701t

Absolute error, 568

Absolute extreme value, 241

on any interval, 262

on closed intervals, 246

Absolute growth rate, 493, 499e

Absolute maxima, 241–243

on closed bounded sets, 988

on constraint curves, 998

on constraint surfaces, 1000

defining, 987

finding, 990–991

locating, 245–247

Absolute minima, 241–243

on closed bounded sets, 988

on constraint curves, 998

on constraint surfaces, 1000

defining, 987

finding, 990–991

locating, 245–247

Absolute values, B-3, B-4, 335e

functions and, 114e, 249, 352e

integrals of, 360

Acceleration, 179, 255e, 501e, 883

for circular motion, 884–885

on circular path, 908

components of, 907–909, 913e

defining, 180, 884

formulas for, 912

gravity and, 331

position from, 333e, 412e

velocity and, 180, 408, 412e

Accumulation

over finite intervals, 647t

over infinite intervals, 647t

Acid, noise, and earthquakes, GP8

Addition

associative property of, 811

commutative property of, 811

of vectors, 806–807

Addition Rule, 344

Additive identity, 811

Additive inverse, 811

Algebra, B-1, B-8

Algebraic functions, 14

end behavior of, 98

Alternating harmonic series, 688

convergent, 690

Alternating series, remainder in, 691–692

Alternating Series Test, 689–690, 694, 701t

Alternating sums, errors in, 706e

Ampère’s Law, 1146, 1170e

Amplitude, GP6, 43, 50e

Analytical methods, 15, 520

defining, 313

technology and, 279e

Angles

half-angle formula, 533

half-angle identities, 42

between planes, 855

projections and, 866e

sag, 515e

small-angle approximation, 294

of triangle, 833e

Angular coordinates, 767

Angular frequencies, 755

Angular speed, 1145e

Annual growth rate, 496

Annual percentage yield (APY), 495

determining, 499e

Annuities, perpetual, 591

Annular regions, 1030

Annulus, flux across, 1129

Antiderivatives, 321–331, 406

Constant Multiple Rule and, 324

defining, 321

family of, 322

finding, 323, 332e

motion problems and, 329–331

particular, 332e

Sum Rule and, 324

of vector-valued functions, 880

of velocity, 329

Antidifferentiation, 321

Antiquity, problems of, 387e

Anvil of hyperbolas, 800e

Approximation, GP11, GP17, GP22, GP32,

GP33, GP46, GP49, GP51, GP52,

GP63

APY. *See* Annual percentage yield

Arbelos, 291e

Arccosine, 44

Arches, height of, 386e

Archimedes, GP26, GP46

Arc length, 517e, 519e, 595e, 765e,

897–898, GP30, GP54

approximation of, 901e

astroid, 762

calculation of, 456, 901e

of ellipse, 581

of exponential curve, 453

formula, 452, 454, 762, 785

functions from, 456e, 899–900

integrals, 901e

of parabolas, 541, 567e

as parameter, 898–900

parametric equations and, 760–763

for polar curves, 785, 788e

speed and, 901e

spiral, 787e

for vector functions, 897

for $x = g(y)$, 454–455

for $y = f(x)$, 451–453

Arcsine, 44

Area. *See also* Surface area

approximation of, from graphs, 347e, 348e

of circles, 538–539, 544e, 1035e

computation of, 784, 1035e

between curves, 417, 418, 422e

by double integrals, 1023

of ellipses, 544e, 981e, 1126

equal, 397e

of parabolas, 424e, 482e

formula, 417, 418, 1036e

by geometry, 399e

Green’s Theorem for, 1126

of hyperbola sector, 799e

intersection points and, 514e

within lemniscates, 1032

net, 353, 364e

defining, 354

definite integrals and, 373–374

maximum, 380e

minimum, 380e

zero, 381e

of parallelograms, 838, 843e, 866e

of plane region, 1026e, 1126

of polar regions, 1031–1032

of rectangles, 285e

of regions, 379e, 478e, 518e, 555e, 786e,

802e, 1134e

bounded by parametric curves, 766e

bounded by polar curves, 781–784

intersection points and, 783

Riemann sums for, 340–344

signed, 354

of trapezoids, 358, 369

of triangle, 839–840, 843e, 844e, 995e

under curves, 338–346

under velocity curve, 338–340

volume and, 479e, 545e

Area function, 19, 25e, 480e

comparing, 368–369

for constant functions, 378e

cross-sectional, 427

defining, 368

Fundamental Theorem of Calculus and, 367–370

by geometry, 400e

from graphs, 380e

for linear functions, 378e

Area function (*continued*)

- matching functions with, 379e
- working with, 374–375, 380e
- Arguments, C-2
 - defining, 2
- Arithmetic, complex, C-1, C-2
- Arithmetic-geometric mean, 661e
- Arithmetic-mean, 1004e
- Arm torque, 843e
- Associative property
 - of addition, 811
 - of scalar multiplication, 811
- Astroid
 - arc length, 762
 - defining, 762
- Asymptotes, 275
 - end behavior and, 274
 - exponentials and, 114e
 - horizontal, 18, 101e
 - limits at infinity and, 92–93
 - of hyperbolas, 792
 - locating, 271
 - oblique, 95, 793
 - of rational functions, 97–98
 - shared, 800e
 - of sine integral function, 381e
 - slant, 95–96, 98, 129e, 793
 - vertical, 18, 98, 101e
 - defining, 85
 - functions with, 91e
 - location of, 87, 90e
- Atmospheric CO₂, GP7
- Attraction, basins of, 321e
- Autonomous differential equations, 607
- Average circulation, 1166
- Average cost, 184, 188e, 240e
 - defining, 183
- Average distance
 - on parabolas, 386e
 - on triangle, 397e
- Average growth rate, 181
- Average rate of change, 6
- Average value, 385e, 1016e, 1046e, 1085e
 - of constant functions, 383
 - defining, 1014
 - of derivatives, 387e
 - of functions, 383
 - over plane region, 1014
 - of three variables, 1043
 - over polar regions, 1032
 - with parameters, 387e
 - of sine function, 397e
 - zero, 1016e
- Average velocity, 56–59, 61e, 386e, 396e
 - defining, 179
 - instantaneous velocity and, 60
 - secant lines and, 58, 60
- Axes. *See also* *x*-axis; *y*-axis
 - imaginary, C-1
 - major, 765e
 - minor, 765e
 - polar, 767
 - real, C-1
 - translation and rotation of, GP58

B

- Balls, GP69
 - closed, 820
 - open, 820
 - spheres and, 820, 865e
- Basins of attraction, 321e
- Bernoulli, Johann, 301, 457e
- Bernoulli equations
 - solutions to, 626e
- Bernoulli's parabolas, 457e
- Bessel functions, 749e
- Bezier curves, GP64
- Binary numbers, 669e
- Binomial coefficients, 735
- Binomial series, 735, 742e, 751e
 - working with, 736, 741e
- Binormal vector, 909–912
 - completing, 913e
 - computing, 918e
 - unit
 - defining, 910
 - formulas for, 912
- Bioavailability, 588–589
- Bisection
 - of ellipses, 802e
 - of regions, 424e
- Body mass index, 949e
- Boiling-point function, 52e
- Boundary points
 - interior points and, 933
 - limits at, 933–935
- Bounded intervals, B-1
- Bounded sequences
 - above, 651
 - below, 651
 - monotonic, 655
- Bounded sets, 988
- Bounds
 - on definite integrals, 361–362
 - on *e*, 492
 - of functions, 925
 - on integrals, 387e
 - lower, 362
 - of series, 681e, 706e
 - of regions, 1029
 - upper, 362
 - of series, 681e, 706e
- Bowls, sine, 442
- Boxes, volume of, 950e
- Boyle's Law, 235e
- Brachistochrone property, GP54
- Brahe, Tycho, 897
- Buoyancy, GP26
- Business, derivatives in, 182–186
- b^x , derivatives of, 210

C

- Calculator limits, 162e, 491e
- Calculus. *See also* Fundamental Theorem of Calculus
 - geometry and, 419–420, 422e
 - in polar coordinates, 779–785

- of vector-valued functions, 875–881
- volumes without, 450e
- Calories, 414e
- Cardioids, 771
- Carrying capacity, 618
 - differential equations and, 627
- Cartesian coordinates, B-4, B-5, 767
 - polar coordinates and, 768–769, 1085e
- Cartesian graphs, matching, 774
- Cartesian-to-polar method, 771
- Catenaries, 512–513, 515e
- Catenoid, surface area of, 516e
- Cauchy, Augustin-Louis, 116
- Cauchy-Riemann equations, 951e
- Cauchy-Schwarz inequality, 836e
- Cauchy's Mean Value Theorem, 257e
- Cavalieri's principle, 438e
- Ceiling function, 26e, 70e
- Center
 - of circle, B-4
 - of ellipses, 791
 - of mass, 1064, 1065, 1071e, 1087e, 1161e, GP72
 - on edge, 1072e
 - with variable density, 1069
 - points, 720e
- Centered difference quotient, 140e
- Centroid, 1071e
 - calculation of, 1066–1067, 1070e
- Chain Rule, 276, 293, 484, 952–957, 1006e, AP-8
 - application of, 194–195, 198e
 - defining, 192, 952
 - formulas, 191–195
 - functions and, 195–196
 - with one independent variable, 952–953, 958e
 - for powers, 195
 - procedure, 192
 - proof of, 200, 878–879
 - for second derivatives, 200e
 - with several independent variables, 953–954
 - subtleties of, 960e
- Chains, winding, 474e
- Change. *See also* Variables, change of
 - in cone volume, 981e
 - differentials and, 977–980
 - directions of, 965, 972e
 - of functions, approximating, 978–979
- Change of base, 34–35, 38e
- Channel flow, horizontal, 1167
- Chaos, GP41
- Chemical rate equations, 613e, 620e
- Circles, 286e, 967e
 - area of, 538–539, 544e, 1035e
 - center of, B-4
 - chord length of, 282
 - circumference of, parametric equations for, 761
 - curvature of, 904, 914e
 - describing, 825e
 - equations of, B-5, 777e
 - expanding, 231e

- flow through, 835e
- in general, 777e
- hexagonal packing of, 836e
- involutés of, 764
- osculating, 914e
- in polar coordinates, 771
- radius of, B-4
- segments of, 544e
- shrinking, 231e
- slope on, 780
- speed on, 902e
- temperature on, 1100
- tilted, 901e
- trajectories on, 893e
- in triangles, 291e
- Circular motion, 764
 - acceleration for, 884–885
 - velocity for, 884–885
- Circular path, acceleration on, 908
- Circular trajectories, 896e
- Circulation, 1112e
 - average, 1166
 - defining, 1106
 - flux and, 1135e
 - on general regions, 1128–1130
 - Green's Theorem and, 1124–1128
 - on half-annulus, 1128
 - integrals, 1131*t*
 - of three-dimensional flow, 1107
 - of vector fields, 1106–1110
- Circumference
 - of circles, 453–454
 - of shells, 440
- Clock vectors, 816e
- Closed balls, 820
- Closed bounded sets
 - absolute maxima on, 988
 - absolute minima on, 988
- Closed curves, 1106
- Closed intervals, B-1
 - absolute extreme value on, 246
- Closed plane curves, 875e
- Closed sets, 933, 988
- Closest points, 917e
- Cobb-Douglas production function, 205e, 951e
- Coefficients, 13
 - binomial, 735
 - integer, 742e
 - solving for, 548
 - Taylor, 720e
 - in Taylor polynomials, 711
 - undetermined, 547e, 548
- Collinear points, 825e
- Combinations, linear, 816e
- Commutative property of addition, 811
- Comparable growth rates, 308
- Comparison Test, 592e, 701*t*
 - defining, 683
 - for improper integrals, 589–590
 - Limit, 685–686
- Completing the square, 523, 543, 544e
- Complex arithmetic, C-1, C-2
- Complex conjugates, C-3
- Complex numbers, C-1, GP50
 - operations with, C-2
 - polar form of, C-4
- Complex plane, C-1
- Components
 - of acceleration, 907–909, 913e
 - of curl, 1145e
 - equality and, 814e
 - of force, 833–834
 - normal, 908
 - scalar, defining, 831
 - tangential, 908
- Composite functions, 52e
 - continuity of, 115e, 936, AP-7
 - defining, 3
 - evaluation of, 4
 - inverse of, 39
 - limits of, 106
 - notation and, 5, 11e
 - symmetry of, 387e
 - working with, 5
- Compositions
 - continuity of, 115e
 - of three functions, 195–196
- Compound interest, 311e
- Compound regions, 417
- Compound surface, 1170e
- Computer algebra systems, 555e
 - integration with, 566e
- Concavity, 273
 - decreasing functions and, 274
 - defining, 263
 - derivatives and, 262–266
 - detecting, 264
 - increasing functions and, 274
 - interpreting, 264
 - linear approximation and, 295–298, 299e
 - of parabolas, 270e
 - tangent line and, 270e
 - test for, 263
 - Trapezoid Rule and, 581e
- Concentric spheres, 1161e
- Conditional convergence, GP48, 692–694, 695e
- Cones
 - cylinders and, 290e
 - descriptions of, 1155*t*
 - draining, 233e
 - flux across, 1160e
 - light, 864e
 - methods for, 450e
 - parameterized surfaces of, 1148
 - in spherical coordinates, 1055*t*
 - surface area of, 982e
 - formula for, 458
 - volume of, 286e, 438e, 463e
 - changes in, 981e
- Confocal ellipses, 800e
- Confocal hyperbolas, 800e
- Conical sheets, mass of, 1154
- Conical tanks, 475e
- Conic parameters, 802e
- Conic sections, GP58, GP59, GP60, GP65
 - directrix and, 793–795
 - eccentricity and, 793–795
 - ellipses, 791–792
 - focus in, 793
 - graphing, 797
 - hyperbolas, 792–793
 - parabolas, 789–791
 - polar equations of, 795–797, 798e, 799e, 802e
- Conjugates
 - complex, C-3
 - for finding limits, 76
 - in polar form, C-6
- Conservative vector fields, 1114–1121, 1170e, 1183e
 - curl of, 1141
 - properties of, 1143
- Constant density, 1068–1069
 - solids, 1070e
- Constant functions
 - area function for, 378e
 - average values of, 383
 - derivatives of, 154
 - limits of, 931
 - Riemann sums for, 352e
 - zero derivatives and, 253
- Constant multiple
 - of functions, 154–155
 - of limits, 72*t*
- Constant Multiple Rule, 344, 932
 - antiderivatives and, 324
 - defining, 154
 - for derivatives, 154–155
- Constant-rate problems, GP2
- Constant Rule for Derivatives, 153, 878
 - proof, 162e
- Constants
 - doubling time, 501
 - Euler's, 682e
 - in integrals, 359–360
 - of integration, 323
 - introducing, 390
 - rate, 493
- Constant velocity, 415e
- Constrained optimization problem, 988
- Constraint, 280
- Constraint curves
 - absolute maxima on, 998
 - absolute minima on, 998
- Constraint surfaces
 - absolute maxima on, 1000
 - absolute minima on, 1000
- Consumer price index, 493
- Continued fractions, 661e
- Continuity, 938e, 1005e
 - checking, 935–936
 - checklist, 104
 - of composite functions, 115e, 936, AP-7
 - of cosine function, 115e
 - defining, 103, 935
 - of derivatives, 146–148
 - differentiability and, 148, 947
 - at endpoints, 107, 381e
 - of functions, 151e
 - of two variables, 935–937

- Continuity (*continued*)
 on intervals, 107–108, 113e, 873
 of inverse functions, 109
 limits and, 873, 931–937
 of piecewise functions, 177e
 at points, 103–107, 112e
 of polynomial functions, 105
 of rational functions, 105
 with roots, 108
 rules, 105
 of sine function, 115e
 of transcendental functions, 109–110, 113e
 for vector-valued functions, 872–873
- Contour curve, 922–924
- Contour plots, 993e
- Convergence
 absolute, 692–694, 695e, 701*t*
 conditional, 692–694, 695e
 growth rates and, 657
 of infinite series, 645
 interval of, 729e, 751e
 parameters, 699e
 of power series, AP-5
 of p -series, 677
 radius of, 729e, 751e
 of Taylor series, 737–740, 740e
- Convergence Tests
 application of, 703e
 series strategies, 700–703
- Convergent sequences, 313, 658
 defining, 642
- Convergent series, properties of, 666–667
- Cooling coffee, GP31
- Coordinate systems, 469
 switching, 1051
- Coordinate unit vectors, 809, 842
- CORDIC algorithm, GP63
- Corner point, 147
- Cosecant function, 46, 227e
 hyperbolic, defining, 503
 integrals of, 524
- Cosine function
 continuity of, 115e
 graphing of, 53e
 hyperbolic, 102e, 455
 defining, 503
 inverse, 49e, 54e, 227e
 defining, 44–45
 derivative of, 221
 working with, 45
 Law of, 50e
 limits, 81e, 337e
 powers of, 532–533, 538e
 products of, 533
- Cost
 average, 183, 184, 188e, 240e
 fixed, 183
 function, 183
 marginal, 183, 184, 188e, 240e, 415e
 unit, 183
 variable, 183
- Cotangent function, 46, 227e
 hyperbolic, defining, 503
 integrals of, 524
- Critical depth, 477e
- Critical points, 272, 334e, 993e
 defining, 244, 985
 finding, 985
 in graphs, 271
 locating, 245, 248e
 solitary, 996e
- Crossover point, 662e
- Cross Product Rule, 878
 proof of, 883e
- Cross products, 837
 applications of, 840–842
 computation of, 842e
 from definition, 842e
 evaluation of, 839
 geometry of, 838
 magnetic force and, 841–842
 properties of, 838–840
 torque and, 840–841
 of unit vectors, 838–839
- Cross-sectional area function, 427
- Crystal lattice, GP48
- Cubes
 expanding, 231e
 partitioning, 1047e
 shrinking, 231e
 surface area of, 285e
 volume of, 285e
- Cubics, properties of, 270e
- Curl, 1139–1141
 of conservative vector fields, 1141
 divergence and, 1183e
 two-dimensional, 1125
 graphs of, 1170e
 interpretation of, 1166–1167
 of rotation field, 1141, 1144e
 of vector field, 1140
 working with, 1141–1143
 zero, 1171e
- Curvature. *See also* Concavity
 of circles, 904, 914e
 defining, 903
 formulas for, 903, 912
 graphs of, 914e
 of helix, 905–906
 of lines, 904
 maximum, 915e
 normal vectors and, 902–912
 of parabola, 905–906
 radius of, 914e
 of sine function, 915e
 unit tangent vector and, 903
 zero, 915e
- Curves. *See also* Level curves
 arc length for, by parametric equations, 761
 area between, 417, 422e
 area under, 338–346
 Bezier, GP64
 closed, 1106
 closed plane, 875e
 complete, 775, 779e
 contour, 922–924
 elliptic, 280e
 equipotential, 1095, 1097e
 exotic, 873e
 exponential, arc length of, 453
 finger, 778e
 flow, 1095
 gradient fields on, 1097e
 graphs and graphing of, 873e
 indifference, 1001
 of intersection, 874e, 982e
 isogonal, 788e
 Lamé, 280e
 lengths of, 451–455, 896–900, 1111
 Lorenz, 424e
 matching, 765e
 Newton's method and, 320e
 orientation of, 869–870
 oriented, 1102
 parametric, 754
 derivatives for, 759
 tangent line and, 801e
 parametric equations of, 757–758
 plane, 914e
 families of, 918e
 torsion of, 918e
 polar
 arc length of, 785, 788e
 graphing, 771
 regions bounded by, 781–784
 in polar coordinates, 769–770
 pursuit, 279e
 rectangles and, 335e
 regions between, 416–420, 524e, 1114–1115
 related, 456e
 roller coaster, 871
 sketching, 320e, 335
 slinky, 871–872
 slope of, 203–204
 in space, 869–872, 912
 properties of, 918e
 on spheres, 875e
 splitting, 37e
 on surfaces, 875e
 symmetric, 457e
 velocity
 area under, 338–340
 displacement and, 339–340
 Cusps, 147, 883e
 graphing, 276–277
 Cycloids, GP54, 759
 Cylinders, 854, 1016e, 1046e, 1048*t*
 cones and, 290e
 descriptions of, 1155*t*
 elliptic, 857
 flow in, 982e
 parabolic, 856
 parameterized surfaces of, 1147
 partial, surface area of, 1152–1153
 in spherical coordinates, 1055*t*

- surface area of, 1007e, 1151–1152
 - tilted, GP71
 - traces and, 856–857
 - volume of, 287e, 438e, 463e, 1007e
 - in spheres, 1003e
 - Cylindrical coordinates, 1083e, GP78
 - integrals in, 1086e
 - integration in, 1049–1053
 - rectangular coordinates and, 1049
 - sets in, 1049, 1059e
 - volume in, 1060e, 1086e
 - Cylindrical shells, 439–447, 1084t
- D**
- Damped oscillators, 320e
 - Dams
 - force-on-dam problems, 472, 482e
 - parabolic, 476e
 - Data fitting, GP7
 - Deceleration, 412e, 478e
 - Decimal expansions, 668e
 - as geometric series, 664–665
 - Decomposing vectors, 835e
 - Decomposition, of regions, 1023
 - Decreasing functions
 - concavity and, 274
 - defining, 257
 - derivatives and, 257–259
 - intervals of, 257–258
 - sketching, 258–259
 - Decreasing sequences, 651
 - Definite integrals, 353–363, 378e, 748e, 916e
 - approximation of, 365e
 - bounds on, 361–362
 - comparing, 361–362
 - defining, 356
 - evaluating, 357–358, 372–373
 - geometry for, 357–358
 - from graphs, 358–359, 366e
 - integration by parts for, 528–529
 - limit definition of, 399e
 - limits for, 362–363
 - net area and, 373–374
 - notation for, 356
 - properties of, 359–361
 - Substitution Rule for, 393–395
 - Demand function, 23e
 - elasticity and, 270e
 - DeMoivre's Theorem, C-5
 - Density
 - constant, 1068–1069
 - solids, 1070e
 - distribution, 1062e
 - mass and, 1017e, 1036e, 1060e, 1062e, 1111
 - physical applications, 465–466
 - variable, 465, 960e, 1067
 - center of mass with, 1069
 - Dependent variable, 647t
 - defining, 2
 - Depth, critical, 477e
 - Derivatives, 131–135, 160e, 272, 765e.
 - See also* Antiderivatives;
 - Partial derivatives
 - approximating, 139e
 - average value of, 387e
 - in business, 182–186
 - of b^x , 210
 - combining rules, 167
 - computing, 141–142, 212
 - concavity and, 262–266
 - of constant functions, 154
 - Constant Multiple Rule for, 154–155
 - Constant Rule for, 153
 - proof of, 162e
 - continuity of, 146–148
 - decreasing functions and, 257–259
 - by different methods, 199e, 217e
 - directional, 1006e
 - computing, 963e
 - defining, 962
 - gradient and, 961–970
 - interpreting, 966–967
 - in economics, 182–186
 - equal, 256e
 - of even functions, 200e
 - of e^x , 157
 - of exponential functions, 487–488
 - First Derivative Test, 260–261, 268e
 - formulas, 150e, 514e
 - verifying, 176e
 - of functions, 135, 140–152, 148e, 279e
 - functions with equal, 254
 - of general exponential functions, 489
 - of general logarithmic functions, 489
 - graphs of, 144–146, 161e, 170e
 - higher-order, 158–159, 169e, 216e, 882e
 - of implicit functions, 204
 - of hyperbolic functions, 505, 519e
 - increasing functions and, 257–259
 - inflection points and, 262–266
 - of integrals, 374, 379e
 - integration of, 530e
 - interpreting, 136–137, 138e
 - of inverse cosine, 221
 - of inverse functions, 222–223
 - of inverse hyperbolic functions, 509–511
 - of inverse secant, 219–222
 - of inverse sine, 218–219
 - of inverse tangent, 219–222
 - of inverse trigonometric functions, 218–224
 - from limits, 161e
 - of linear functions, 223
 - local maxima and, 260–262, 984
 - local minima and, 260–262, 984
 - of logarithmic functions, 214, 216e
 - matching, 269e
 - of natural logarithm, 209, 484
 - notation, 142–143
 - of odd functions, 200e
 - one-sided, 152e
 - for parametric curve, 759
 - parametric equations and, 759–760
 - at points, 194
 - of polynomials, 156
 - Power Rule for, 153
 - extending, 162e
 - general form of, 166
 - for negative integers, 162e, 166–167
 - proof, 162e
 - using, 166–167
 - power series for, 748e
 - properties, 266
 - as rate of change, 178–186
 - Rules, 878
 - second, 204
 - Chain Rule for, 200e
 - linear approximation and, 300e
 - Second Derivative Test, 265, 268e
 - second-order, 175
 - sketching, 149e
 - from tables, 169e
 - tables for, 197e
 - tangent lines and, 136
 - of tower functions, 215e, 490
 - for trigonometric functions, 171–175
 - with two variables, 940–943
 - of vector-valued functions, 875–879, 881e
 - zero, constant functions and, 253
- Derivatives, uses and applications of, GP11, GP12, GP13, GP15, GP16, GP17, GP21, GP22, GP60, GP61, GP65
- Descartes' four-circle solution, 915e
 - Descent time, 546e
 - Designer functions, 50e, 334e
 - Designer series, 742
 - Diagonals
 - lengths of, 825e
 - of parallelograms, 837e
 - Difference
 - equations, 623
 - of even functions, 423e
 - of limits, 72t
 - of odd functions, 423e
 - quotients, 6, 12e, 178e
 - limits of, 200e
 - of squares, 538
 - Differentiability, 141, 147
 - conditions for, 947, AP-7
 - continuity and, 148, 947
 - defining, 135, 947
 - of functions, 151e
 - Differential equations, 327, 597–638, GP15, GP31, GP33, GP36, GP37, GP38, GP39, GP40
 - autonomous, 607
 - carrying capacity and, 627
 - defining, 597
 - domain of solutions to, 598
 - Euler's method for, GP33
 - first-order, GP31, GP33, GP36, GP38, GP39, 620–625
 - general solutions, 598
 - growth rate functions and, 627
 - initial value problems and, 328, 598–600

Differential equations (*continued*)

- linear, 597–598
 - motion and, 601–602
 - nonlinear, 597–598
 - order of, 597
 - population models and, 627–629
 - predator-prey models and, 631–633
 - second order, GP27, GP40, GP65, GP66, GP75, GP76, 605e. *See also* Second-order differential equations
 - separable, 614–618
 - implicit solutions, 616
 - stirred tank reactions and, 629–631
 - verifying solutions to, 598–599
- Differentials, 300
- as change, 298
 - change and, 977–980
 - defining, 298, 978
 - linear approximation and, 292–298
 - logarithmic, 983e
- Differentiation
- implicit, 205e, 945, 955–957, 959e, 1006e
 - defining, 201
 - geometry of, 960e
 - slope of tangent line and, 203–204
 - integration and, 372
 - logarithmic, 214–215
 - numerical, GP11
 - of power series, 730e, 743–744
 - rules of, 152–162
- Dipstick problem, GP27
- Direction
- of change, 965, 972e
 - of steepest ascent, 965, 971e
 - of steepest descent, 965, 971e
- Directional derivatives, 1006e
- computing, 963e
 - defining, 962
 - gradient and, 961–970
 - interpreting, 966–967, 971e
- Direction fields, 637e
- analysis, 613e, 622–623
 - defining, 606–611
 - equilibrium solutions to, 608
 - matching, 611e
 - sketching, 608, 612e
 - with technology, 612e
- Directrix
- conic sections and, 793–795
 - of parabolas, 789
- Discontinuity
- infinite, 104, 115e
 - jump, 104, 115e
 - points of, 104
 - removable, 104, 115e
- Discriminants, 986
- Disk method, 443
- defining, 428
 - about x -axis, 428, 446
 - about y -axis, 431, 446
- Disk/washer method, GP25
- Displacement, 178
- approximation of, 339, 348e

- defining, 404
 - distance and, 478e
 - oscillator, 531e
 - from velocity, 350e, 403–405, 410e, 411e, 478e
 - velocity curve and, 339–340
- Distance, 1060e
- average
 - on parabolas, 386e
 - on triangle, 397e
 - displacement and, 478e
 - to ellipse, 1003e
 - formulas, B-4, 819
 - function, 972e
 - to paraboloid, 1007e
 - to planes, 1003e
 - from plane to ellipsoid, 983e
 - from point to line, 846–849
 - position and, 478e
 - to sphere, 1003e
 - to surface, 1003e
 - traveled, 404
- Distribution, normal, 275–276
- Distribution of wealth, 424e, GP19
- Distributive property, 811, 836e
- Divergence, 1145e
- computing, 1137
 - curl and, 1183e
 - of gradient fields, 1146e
 - from graphs, 1138
 - of infinite series, 645
 - Product Rule for, 1142
 - proof, 682e
 - of radial vector field, 1138–1139, 1144e, 1146e
 - two-dimensional, 1127
 - of vector field, 1137–1138, 1143e
 - working with, 1141–1143
- Divergence test, 672–673, 680e, 689, 701*t*
- application of, 700–702
- Divergence Theorem, 1171–1179, 1180e
- for hollow regions, 1176–1177
 - mass transport and, 1174
 - proof of, 1175–1176
 - with rotation field, 1173
 - verification of, 1172
- Divergent sequences, 313, 658
- defining, 642
- Division
- long, 553–554
 - with rational functions, 522
- Domain, 53e, 272, 274, 869, 874e, 927e, 1005e
- in context, 3
 - defining, 1, 920
 - determining, 600
 - of differential equation solutions, 598
 - finding, 920
 - of functions, 2
 - in graphing, 271
 - with independent variables, 926
 - of natural logarithm, 484
 - of two variables, 920

- Dot Product Rule, 878
- proof of, 879
- Dot products, 827–833
- applications of, 832–833
 - defining, 827
 - force and, 832
 - forms of, 827
 - properties of, 830, 846e
 - work and, 832
- Double-angle identities, 42
- Double integrals, 1015e, 1082e, 1085, 1088e
- area by, 1023
 - with change of variables, 1076
 - defining, 1010
 - orders of, 1011
 - over general regions, 1017–1023
 - over polar rectangle, 1028
 - over rectangular regions, 1008–1014
 - in polar coordinates, 1027–1032
- Doubling time, 494
- constant, 501
- Drug dosing, GP43, 625e
- half-lives and, 498
 - initial value problems for, 622
 - periodic, 660e
 - sequences for, 655
- Dummy variables, 344
- E**
- e (number)
- bounds on, 492
 - computing, 490
 - defining, 486
 - properties of, 487
- e^x
- derivative of, 157
 - inverse properties for, 208
- Earth, speed of, 901e
- Earth-Mars system, 778e
- Earthquakes, GP8
- Eccentricity, conic sections and, 793–795
- Eccentricity-Directrix Theorem, 794, 798e
- proof of, AP-6
- Ecological diversity, GP67
- Economics
- derivatives in, 182–186
 - elasticity in, GP13, 185
 - models, 1001–1002
 - production functions, GP68
 - stimulus packages, GP44
- Edges, center of mass on, 1072e
- Eiffel Tower, GP72
- Eigenvalues, defining, 321e
- Elasticity
- defining, 185
 - in economics, GP13, 185
 - inelasticity, 186
- Electric field, GP70, 545e
- point charges and, 1097e
- Electric potential, 1146e
- function, 925, 929, 950e

- Ellipses, 765e, 1083e
 - arc length of, 581e, 898
 - area of, 544e, 981e, 1126
 - bisection of, 802e
 - center of, 791
 - confocal, 800e
 - construction of, 802e
 - defining, 760
 - distance from plane to, 983e
 - distance to, 1003e
 - equations of, 792, 794–795, 797e
 - foci of, 791
 - graphing of, 792
 - parabolas and, 799e
 - parametric equations of, 760, 766e
 - properties of, 794
 - rectangles in, 1003e
 - speed on, 894e
 - tangent line for, 207e, 798e
 - tilted, 896e
 - trajectories on, 887
 - vertices of, 791
- Ellipsoid, 1047e
 - defining, 858
 - inside tetrahedron, 996e
 - surface area of, 464
 - volume of, 799e, 1007e
- Elliptic cones, 861
- Elliptic curves, 280e
- Elliptic cylinders, 857
- Elliptic integrals, 749e, 898, GP37
- Elliptic paraboloids, 859
- End behavior
 - of algebraic functions, 98
 - asymptotes and, 274
 - determining, 271
 - of exponential functions, 102e
 - of limits at infinity, 95–98, 272
 - of rational functions, 95–96, 102e
 - of transcendental functions, 99–100, 102e
- Endowment models, 626e
- Endpoints, continuity at, 107, 381e
- Energy
 - changes in, 336e
 - conservation of, 959e, 1123e
 - consumption, 496, 499e
 - power and, 414e
- Enzyme kinetics, GP12
- Epidemic models, 307
 - logistic equation for, 619e
- Equal areas, 397e
 - for parabolas, 424e, 482e
 - property, 896e
- Equal derivatives, 256e
- Equality
 - components and, 814e
 - of partial derivatives, 944–945
- Equal vectors, 805
- Equal volumes, 450e, 1035e
- Equations. *See also* Differential equations; Parametric equations
 - Bernoulli, 626e
 - Cauchy-Riemann, 951e
 - chemical rate, 613e, 620e
 - of circles, B-5, 777e
 - difference, 623
 - of ellipses, 792, 794–795, 797e
 - equivalent, 402e
 - general, of planes, 850
 - Gompertz, 629
 - solutions to, 634e
 - graphing, 52e
 - of hyperbolas, 793, 797e, 799e
 - identification of, 820
 - integrals, 381e
 - Laplace's, 951e, 1005e, 1130
 - of lines, B-6, B-7, 845, 853e
 - logistic, direction field for, 608–609
 - Lotka-Volterra, 631
 - Maxwell's, GP76
 - of motion, 894e
 - of parabolas, 797e, 799e
 - of planes, 849–851, 854e, 867e
 - polar
 - of conic sections, 795–797, 798e, 799e, 802e
 - parametric equations and, 801e
 - symmetry in, 772
 - RC circuit, 635e
 - series in, 671e, 706e
 - simple nonlinear, 608
 - of spheres, 819–820, 826e
 - of standard parabolas, 790
 - of tangent lines, 134, 138e, 168e
 - of tangent planes, 975–976
 - vector, 815e
 - of lines, 845
 - wave, 951e
- Equidistant sets, 799e
- Equilibrium, 466
 - maintenance of, 825e
 - stable, 608
- Equilibrium solutions, 613e, 618
 - to direction fields, 608
 - stable, 622, 625e
 - unstable, 622
- Equipotential curves, 1095, 1097e
- Equipotential surfaces, 1095
- Equivalent equations, 402e
- Errors
 - absolute
 - defining, 568
 - in numerical integration, 568
 - in approximations, 720e
 - dependence of, 720e
 - estimating, 581e, 719e
 - in Euler's method, 612e
 - function, 749e
 - manufacturing, 980
 - maximum, 719e
 - in Midpoint Rule, 571
 - in numerical integration, 577
 - in partial sums, 695e
 - relative
 - defining, 568
 - in numerical integration, 568
 - in Simpson's Rule, 576–577
 - time steps and, 610e
 - in Trapezoid Rule, 571, 576–577
- Escape velocity, 593e
- Estimates, 681e
- Euler, Leonhard, 28
- Euler approximation, 609
 - computation of, 613e
- Euler's constant, 682e
- Euler's formula, C-3, C-4, C-6, GP50
- Euler's method, 606–611, 637e, GP33
 - convergence of, 613e
 - errors in, 612e
 - first step of, 609
 - on grids, 613e
 - procedure, 610
 - stability of, 613e
 - time steps in, 609
- Euler spiral, 914e
- Euler steps, 609–610
- Even functions
 - composition of, 13e
 - defining, 8, 381
 - derivatives of, 200e
 - differences of, 423e
 - integrals of, 382
 - limits of, 70e, 82e
 - odd functions and, 381–382
- e^x , derivative of, 157
- Exotic curves, 873e
- Explicit formulas, 640–641, 647e
 - for sequences, 648e
- Explicitly defined surfaces, surface integrals
 - on, 1153–1154
- Exponential curve, arc length of, 453
- Exponential decay, 497–498
- Exponential distribution, GP34
- Exponential functions, 109, 486, 730e, GP14, GP28, GP31, GP43
 - defining, 14, 27, 487
 - derivatives of, 208–215, 487–488
 - with different bases, 312e
 - end behavior of, 102e
 - family of, 457e
 - general, 212–213
 - defining, 488
 - derivatives of, 489
 - integrals of, 489
 - graphing, 38e
 - integrals of, 487–488
 - inverse relations for, 33
 - natural, 14
 - defining, 28
 - derivatives of, 156
 - powers and, 309, 312e
 - properties of, 27–28
- Exponential growth, 492
 - functions
 - defining, 494
 - designing, 499e
 - initial value in, 494
 - rate constant and, 494

Exponential growth (*continued*)
 linear growth and, 493–494
 rates, 312e

Exponential inequalities, 402e

Exponential integrals, 530e

Exponential models, 211, 216e

Exponentials, asymptotes and, 114e

Extraneous roots, 1029

Extreme points, local, 721

Extreme values

- absolute, 241, 246, 262
- inflection points and, 272, 273, 275
- local maxima and, 261–262
- local minima and, 261–262

Extreme Value Theorem

defining, 243

Local, 244

F

Factorial function, 27e

growth rate of, 308

Factorial growth rate, 312e

Factorial sequence, 656

Factorization formula, 82e

Fermat's Principle, 288e

Fermat's volume calculation, 437

Fibonacci sequence, harmonic series and,
 682e

Finance, GP42

Financial models, 495–496

Finger curves, 778e

Finite intervals, accumulation over, 647t

First Derivative Test, 260–261, 268e

First-order linear differential equations,
 620–625

solutions to, 621–623

Fixed cost, 183

Fixed point iteration, GP9, GP41, 649e

Fixed points

- of quadratic functions, 320e
- of quartics, 320e

Floating point operations, 983e

Floor function, 70e

Flow

- ideal, 1135e
- three-dimensional, circulation of, 1107
- two-dimensional, 1109–1110

Flow curves, 1095

Flow rate, 333e, 351e

- through circle, 835e
- in cylinders, 982e
- fluid, 956e
- variable, 478e

Fluid flow, GP75, 956

Flux, 1112e

- across annulus, 1129
- computing, 1179e, 1184e
- across cone, 1160e
- defining, 1107, 1109
- Green's Theorem and, 1127–1128, 1134e
- heat, 1113e, 1145e
- across hemispheres, 1184e

integrals, 1131t, 1156–1157
 for inverse square field, 1176–1177
 outward, 1109

of radial field, 1128

across paraboloids, 1184e

of radial field, 1158

across spheres, 1180e

on tetrahedrons, 1160e

of two-dimensional flows, 1109–1110

of vector fields, 1107–1108

Focal chords, 800e

Focus

- in conic sections, 793
- of ellipses, 791
- of hyperbolas, 792
- of parabolas, 789

Force

- balancing, 813
- components of, 833–834
- dot products and, 832
- on inclined plane, 834e
- inverse square, 1105
- magnetic, 841–842
- net, 866e
- normal, 832–833
- orientation and, 477e
- parallel, 832–833
- physical applications, 471–473
- pressure and, 471–473
- on tanks, 476e
- variable, 466
- vectors, 812–813
- work from, 474e

Force field, work in, 1105, 1122e

Force-on-dam problems, 472, 482e

Formulas

- for acceleration, 912
- arc length, 452, 454, 762, 785
- area, 417, 418, 1036e
- Chain Rule, 191–195
- for cone surface area, 458
- for curvature, 903, 912
- for curves, 912
- derivation of, 566e
- derivatives, 150e, 176e, 514e
- distance, B-4
- Euler's, C-3, C-4, C-6
- explicit, 640–641, 647e
- for sequences, 648e
- factorization, 82e
- Gauss', 1181e
- half-angle, 533
- Heron's, 982e
- implicit, 640
- for indefinite integrals, 392, 510, 520
- integration, 392t
- midpoint, 826e
- reduction, 527
- application of, 530e
- defining, 534
- sine, 537e
- for shells, 442
- for Simpson's Rule, 582e

Stirling's, 700e

surface area, 459–462

for surface area of cones, 458

for torsion, 912, 915e

for unit binormal vector, 912

volume, 1047e, 1063e

Fourier series, GP53

Four-leaf rose, 773

Fractals, 670e

Fractions

- continued, 661e
- partial, 546–554
- decomposition, 547, 554
- form of, 551
- integration with, 548–549
- method of, 547–548
- with simple linear factors, 548–550
- splitting up, 522

Free fall, 181, 349e, 619e

Frequencies, angular, 755

Fresnel integral, 381e

Frustum, surface area of, 458, 464e

Fubini's Theorem, 1012, 1038

Fuel consumption, GP29

Functions, GP3, GP4, 178. *See also specific types*

- absolute values and, 114e, 249, 352e
- algebraic, 14
- end behavior of, 98

approximation of, with polynomials,
 708–718

arc length, 899–900

average values of, 383

over plane region, 1014

Bessel, 749e

boiling-point, 52e

bounds of, 925

ceiling, 26e, 70e

Chain Rule and, 195–196

change of, approximating, 978–979

Cobb-Douglas production, 205e, 929e,
 951e

compositions of three or more,
 195–196

constant multiple of, 154–155

continuity of, 151e

cosecant, 46, 227e

hyperbolic, 503

integrals of, 524e

cost, 183

cotangent, 46, 227e

hyperbolic, 503

integrals of, 524e

decreasing

concavity and, 274

defining, 257

derivatives and, 257–259

intervals of, 257–258

defined by series, 671e

defining, 1

demand, 23e

elasticity and, 270e

derivatives of, 135, 148e, 279e

designer, 50e, 334e
 differentiability of, 151e
 distance, gradient of, 972e
 domain of, 2
 electric potential, 925, 950e
 with equal derivatives, 254
 error, 749e
 factorial, 27e
 growth rate of, 308
 floor, 70e
 gamma, 593e
 graphing, 24e, 52e, 271–277, 927e
 growth rate of, 307–310, 634e, 635e
 differential equations and, 627
 harmonic, 1181e
 Heaviside, 69e
 identifying, 2
 implicit
 higher-order derivatives of, 204
 tangent line with, 203–204
 increasing
 concavity and, 274
 defining, 257
 derivatives and, 257–259
 intervals of, 257–258
 sketching, 258–259
 with independent variables, 926
 with infinite limits, 91e
 integrability of, 357, 1010
 integrals and, 380e, 401e, 402e
 with jumps, 66–67
 limits of, 118
 of two variables, 931–932
 logistic, designing, 634e
 matching, 54e, 269e
 with area function, 379e
 with graphs, 874e
 with polynomials, 720e
 monotonic, 257
 nondecreasing, 257
 nondifferentiability of, 947–948
 nonincreasing, 257
 objective, 280
 one-to-one, 29–30, 52
 periodic, 42
 piecewise, 53e, 939e
 continuity of, 177e
 defining, 16, 24e
 graphing, 16–17
 solid from, 438e
 polynomial
 continuity of, 105
 limits of, 73–74
 population, 52e
 position, 178, 181
 as antiderivative, 329
 potential, 973e
 defining, 1094
 gradient fields and, 1094
 power, 983e
 derivatives of, 154
 graphing, 16–17
 as power series, 746–747

to power series, 730e
 probability, 924
 Product Rule for, 170e
 quadratic, 240
 fixed points of, 320e
 Mean Value Theorem for, 256e
 sine function and, 386e
 range of, 2
 representing, 13–27
 revenue, 190
 review of, 1–13
 with roots, 17, 53e, 108, 113e
 scalar-valued, surface integrals of,
 1149–1155
 secant, 46
 hyperbolic, 503
 products of, 535–536
 sine
 curvature of, 915e
 hyperbolic, 455, 503
 linear approximation for, 294
 quadratic functions and, 386e
 sine integral, 375, 581
 slope, 18, 25e
 smooth, 877
 step, 70e
 stream, 1130, 1146e
 superexponential, 312e, 337e
 symmetric, integrating, 382–383
 symmetry in
 defining, 8–9
 identifying, 9
 tangent, 46
 hyperbolic, 503, 564
 powers of, 535
 products of, 535–536
 of three variables, 937, 945–946
 average value of, 1043
 tower, derivatives of, 215e, 490
 transcendental, 14, 101e
 continuity of, 109–110, 113e
 end behavior of, 99–100, 102e
 limits involving, 110
 transformations of, 20–22
 of two variables, 920
 applications of, 924–925
 continuity of, 935–937
 graphs of, 921–924
 limit laws for, 932
 utility, 959e
 velocity, 138e, 149e
 with vertical asymptotes, 91e
 work, 474e
 zeta, 681e
 Fundamental Theorem of Calculus,
 367–377, 484, 567e, 1179t
 area function and, 367–370
 consequences of, 408
 discrete version of, 381e
 proof of, 376
 Fundamental Theorem of Line Integrals,
 1121e, 1122e, 1179t
 Funnels, surface area of, 460–461

G

Gabriel's horn, 585
 Gamma function, 593e
 Gauss' formula, 1181e
 Gaussians, 593e
 Gauss' Law, 1177–1179, 1180e
 General equations
 linear, B-6
 of planes, 850
 General exponential functions,
 212–213
 defining, 488
 derivatives of, 489
 integrals of, 489
 Generalized Mean Value Theorem, 257e,
 301, 385, 387e
 Generalized sum rule, 155
 General logarithmic functions, 213–214
 defining, 488
 derivatives of, 489
 integrals of, 489
 General partition, 355
 General Power Rule, 489–490
 defining, 211
 General regions
 circulation on, 1128–1130
 description of, 1034e
 double integrals over,
 1017–1023
 flux on, 1128–1130
 General Riemann sum, 355
 General rotation vector field, 1140
 General solutions, 598
 differential equations, 598
 finding, 605e
 to initial value problems, 328
 verification of, 604e
 Geometric-arithmetic mean, 836e
 Geometric means, 1004e
 Geometric probability, GP23
 Geometric sequences, 652–654
 defining, 653
 limit laws and, 653
 Geometric series, 639, 663,
 701t, 742e
 decimal expansions as, 664–665
 evaluating, 668e, 700
 power series from, 751e
 Geometric sums, 663–665, 668e
 Geometric sums/series, GP42, GP43,
 GP44, GP45
 Geometry, 565e
 area by, 399e
 area function by, 400e
 calculus and, 419–420, 422e
 of cross product, 838
 for definite integrals, 357–358
 Lagrange multipliers and, 1000–1001
 of l'Hôpital's Rule, 302
 of Substitution Rule, 395
 Gini index, GP19, 424e
 Gompertz equation, 629
 solutions to, 634e

Gompertz growth model, 628

Gradient

- computing, 964
- defining, 964
- directional derivatives and, 961–970
- of distance function, 972e
- interpretations of, 965–967
- level curves and, 967–968
- rules, 973e
- in three-dimensions, 969–970
- vector, 964–965

Gradient fields, 1096e, 1182e

- on curves, 1097e
- defining, 1094
- divergence of, 1146e
- potential functions and, 1094

Graphs and graphing

- analysis of, 269e, 270
- approximation of area from, 347e, 348e
- area function from, 380e
- Cartesian, matching, 774
- confirmation of, 515e
- of conic sections, 797
- of cosine, 53e
- critical points in, 271
- of curl, 1170e
- of curvature, 914e
- of curves, 873e
- of cusps, 276–277
- defining, 2
- definite integrals from, 358–359, 366e
- of derivatives, 144–146, 161e, 170e
- divergence from, 1138
- domain in, 271
- of ellipses, 792
- equations, 52e
- exponential functions, 38e
- finding limits from, 63–64
- of functions, 24e, 52e, 271–277, 927e
 - of two variables, 921–924
- functions matched with, 874e
- guidelines, 271–277
- of hyperbolic functions, 502–507
- inverse functions, 37e, 45–46
- levels curves and, 919–927
- limits by, 70e
- linear functions, 15, 23e
- logarithmic functions, 38e
- Mean Value Theorem, 255e–256e
- piecewise functions, 16–17
- polar
 - matching, 774
 - plotting, 771–773
 - symmetry in, 772–773
- polar coordinates in, 771–774
- power functions, 16–17
- root functions, 17
- of roots, 276–277
- semicircles, 13e
- of sequences, 652
- of sine, 53e

- sketching, 102e
- of spirals, 870
- of surfaces, 856
- symmetry in, 271
 - defining, 8
- of Taylor polynomials, 719e
- transformations of, 20–22, 43–44
- using, 14–20
- utilities, 774–775
- velocity, 411e
- windows, 271

Gravitational field, GP74

Gravitational fields, 1063e

- mass and, 1097e
- motion in, 181, 408, 601
- two-dimensional motion in, 887–892
- work in, 477e

Gravitational potential, 973e

Gravity

- acceleration due to, 331
- motion with, 330–331, 333e
- variable, 415e

Grazing goat problems, GP57

Green's Theorem, 1168, 1179t, 1183e

- area calculated by, 1126
- circulation form of, 1124–1128
- flux form of, 1127–1128, 1134e
- proof of, 1131

Grids

- Euler's method on general, 613e
- nonuniform, 579e
- points, 341

Growth models, 181–182

- of functions, 925
- Gompertz, 628

Growth rates, 336e

- absolute, 493, 499e
- annual, 496
- average, 181
- comparable, 308, 312e
- convergence and, 657
- exponential, 312e
- factorial, 312e
- of factorial function, 308
- functions, 627, 634e, 635e
 - differential equations and, 627
- of functions, 307–310
- instantaneous, 182
- l'Hôpital's Rule and, 307–310
- oscillating, 414e
- powers and, 308–309
- ranking, 309
- relative, 493, 499e
- of sequences, 656–657, 660e

H

- Hadamard, Jacques, 307
- Hailstone sequence, 662e
- Half-angle formula, 533
- Half-angle identities, 42
- Half-annulus, circulation on, 1128
- Half-life, 497

of common drugs, 498

Hand tracking, 864e

Harmonic functions, 1181e

Harmonic series, 673, 951e

alternating, 688

defining, 674

Fibonacci sequence and, 682e

Harmonic sums, 492e

Harvesting models, 602

Heat

- flux, 836e, 1113e, 1145e
- transfer, 1181e

Heaviside function, 69e

Height

- of arches, 386e
- of shells, 440
- volume and, 279e
- of waves, 386e

Helical trajectories, 896e

Helix

- curvature of, 905–906
- parameterization for, 898
- principal unit normal vectors for, 907

Hemispheres

- flux across, 1184e
- parabolic, volume of, 427
- volume of, 233e, 438e

Heron's formula, 982e

Hessian matrix, 986

Hexagonal circle packing, 836e

Hexagonal sphere packing, 836e

Higher-order derivatives, 158–159, 169e, 216e, 882e

- of implicit functions, 204
- partial, 944–945

Hollow regions, Divergence Theorem for, 1176–1177

Hooke, Robert, 467

Hooke's law, 466–467

Horizontal asymptotes, 18, 101e

- limits at infinity and, 92–93

Horizontal channel flow, 1167

Horizontal lines, B-6

Horizontal line test, defining, 29

Horizontal motion, 238

Horizontal plane, 1049t

- in spherical coordinates, 1055t

Horizontal tangents, 217e

- planes, 982e

Hydrostatic pressure, 472

Hyperbolas, 803e, 792–793

- anvil of, 800e
- area of, 799e
- asymptote of, 792
- confocal, 800e
- constructions for, 799e
- equations of, 793, 797e, 799e
- foci of, 792
- properties of, 794
- sectors of, 800e
- tangent line for, 798e
- tracing, 798e
- vertex of, 792

- Hyperbolic cap, volume of, 799e
 - Hyperbolic field, 1113e
 - Hyperbolic functions, GP28
 - applications of, 512–513
 - Chain Rule and, 506
 - defining, 502–507
 - derivatives of, 505–507, 519e
 - evaluation of, 515e
 - graphs of, 502–507
 - identities of, 502–507
 - integrals of, 505–507
 - inverse, 507–508
 - derivatives of, 509–511
 - intersection points in, 509
 - as logarithms, 508
 - Product Rule and, 506
 - trigonometric functions and, 502
 - Hyperbolic paraboloid, 860, 1035e
 - Hyperboloid
 - of one sheet, 859
 - solids bounded by, 1033e
 - of two sheets, 861
 - Hypervolume, 1047e
 - Hypocycloid, 762
- I**
- Ice cream, GP16
 - Ideal flow, GP28, 1135e
 - Ideal Gas Law, 299e, 930e, 945
 - calculation of, 949e
 - Identical limits of integration, 359
 - Identities, 514e, 844e, 951e
 - double-angle, 42
 - half-angle, 42
 - of hyperbolic functions, 502–507
 - inverse, 517e
 - inverse tangent, 255e
 - proofs of, 517e
 - Pythagorean, 42, 532
 - reciprocal, 42
 - trigonometric, C-4, 41–42
 - deriving, 200e
 - Image of transformation, 1073–1074
 - Imaginary axis, C-1
 - Imaginary part, C-1
 - Implicit differentiation, 205e, 945, 955–957, 959e, 1006e
 - defining, 201
 - geometry of, 960e
 - slope of tangent line and, 203–204
 - Implicit formula for sequences, 640
 - Implicit functions
 - higher-order derivatives of, 204
 - tangent line with, 203–204
 - Improper integrals, 582–586, 1027e, 1035e
 - comparison test for, 589–590
 - convergence of, 587
 - divergence of, 587
 - over infinite intervals, 583
 - with unbounded integrand, 587
 - Inclined planes, 866e
 - force on, 834e
 - Increasing functions
 - concavity and, 274
 - defining, 257
 - derivatives and, 257–259
 - intervals of, 257–258
 - sketching, 258–259
 - Increasing sequences, 651
 - Indefinite integrals, 332e, 395e, 880
 - antiderivatives and, 323–324
 - general formulas for, 391–392, 520
 - integration by parts for, 525–527
 - Power Rule for, 323
 - Substitution Rule and, 388–391
 - of trigonometric functions, 324–326
 - verification of, 334e
 - Independent variables, 647*t*, 899
 - Chain Rule
 - with one, 952–953, 958e
 - with several, 953–954
 - defining, 2
 - domain with, 926
 - function with, 926
 - Lagrange multipliers
 - with three, 999–1000*t*
 - with two, 997–999
 - range with, 926
 - Indeterminate forms, 301
 - l'Hôpital's Rule and, 303–304
 - procedures for, 306
 - Index, 344
 - defining, 639
 - Indifference curves, 1001
 - Individual objects
 - on lines, 1064
 - sets of, 1063–1064
 - Induction, mathematical, 656
 - Inelasticity, 186
 - Inequalities
 - exponential, 402e
 - solutions to, B-2
 - test values and, 259
 - triangle, 122, 816e, 836e
 - trigonometric, 82e
 - Infinite discontinuity, 104, 115e
 - Infinite integrands, 587
 - at interior point, 588
 - Infinite intervals, 582–586
 - accumulation over, 647*t*
 - evaluation of, 583
 - improper integrals over, 583
 - solids of revolution and, 585
 - Infinite limits, 83–91
 - analytical determination of, 86–88
 - defining, 84, 122–124
 - functions with, 91e
 - at infinity, 93–94
 - one-sided, 85
 - Infinite products, 688e
 - Infinite series, GP47, GP48, GP49, GP53, 639, 662–667, 748e
 - convergence of, 645
 - defining, 645
 - estimating, 695e
 - evaluation of, 669e, 745–746
 - sequence of partial sums and, 644–647
 - steady states and, 705e
 - tail of, 666
 - value of, 677–680
 - working with, 644–645
 - Infinity
 - infinite limits at, 93–94
 - limits at, 91–102
 - defining, 124, 127e
 - end behavior of, 95–98, 272
 - of polynomials, 94
 - of powers, 94
 - Inflection points, 335e
 - defining, 263
 - derivatives and, 262–266
 - extreme values and, 272, 273, 275
 - in graphs, 271
 - Initial conditions, 328, 601
 - Initial value problems
 - differential equations and, 328, 598–600
 - for drug dosing, 622
 - in exponential growth functions, 494
 - general solutions to, 328
 - for position, 330
 - solutions to, 332e, 605e, 636e
 - for velocity, 330
 - Initial velocity, 188e, 895e
 - Inner function, 192
 - Inner integrals, 1039, 1051
 - Instantaneous growth rate, 182
 - Instantaneous velocity, 58–59, 61e, 132–133
 - defining, 179
 - tangent line and, 60
 - Integers
 - coefficients, 742e
 - squared, 54e
 - sum of, 54e
 - powers, 345
 - Integrability
 - defining, 356, 1010
 - Integrals, 595e. *See also specific types*
 - of absolute values, 360
 - approximation of, 579e
 - bounds on, 387e
 - change of variables in multiple, 1072–1081
 - circulation, 1131*t*
 - computation of, 511
 - constants in, 359–360
 - of cosecant, 524e
 - of cotangent, 524e
 - in cylindrical coordinates, 1086e
 - derivatives of, 374, 379e
 - elliptic, 749e
 - equations, 381e
 - of even functions, 382
 - existence of, 1036e
 - exponential, 530e
 - of exponential functions, 487–488
 - flux, 1131*t*, 1156–1157

Integrals (*continued*)

formulas for, 510
 Fresnel, 381e
 functions and, 380e, 401e, 402e
 of general exponential functions, 489
 general formulas for, 391–392, 520
 of general logarithmic functions, 489
 improper, 582–586, 1027e, 1035e
 comparison test for, 589–590
 convergence of, 587
 divergence of, 587
 over infinite intervals, 583
 with unbounded integrand, 587
 iterated, 1010–1014, 1015e
 evaluation of, 1011–1012
 limits with, 402e
 log, 530e
 for mass calculations, 1063–1069
 Mean Value Theorem for, 384–385, 386e
 of natural logarithms, 485
 of odd functions, 382
 over subintervals, 360
 possible, 531e
 probabilities as, 492e
 properties of, 360–361, 365e, 399e
 related, 1026e
 sine, Riemann sums and, 387e
 solids from, 438e
 in spherical coordinates, 1087e
 of sums, 359
 surface, 1146
 on explicitly defined surfaces, 1153–1154
 of scalar-valued functions, 1149–1155
 Stokes' Theorem for evaluation of, 1165–1166
 of vector fields, 1155–1159, 1160e
 symmetry in, GP20, 382, 385e
 trigonometric, 532–536
 units in, 467
 using tables of, 562–563, 594e
 of vector-valued functions, 880–881
 volume, 1040
 work, 1104–1105, 1112e
 working with, 381–385
 zero log, 595e
 Integral Test, 680e, 701t, 702
 application of, 676
 defining, 674
 Integrand, 323, 356
 change of variables and, 1077–1078
 infinite, 587
 at interior point, 588
 nonnegative, 361–362
 patterns in, 557–558
 rewriting, 557
 substitution and, 557–558
 unbounded, 586–590
 improper integrals with, 587
 Integrating factors, 627
 Integration
 basic approaches, 520–523
 with computer algebra systems, 566e

constant of, 323
 in cylindrical coordinates, 1049–1053
 of derivatives, 530e
 differentiation and, 372
 formulas, 392t, 520t
 of inverse functions, 531e
 limits of, 356, 1037–1038, 1050, 1056–1057
 methods of, 562–565
 numerical, 562, 567–576, 596e
 absolute error in, 568
 Midpoint Rule in, 568–570
 relative error in, 568
 order of, 1014, 1025e, 1041–1042, 1045e, 1062e, 1084e
 with partial fractions, 548–549
 by parts, 559–560
 defining, 525
 for definite integrals, 528–529
 for indefinite integrals, 525–527
 solids of revolution and, 528–529
 of powers, 533–534
 with quadratic factors, 552–553
 regions of, 1024e
 with respect to y , 418–420
 review of, 523e
 by Riemann sums, 399e
 in spherical coordinates, 1056–1058
 strategies, 556–560
 substitution for, 388–398, 557
 symbolic methods of, 562, 564–565
 tabular, 531e
 techniques, 520–596, 594e
 variable of, 323
 Intercepts, 274, 275
 finding, 271
 of quadric surfaces, 858
 Interest, compound, 311e
 Interior points, boundary points and, 933
 Intermediate Value Theorem, 110–112, 113e
 Intermediate variables, 953
 Intersecting lines, 853e
 Intersection
 curves of, 874e, 982e
 line-plane, 854e
 of planes, 852, 854e, 867e
 Intersection points, 53e, 217e, 786e, 802e
 area and, 514e
 of regions, 783
 finding, 319e
 in inverse hyperbolic functions, 509
 lines and, 849
 in Newton's method, 316–317
 Intervals, 782
 absolute extreme values on any, 262
 bounded, B-1
 closed, B-1
 continuity on, 107–108, 113e, 873
 of convergence, 729e, 751e
 finite, accumulation over, 647t
 infinite, 582–586
 accumulation over, 647t
 improper integrals over, 583

solids of revolution and, 585
 open, B-1
 symmetric, 119–120, 125e
 unbounded, B-1
 Inverse cosine, 49e, 54e, 227e
 defining, 44–45
 derivative of, 221
 working with, 45
 Inverse functions, 28, 239e
 continuity of, 109
 defining, 29
 derivatives of, 222–223
 existence of, 30–31
 finding, 31–32, 37e
 graphing, 32–33, 37e, 45–46
 integration of, 531e
 properties, 45–46
 right-triangle relationships, 46, 49e, 54e
 Inverse hyperbolic functions, 507–508
 derivatives of, 509–511
 intersection points in, 509
 as logarithms, 508
 Inverse identities, 517e
 Inverse properties
 for e^x , 208
 for $\ln x$, 208
 Inverses, 53e
 linear functions and, 223
 Inverse secant, derivative of, 219–222
 Inverse sine, GP30, 49e, 54e, 730e
 defining, 44–45
 derivative of, 218–219
 working with, 45
 Inverse square field, flux for, 1176–1177
 Inverse square force, 1105
 Inverse tangent
 derivative of, 219–222
 identities, 255e
 Irreducible quadratic factors, 551–554
 partial fractions with, 552
 repeated, 554
 simple, 554
 Irrational, 1125
 Isogonal curves, 788e
 Isosceles triangle
 expanding, 231e
 shrinking, 231
 Iterated integrals, 1010–1014, 1015e
 evaluation of, 1011–1012
 Iterations, 314
 fixed-point, 649e
J
 Jacobian determinants, 1084e
 of polar-to-rectangular transformation, 1075–1076
 of transformation of two variables, 1075
 of transformations of three variables, 1075
 Joules, 496
 Jump discontinuity, 104, 115e
 Jumps, functions with, 66–67
 Jupiter, speed of, 901e

K

Kepler, Johannes, 897
 Kepler's laws, GP59, GP65
 Kilowatt-hour (kWh), 496
 Kilowatts, 414e

L

Lagrange multipliers, 996
 applications of, 1003
 geometry problems and, 1000–1001
 with three independent variables, 999–10002
 with two independent variables, 997–999
 Lamé curves, 803e
 Laplace's equation, 951e, 1005e, 1130
 Laplace transforms, 593e
 Laplacian, 1142, GP78
 Lapse rate, 198e, 252
 atmospheric, 255e
 Latitude, 1060e
 Latus rectum, 800e
 Law of Cosines, 50e, 981e
 Law of Sines, 50e
 Leading terms, 666
 Least squares approximation, 995e
 Least Upper Bound Property, AP-5
 Left-sided limits
 calculating, 74–75
 defining, 64
 Leibniz, Gottfried, 116, 356
 on differentials, 298
 Leibniz Rules, 170e
 Lemniscates, 774, 778
 area within, 1032
 Lengths. *See also* Arc length
 of curves, 451–455, 896–900, 1111
 of diagonals, 825e
 of ellipses, 898
 of planetary orbits, 897–898
 Level curves, 972e
 defining, 922
 gradient and, 967–968
 graphs and, 919–927
 line integrals and, 1122e
 of planes, 930e
 of savings plans, 929e
 sketching, 923
 Level surfaces, 926
 l'Hôpital, Guillaume François, 301
 l'Hôpital's Rule, 301–310, 686, 742
 geometry of, 302
 growth rate and, 307–310
 indeterminate forms and, 303–304
 loopholes in, 516e
 loops, 312e
 pitfalls of, 309–310
 proof of, 301–302
 by Taylor series, 750e
 using, 302
 Lifting problems, 481
 pendulums and, 477e
 physical applications, 467–469

Light
 cones, 864e
 transmission of, 287e
 Limaçon family, 777e
 Limit Comparison Test, 685–686, 701e
 Limit definition of definite integrals, 356, 399e
 Limits, 56–62, 336e
 at boundary points, 933–935
 calculating, 129e
 calculator, 162e, 491e
 of composite functions, 106
 computing, 71–82
 of constant functions, 931
 constant multiple of, 72
 continuity and, 873, 931–937
 cosine, 81e, 337e
 defining, 58, 63–71, 116–124, 931
 derivatives from, 161e
 difference of, 72
 of difference quotients, 200e
 estimating, 69e
 evaluating, 69e, 72–73, 80e
 of even functions, 70e, 82e
 examining, 64–65
 finding
 factor and cancel method, 75–76
 from graphs, 63–64
 from tables, 64
 of functions, 118
 of two variables, 931–932
 by graphs, 70e
 identical, 359
 important, 162e
 infinite, 83–91
 analytical determination of, 86–88
 defining, 84, 122–124
 functions with, 91e
 at infinity, 93–94
 at infinity, 91–102
 defining, 124, 127e
 horizontal asymptotes and, 92–93
 of polynomials, 94
 of powers, 94
 with integrals, 402e
 of integration, 356, 1037–1038, 1050, 1056–1057
 laws, 72, 939e
 for functions of two variables, 932
 justifying, 122–123
 for one-sided limits, 74–75
 proofs of, AP-1
 sequences and, 650–651
 left-sided
 calculating, 74–75
 defining, 64
 of linear functions, 71, 931
 miscellaneous, 939e
 nonexistence of, 934–935, 938e
 of odd functions, 70e, 82e
 one-sided, 81e
 defining, 64–67

infinite, 85, 127e
 limit laws for, 74–75
 proof, 127e
 two-sided limits and, 66
 optic, 311e
 with parameters, 749e
 of polynomial functions, 73–74
 power of, 72
 product of, 72
 proofs, 120–122, 939e
 quotient of, 72
 of rational functions, 73–74
 of Riemann sums, GP18, 416–417
 right-sided
 calculating, 74–75
 defining, 64
 roots of, 72
 of sequences, 641–644, 648e
 defining, 658
 sine, 81e
 sum of, 72, 357–358
 by Taylor series, 749e
 tetrahedron, 1086e
 transcendental functions and, 110
 trigonometric, 78–79, 176e
 of trigonometric functions, 88
 two-sided, one-sided limits and, 66
 using polar coordinates, 939e
 for vector-valued functions, 872–873, 875e
 Linear approximation, 518e, 709–712
 change estimated with, 297
 concavity and, 295–298, 299e
 defining, 293, 977
 differentials and, 292–298
 errors and, 293–294, 300e
 estimations with, 299e, 336e
 second derivatives and, 300e
 for sine function, 294
 tangent planes and, 973–980
 underestimation with, 295
 uses of, 297
 variation on, 296
 Linear combinations, 816e
 Linear differential equations, 597
 first-order, 620–625
 solutions to, 621–623
 Linear factors
 repeated, 550–551, 554
 simple, 554
 partial fractions with, 548–550
 Linear functions
 area function for, 378e
 derivatives of, 223
 graphing, 15, 23e
 limits of, 71, 121, 931
 Mean Value Theorem for, 256e
 piecewise, 16–17, 24e
 Linear growth, exponential growth and, 493–494
 Linear transformations, 1083e
 Linear vector fields, 1123e

Line integrals, 1114e, 1182e
 evaluation of, 1099–1100
 Stokes' theorem for, 1164–1165
 Fundamental Theorem of, 1121e, 1122e, 1179t
 level curves and, 1122e
 scalar, 1098–1099
 evaluation of, 1100
 of vector fields, 1102–1106
 Lines. *See also* Tangent line
 curvature of, 904
 distance to, from points, 846–849
 equations of, B-6, B-7, 845, 853e
 horizontal, B-6
 intersecting, 853e
 intersection points and, 849
 mass on, 1070e
 motion on, 337e
 normal, 207e
 parallel, B-7, 853e
 parametric equations of, 756, 845
 perpendicular, B-7
 revolution about, 433, 444
 secant, 6–9, 91e
 average velocity and, 58, 60
 slope of, 7
 skew, 849, 853e
 slopes of, 150e
 in space, 844–852, 866e
 as vector-valued functions, 869
 vertical, B-6
 Line segments, 757, 853e
 length of, 901e
 parametric equations for, 758
 LIPET, 527
 Lissajous curves, 766e
 $\ln x$
 inverse properties for, 208
 powers of, 308
 unbounded, 492e
 Loan payments, 623, 670e
 Local Extreme Value Theorem, 244
 Local maxima, 243–247, 984–985
 defining, 244, 984
 derivatives and, 260–262, 984
 extreme values and, 261–262
 First Derivative Test for, 260–261
 location of, 334e
 in Newton's method, 317
 Local minima, 243–247, 985
 defining, 244, 984
 derivatives and, 260–262, 984
 extreme values and, 261–262
 First Derivative Test for, 260–261
 location of, 334e
 in Newton's method, 317
 Logarithmic differentials, 983e
 Logarithmic differentiation, 214–215
 Logarithmic functions
 defining, 14, 33
 derivatives of, 208–215, 216e
 with different bases, 312e
 general, 213–214

defining, 488
 derivatives of, 489
 integrals of, 489
 graphing, 38e
 inverse relations for, 33
 natural, 14
 properties of, 34, 37e
 Logarithmic scales, GP8
 Logarithms, hyperbolic functions expressed as, 508
 Log integrals, 530e
 Logistic equation, 616–618
 direction field for, 608–609
 for epidemics, 619e
 for populations, 619e
 Logistic functions, designing, 634e
 Logistic growth, GP39, 637e
 Logistic map, GP41
 Logistic models, designing, 628
 Log-normal probability distribution, 518e
 Long division, 553–554
 Longitude, 1060e
 Lorenz curves, 424e
 Lotka, Alfred, 631
 Lotka-Volterra equations, 631
 Lower bounds, 362
 of series, 681e, 706e
M
 Maclaurin series, 738, 746
 Magnetic field, 545e
 toroidal, 902e
 Magnetic force, cross products and, 841–842
 Magnitude
 unit vectors and, 810, 822–823, 825e
 of vectors, 808, 822–823
 Major axis, 765e
 defining, 792
 vertex, 792
 Manufacturing errors, 980
 Marginal cost, 184, 188e, 240e, 415e
 defining, 183
 Mass, 476e
 center of, 1064, 1065, 1071e, 1087e, 1161e
 on edge, 1072e
 with variable density, 1069
 of conical sheets, 1154
 density and, 1017e, 1036e, 1060e, 1062e, 1111
 gravitational fields and, 1097e
 integrals for calculations of, 1063–1069
 on lines, 1070e
 of one-dimensional objects, 465, 473e
 of paraboloids, 1052
 physical applications, 465–466
 on plane, 834e
 from variable density, 465

Mass transport, Divergence Theorem and, 1174
 Matching, surfaces, 867e
 Matching curves, 765e
 Matching functions, 54e
 with area function, 379e
 with polynomials, 720e
 Mathematical induction, 656
 Maximum curvature, 915e
 Maximum error, 719e
 Maxwell's equations, GP76
 Mean
 arithmetic, 1004e
 arithmetic-geometric, 661e
 defining, 387e
 geometric, 1004e
 geometric-arithmetic, 836e
 Means by tangent lines, GP21
 Mean Value Theorem, 250–254, 336e
 Cauchy's, 257e
 consequences of, 253–254
 Generalized, 257e, 301, 385, 387e
 graphs of, 255e–256e
 for integrals, 384–385, 386e
 for linear functions, 256e
 police and, 256e
 proof of, 252
 for quadratic functions, 256e
 Median, of triangle, 826e, 1072e
 Megawatts, 414e
 Mercator projections, GP35
 Michaelis-Menton kinetics, GP13
 Midpoint Rule, 580e
 application of, 569–570
 approximation, 578e
 defining, 569
 errors in, 571
 in numerical integration, 568–570
 Riemann sums and, 569
 Midpoints, 819
 formulas, 826e
 spheres and, 824e
 Minor axis, 765e
 defining, 792
 vertices, 792
 Mixed partial derivatives, 944–945
 Modified Newton's method, 321e
 Modulus, C-2
 Moments of inertia, GP73
 Monotonic functions, 257
 Monotonic sequences, 651
 bounded, 655
 Moore's Law, 518e
 Morphing, parabolas, 397e
 Motion
 analyzing, 918e
 antiderivatives and, 329–331
 circular, 764
 acceleration for, 884–885

straight-line motion and, 886–887
 velocity for, 884–885
 equations of, 894e
 in gravitational fields, 181, 408, 601
 with gravity, 330–331, 333e
 horizontal, 238
 on lines, 337e
 modeling, 917e
 Newton's First Law of, 180
 one-dimensional, 178–182
 oscillating, 412e
 periodic, 396e
 of projectiles, 337e
 in space, 883–892
 on sphere, 883e
 straight-line
 circular motion and, 886–887
 uniform, 886
 three-dimensional, 894e
 two-dimensional, 894e
 in gravitational field, 887–892
 vertical, 337e
 Multiple integration, GP69, GP70, GP71,
 GP73, GP74
 Multiplication
 for integration, 521
 scalar, 805–806, 814e, 865e
 associative property of, 811
 by zero scalar, 811
 by zero vector, 811
 Multiplicative identity, 811
 Multiplier effect, 707e
 Mystery series, 746–747

N

$n!$, GP51
 Natural exponential function
 defining, 28
 derivative of, 156
 Natural logarithms
 defining, 483
 derivative of, 484
 domain of, 484
 integral of, 485
 of power, 485
 of products, 485
 properties of, 483–485, 491e
 range of, 484
 sign of, 484
 n -balls, GP69
 Net area, 353, 364e
 defining, 354
 definite integrals and, 373–374
 maximum, 380e
 minimum, 380e
 zero, 367e, 381e
 Net change
 future values and, 408–410
 velocity and, 402–410
 Net force, 866e
 Newton, Isaac, 116, 312
 Newtons, 832, GP30
 Newton's First Law of Motion, 180
 Newton's Law of Cooling, 623–625, 637e
 Newton's method, GP17, 312, 336e, 516e
 application of, 314
 curves and, 320e
 derivation of, 313–315
 intersection points in, 316–317
 modified, 321e
 pitfalls of, 317–318, 320e
 procedure for, 314
 programs for, 316
 residuals in, 316
 for roots, 319e
 Noncusps, 883e
 Nondecreasing functions, 257
 Nondecreasing sequences, 651
 Nondifferentiability, 950e, 951e
 conditions for, 257e
 of functions, 947–948
 Nonincreasing functions, 257
 Nonincreasing sequences, 651
 Nonlinear differential equations,
 597–598
 Nonlinear equations, 608
 Nonlinear springs, 476e
 Nonnegative integrand, 361–362
 Nonorientable surfaces, 1149
 Nonuniform grids, 579e
 Nonzero constants, 756
 Normal components, 908
 Normal distribution, GP52, 275–276, 745
 Normal forces, 832–833
 Normal lines, visualizing, 207e
 Normal planes, 911
 Normals, tangents and, 788e
 Normal vectors, 849, 1092–1093
 curvature and, 902–912
 principal unit
 defining, 906
 for helix, 907
 properties of, 907
 Norms, 808
 Notation
 for definite integrals, 356
 derivatives, 142–143
 of partial derivatives, 942
 sigma, 344–345, 350e
 for Riemann sums, 351e
 summation, 344–345
 Notches, 291e
 n th-order Taylor polynomials, 711
 Numbers
 binary, 669e
 complex, C-1
 operations with, C-2
 polar form of, C-4
 prime, 681e
 real, 748e
 representing, 745–746
 sets of, B-1, B-2
 Taylor polynomials and, 713–714
 Numerator, degree of, 97
 Numerical differentiation, GP11

Numerical integration, GP32, 562,
 567–576, 596e
 absolute error in, 568
 errors in, 577
 Midpoint Rule in, 568–570
 relative error in, 568
 Simpson's Rule in, 574–577
 Trapezoid Rule in, 570–574

O

Objective functions, 280, 996
 Oblique asymptotes, 95, 793
 Octants, 817
 Odd functions
 composition of, 13e
 defining, 8, 381
 derivatives of, 200e
 differences of, 423e
 even functions and, 381–382
 integrals of, 382
 limits of, 70e, 82e
 Odd squares, 682e
 One-dimensional motion, 178–182
 One-dimensional objects, mass of, 465, 473e
 One-sided derivatives, 152e
 One-sided limits, 81e
 defining, 64–67
 infinite, 85, 127e
 limit laws for, 74–75
 proof, 127e
 two-sided limits and, 66
 One-to-one functions, 29–30, 52
 One-to-one transformations, 1074
 Open balls, 820
 Open intervals, B-1
 Open regions, 992–993
 Open sets, 933
 Operations, floating point, 983e
 Optics limits, 311e
 Optimization problems, GP16, GP68,
 280–284, 335e
 fuel use, GP29
 guidelines for, 282
 Orientable surfaces, 1149
 Orientation, 1112e
 of curves, 869–870
 force and, 477e
 Oriented curves, 1102
 Origin
 behavior at, 491e
 symmetry about, 8, 772
 tangent line at, 786e
 in xyz -coordinate system, 818
 Orthogonal planes, 851
 defining, 852
 Orthogonal projections, 830–832
 defining, 831
 sketching, 834e
 Orthogonal trajectories, 207e, 619e
 Orthogonal unit vectors, 835e
 Orthogonal vectors, 840
 defining, 828

Oscillating growth rates, 414e
 Oscillating motion, 412e
 Oscillators, GP15
 damped, 320e
 displacement of, 531e
 Osculating circle, 914e
 Osculating plane, 909, 911
 Outward flux, 1109
 of radial field, 1128

P

Parabolas, 789–791
 Archimedes' quadrature of, 670e
 arc length of, 541, 567e
 average distance on, 386e
 Bernoulli's, 457e
 concavity of, 270e
 curvature of, 905–906
 directrix of, 789
 ellipses and, 799e
 equal area properties for, 424e, 482e
 equations of, 797e, 799e
 focus of, 789
 morphing, 397e
 parametric, 754
 properties, 27e
 reflection property of, 791–792, 799e
 standard, equations of, 790
 tracing, 798e
 transformations of, 21
 vertex of, 790
 vertex property, 27e
 Parabolic coordinates, 1083e
 Parabolic cube, volume of, 426
 Parabolic cylinder, 856
 Parabolic dam, 476e
 Parabolic hemisphere, volume of, 427
 Parabolic regions, 1071e
 Parabolic trajectories, 895e, 915e
 Paraboloids
 descriptions of, 1155*t*
 distance to, 1007e
 elliptic, 859
 flux across, 1184e
 hyperbolic, 860, 1035e
 mass of, 1052
 solid, 1052
 solids bounded by, 1033e
 volume of, 799e, 981e, 1029
 Parallelepiped, 866e
 Parallel forces, 832–833
 Parallel lines, B-7, 853e
 Parallelogram Rule, 806, 811, 820
 Parallelograms
 area of, 838, 843e, 866e
 diagonals of, 837e
 Parallel planes, 819, 851
 defining, 852
 Parallel vectors, 805, 816e, 825e
 Parameterized surfaces
 of cones, 1148
 of cylinders, 1147
 of spheres, 1148
 Parameters, 753
 arc length as, 898–900
 conic, 802e
 eliminating, 764, 801e
 limits with, 749e
 Parametric circle, 755
 Parametric curve, GP55, 754
 derivatives for, 759
 tangent line and, 801e
 Parametric descriptions, 801e, 1159e
 Parametric equations
 arc length and, 760–763
 curves by, 761
 basic ideas, 753–759
 of circles, 756
 of curves, 757–758
 curves to, 764
 derivatives and, 759–760
 of ellipse, 760, 766e
 of lines, 756, 845
 for line segments, 758
 for polar curves, 775
 polar equations and, 801e
 working with, 763e
 Parametric parabolas, 754
 Parametric surfaces, 1149
 Partial cylinder, surface area of, 1152–1153
 Partial derivatives, 940–948, 1005e
 applications of, 945–946
 calculation of, 942
 defining, 941
 equality of, 944–945, 949e
 estimation of, 949e
 evaluation of, 948e
 higher-order, 944–945
 mixed, 944–945
 notation of, 942
 second, 944–945, 949e
 Partial fractions, 546–554
 decomposition, 547, 554
 form of, 551
 from integrals, 556e
 integration with, 548–549
 with irreducible quadratic factors, 552
 method of, 547–548
 with simple linear factors, 548–550
 Partial sums, 705e
 errors in, 695e
 sequence of, infinite series and, 644–647
 Particle position, 537e
 Partition
 defining, 1009
 general, 355
 regular, 341
 Path length, of projectiles, 546e
 Paths
 circular, acceleration on, 908
 on spheres, 886–887
 Pendulums
 lifting, 477e
 period of, GP37, 567e, 580e
 Perimeter, of rectangles, 285e
 Period, 50e
 of pendulums, GP37, 567e, 580e
 of trigonometric functions, 42
 Periodic functions, 42
 Periodic motion, 396e
 Perpendicular lines, B-7
 Perpendicular vectors, 816e
 Perpetual annuity, 591
 Pharmacokinetics, GP14, GP43, 498–499
 Phase and amplitude, GP6
 Phase shift, 43
 Physical applications
 density, 465–466
 force, 471–473
 lifting problems, 467–469
 mass, 465–466
 pressure, 471–473
 pumping problems, 469–473
 Pi (π), GP46, GP49
 Piecewise functions, 939e
 continuity of, 177e
 defining, 16, 24e
 graphing, 16–17
 linear, 16–17, 24e, 53e
 solid from, 438e
 Piecewise velocity, 412e
 Planar polar regions, 1032
 Plane curves, 914e
 families of, 918e
 torsion of, 918e
 Plane region
 area of, 1023, 1026e, 1126
 average value of function over, 1014
 Planes
 angle between, 855e
 complex, C-1
 defining, 849
 distance from ellipsoid to, 983e
 distance to, 1003e
 equations of, 849–851, 854e, 867e
 general equations of, 850
 horizontal, 1049*t*
 in spherical coordinates, 1055*t*
 inclined, 866e
 force on, 834e
 intersection of, 852, 854e, 867e
 mass on, 834e
 normal, 911
 orthogonal, 851
 defining, 852
 osculating, 909, 911
 parallel, 819, 851
 defining, 852
 properties of, 851, 854e
 simple, equations of, 818–819
 sketching, 824e
 in space, 844–852
 tangent, 972e, 980e
 equations of, 975–976
 horizontal, 982e
 linear approximation and, 973–980
 through three points, 850–851
 transformations in, 1073–1078
 uv-plane, 1080

- vectors in, 804–813
- vertical half-plane, 1048*t*
 - in spherical coordinates, 1055*t*
- Planetary orbits, GP59, 386e
 - lengths of, 897–898
- Planimeters, GP77
- Pluto, 897
- Point charges, electric fields and, 1097e
- Points. *See also* Critical points; Intersection points
 - boundary
 - interior points and, 933
 - limits at, 933–935
 - closest, 917e
 - collinear, 825e
 - continuity at, 103–107, 112e
 - corner, 147
 - crossover, 662e
 - derivatives at, 194
 - of discontinuity, 104
 - distance from, to lines, 846–849
 - extreme, 721
 - fixed
 - of quadratic functions, 320e
 - of quartics, 320e
 - grid, 341
 - inflection, 335e, 721, 516e
 - defining, 263
 - derivatives and, 262–266
 - extreme values and, 272, 273, 275
 - in graphs, 271
 - interior, boundary points and, 933
 - locating, 245
 - planes through three, 850–851
 - in polar coordinates, 768, 775
 - saddle, 860
 - defining, 985–987
 - sets of, 825e, 866
 - solitary, 996e
- Point-slope form, B-6, 314
- Polar axis, 767
- Polar coordinates, GP56, GP65
 - calculus in, 779–785
 - Cartesian coordinates and, 768–769, 1085e
 - circles in, 771
 - curves in, 769–770
 - defining, 767–768
 - double integrals in, 1027–1032
 - graphing in, 771–774
 - limits using, 939e
 - points in, 768, 775
 - rectangular coordinates and, 1027–1030
 - sets in, 801e
 - unit vectors in, 1098e
 - vector fields in, 1098e
 - vectors from, 815e
- Polar curves
 - arc length for, 785, 788e
 - area of regions bounded by, 781–784
 - graphing, 776e
- Polar equations
 - of conic sections, 795–797, 798e, 799e, 802e
 - parametric equations and, 801e
 - symmetry in, 772
- Polar form
 - conjugates in, C-6
 - quotients in, C-6
- Polar graphs
 - matching, 774
 - plotting, 771–772, 773
 - symmetry in, 772–773
- Polar rectangle, 1027
 - double integrals over, 1028
- Polar regions
 - area of, 1031–1032
 - average value over, 1032
 - general, 1030–1032
 - planar, 1032
- Polar-to-rectangular transformation,
 - Jacobian of, 1075–1076
- Pólya's method, GP1
- Polynomial approximation, 294, GP22, GP49, GP52
 - defining, 709
 - with functions, 708–718
 - Taylor, 712–713
- Polynomial equations, C-6
- Polynomial functions
 - continuity of, 105
 - limits of, 73–74
- Polynomials. *See also* Taylor polynomials
 - composition of, 27
 - deceptive, 272
 - degree of, 13
 - derivatives of, 156
 - limits at infinity of, 94
- Population function, 52e
- Population models
 - differential equations and, 627–629
 - logistic equation for, 619e
- Position, 178–179, 501e, 883
 - from acceleration, 333e, 412e
 - defining, 404, 884
 - distance and, 478e
 - initial value problems for, 330
 - particle, 537e
 - vectors, 807
 - velocity and, 333e, 403–405, 411e
- Position function, 178, 181
 - as antiderivative, 329
 - values of, 405–407
- Positive orientation, defining, 755
- Positive terms, series with, 667
- Possible integrals, 531e
- Potential functions, 973e
 - defining, 1094
 - gradient fields and, 1094
- Power, defining, 496
 - energy and, 414e
- Power functions, 983e
 - derivatives of, 154
 - graphing, 16–17
- Power Rule, 943
 - for indefinite integrals, 323
- Power Rule for Derivatives, 153
 - extending, 162e
 - general form of, 166
 - for negative integers, 162e, 166–167
 - proof of, 162e
 - using, 166–167
- Powers
 - Chain Rule for, 195
 - of cosine, 532–533, 538e
 - exponential functions and, 309
 - growth rates and, 308–309
 - integration of, 533–534
 - limits of, 72
 - limits at infinity of, 94
 - of $\ln x$, 308
 - natural logarithm of, 485
 - product of, 533–536
 - by Riemann sums, 367e
 - roots and, 424e
 - of sine, 532–533, 538e
 - sums of, of integers, 345
 - symmetry of, 387e
 - of tangent function, 535
- Power series, 700e
 - combining, 729e
 - convergence of, AP-5
 - defining, 708–709
 - for derivatives, 748e
 - differentiation of, 730e, 743–744
 - functions as, 730e, 746–747
 - from geometric series, 751e
 - integrating, 745
 - product of, 730e
 - properties of, 722–729
 - scaling, 730e
 - shifting, 730e
- Predator-prey models, GP36, 635e
 - differential equations and, 631–633
 - solutions to, 636e
- Pressure
 - forces and, 471–473
 - hydrostatic, 472
 - physical applications, 471–473
- Price elasticity of demand. *See* Elasticity
- Prime numbers, 681e
- Prime Number Theorem, 307
- Principal unit normal vector, 906
 - for helix, 907
- Prisms, volume of, 1039
- Probabilities
 - of encounters, 983e
 - geometric, GP23
 - as integrals, 492e
- Probability functions, 924
- Problem-solving skills, GP1, GP2
- Product Law, proof of, AP-1
- Product property, proof of, 492
- Product Rule, 878
 - defining, 163–164
 - for divergence, 1142
 - for functions, 170e
 - proof of, 883e
 - using, 164

Production functions, GP68
 Products. *See also* Cross products; Dot products
 of cosine, 533
 infinite, 688e
 limits of, 72
 natural logarithm of, 485
 of secant function, 535–536
 of sine, 533
 of tangent function, 535–536
 Projectiles
 motion of, 337e
 path length of, 546e
 Projections
 angles and, 866e
 Mercator, GP35
 Proofs, limits, 120–122
p-series, 701*t*
 alternating, 695e
 approximation of, 677
 convergence of, 677
 log, 681e, 706e
 shifting, 682e
 test, 677
 Pumping problems, 481e
 physical applications, 469–473
 solutions to, 470
 Pursuit curve, 279e
 Pursuit problem, GP40
 Pyramids, volume of, 958e
 Pythagorean identities, 42, 532
 Pythagorean Theorem, 538

Q

Quadratic approximation, 709–712
 Quadratic factors
 integration with, 552–553
 irreducible, 551–554
 partial fractions with, 552
 repeated, 554
 simple, 554
 Quadratic functions, 240
 fixed points of, 320e
 Mean Value Theorem for, 256e
 sine function and, 386e
 Quadratic vector fields, 1123e
 Quadric surfaces, 855, 857–862
 identifying, 863
 intercepts of, 858
 traces of, 858
 Quadrilateral property, 826e
 Quartics
 fixed points of, 320e
 general, 270e
 Quotient Law, proof of, AP-2
 Quotient Rule, 164–165, 309
 Tangent line and, 165
 using, 165
 Quotients
 centered difference, 140e
 difference, 178e
 limits of, 200e
 working with, 6, 12e

limits of, 72
 in polar form, C-6

R

Radial coordinates, 767
 Radial field, 1171e, 1180e, 1183e
 divergence of, 1146e
 flux of, 1158
 outward flux of, 1128
 spheres and, 1161e
 Radial vector fields
 defining, 1090–1091
 divergence of, 1138–1139, 1144e
 in three dimensions, 1093
 Radian measure, 39–40
 Radiometric dating, 497–498
 Radius
 of circle, B-4
 of convergence, 729e, 751e
 of curvature, 914e
 of spheres, 820
 Range, 53e
 in context, 3
 defining, 1, 920
 of functions, 2
 with independent variables, 926
 of natural logarithm, 484
 of two variables, 920
 Rate constant, 493
 exponential growth functions
 and, 494
 Rate of change
 derivatives as, 178–186
 instantaneous, 136
 tangent line and, 133–134
 Ratio, 663
 Rational functions, 14, 17–18
 asymptotes of, 97–98
 continuity of, 105
 division with, 522
 end behavior of, 95–96, 102e
 limits of, 73–74
 reduced proper, 551
 surprises of, 274
 Rational terms, 702–703
 Ratio Test, 701*t*, 702
 defining, 696
 proof, 697, AP-2
 using, 697
 RC circuit equation, 635e
 Real axis, C-1
 Real numbers, 748e
 approximation of, 751e
 representing, 745–746
 sets of, B-1, B-2
 Taylor polynomials and, 713–714
 Real part, C-1
 Reciprocal identities, 42
 Rectangles, 286e
 area of, 285e
 curves and, 335e
 in ellipse, 1003e
 expanding, 231e
 perimeter of, 285e
 polar, 1027
 double integrals over, 1028
 triangles in, 290e
 volume of, 235e
 Rectangular coordinates, 767
 cylindrical coordinates and, 1049
 polar coordinates and, 1027–1030
 spherical coordinates and, 1054
 Rectangular regions, double integrals over, 1008–1014
 Recurrence relation, 641, 648e
 applications of, 655–656
 defining, 640
 sequences by, 660e
 Recursively defined sequences, 705e
 Reduced form, of rational function, 548
 Reduction formula, 527
 application of, 530e
 defining, 534
 sine, 537e
 Reflection property, of parabolas, 791–792, 799e
 Regions
 annular, 1030
 area of, 379e, 478e, 518e, 555e, 786e, 802e, 1134e
 bounded by polar curves, 781–784
 bisection of, 424e
 bounded by spirals, 788e
 bounded by surfaces, 1029
 compound, 417
 between curves, 416–420, 524e, 1114–1115
 decomposition of, 1023
 general
 circulation on, 1128–1130
 description of, 1034e
 double integrals over, 1017–1023
 flux on, 1128–1130
 hollow, Divergence Theorem for, 1176–1177
 of integration, 1024e
 open, 992–993
 parabolic, 1071e
 plane
 area of, 1023, 1026e
 average value of function over, 1014
 polar
 area of, 1031–1032
 average value over, 1032
 general, 1030–1032
 planar, 1032
 rectangular, double integrals over, 1008–1014
 specification of, 1030–1031
 between spheres, 1005e
 square, 1026e
 unbounded, 992–993
 Regular partition, 341
 Reindexing, 662e

- Related rates, 227–230
 - steps for, 228
 - Related sequences, 705e
 - Relative error, in numerical integration, 568
 - Relative growth rates, 493, 499e
 - Remainders, 681e, 719e
 - in alternating series, 691–692
 - estimation of, 715–718
 - Taylor polynomials and, 713
 - Removable discontinuity, 104, 115e
 - Repeated irreducible quadratic factors, 554
 - Repeated linear factors, 550–551, 554
 - Repeated square root, 661e
 - Residuals, in Newton's method, 316
 - Resistors, 929e
 - Resource consumption, 496
 - Revenue function, 190
 - Reversing limits, 359
 - Revolution, 451e
 - about lines, 433, 444
 - solid of, 428, 435
 - infinite intervals and, 585–586
 - integration by parts and, 528–529
 - solids of, 516e, 864e
 - surface area of, 460
 - surfaces of, 766e, 801e, 1161e
 - about y-axis, 430–431
 - Riemann sums, 1010
 - for area, 340–344
 - for constant functions, 352e
 - defining, 341
 - evaluating, 399e
 - general, 355
 - identifying, 351e
 - integration by, 399e
 - left, 341, 342, 349e
 - limits of, 416–417
 - midpoint, 341, 343, 349e, 350e
 - Midpoint Rule and, 569
 - powers by, 367e
 - right, 341, 342, 349e
 - for semicircles, 351e
 - sigma notation for, 345–346, 351e
 - sine integrals and, 387e
 - from tables, 344
 - Right-handed coordinate systems, 817
 - Right-hand rule, 837
 - Right-sided limits
 - calculating, 74–75
 - defining, 64
 - Right-triangle relationships, 46, 49e, 54e
 - Roller coaster curves, 871
 - Rolle's Theorem, 251, 255e
 - proof of, 252
 - Root functions, 53e
 - graphing, 17
 - Root mean square, 401e
 - Roots, 13
 - continuity with, 108
 - estimation of, 320e
 - finding, 312, 319e
 - functions with, 17, 53e, 108, 113e
 - graphing, 276–277
 - limits of, 72
 - Newton's method for, 319e
 - powers and, 424e
 - square, 649e
 - Root Test, 701t, 702
 - defining, 698
 - proof, 698, AP-4
 - using, 698
 - Rose family, 778e
 - Rotation field, 1145e, 1179e
 - curl of, 1141, 1144e
 - Divergence Theorem with, 1173
 - Rotations, 1077
 - fields, 1113e
 - Rounded values, 898
 - Rule of 70, 501e
- S**
- Saddle point, 860
 - defining, 985–987
 - Sag angle, 515e
 - SAV. *See* Surface-area-to-volume ratio
 - Scalar components, defining, 831
 - Scalar line integrals, 1098–1099
 - evaluation of, 1100
 - Scalar multiplication, 805–806, 814e, 865e
 - associative property of, 811
 - Scalars, defining, 805
 - Scalar-valued functions, surface integrals
 - of, 1149–1155
 - Scaling
 - power series, 730e
 - transformations, 22
 - Searchlight problem, 236e, 289e
 - Secant function, 46
 - hyperbolic, defining, 503
 - inverse, derivative of, 219–222
 - products of, 535–536
 - Secant lines, 6–9, 91e
 - average velocity and, 58, 60
 - slope of, 7
 - Secant substitution, 540, 543, 545e
 - Second-order differential equations topics covered online at bit.ly/2Od5VE4*
 - Amplitude-phase form*
 - Basic ideas and terminology*
 - Cauchy-Euler equations*
 - Electrical circuits*
 - Filters, high-pass and low-pass*
 - Forced oscillator equations*
 - Gain function*
 - Homogeneous equations and solutions*
 - Linear independence*
 - Mechanical oscillators*
 - Nonhomogeneous equations and solutions*
 - Phase lag function*
 - Phase plane*
 - Resonance*
 - Superposition principle*
 - Transfer function*
 - Undetermined coefficients*
 - Variation of parameters*
 - Second derivatives, 204, 956–957
 - Chain Rule for, 200e
 - linear approximation and, 300e
 - Second Derivative Test, 265, 268e, 985–986, AP-8
 - inconclusive, 987
 - Second-order derivatives, 175
 - Second-order equations, 605e
 - Second partial derivatives, 944–945, 949e
 - Segments
 - of circles, 544e
 - line, 757, 853e
 - parametric equations for, 758
 - Semicircles, graphing, 13e
 - Separable differential equations, 614–618
 - implicit solutions, 616
 - Sequences, GP9, GP41, GP42, GP43, GP44, GP45, GP46, GP63
 - bounded
 - above, 651
 - below, 651
 - monotonic, 655
 - convergent, 313, 658
 - defining, 642
 - decreasing, 651
 - defining, 640
 - divergent, 313, 658
 - defining, 642
 - examples of, 639–640
 - explicit formulas for, 648e
 - factorial, 656
 - Fibonacci, 682e
 - geometric, 652–654
 - graphing, 652
 - growth rates of, 656–657, 660e
 - hailstone, 662e
 - increasing, 651
 - limit laws and, 650–651
 - limits of, 641–644, 648e
 - defining, 658
 - monotonic, 651
 - bounded, 655
 - nondecreasing, 651
 - nonincreasing, 651
 - of partial sums, infinite series and, 644–647
 - by recurrence relation, 660e
 - recursively defined, 705e
 - related, 705e
 - series and, 705e
 - Squeeze Theorem and, 654–655
 - steady states and, 705e
 - of sums, 682e
 - terminology for, 651–652
 - working with, 641–642
 - Series, 662–667. *See also* Infinite series;
 - Power series; *p*-series
 - alternating, remainder in, 691–692
 - binomial, 735, 742e, 751e
 - working with, 736, 741e
 - categorizing, 700
 - composition of, 742e
 - Convergence Tests and, 700–703

Series (*continued*)

convergent, properties of, 666–667
 designer, 742e
 in equations, 671e, 706e
 evaluation of, 705e
 functions defined by, 671e
 geometric, 663, 742e
 decimal expansions as, 664–665
 evaluating, 668e
 power series from, 751e
 harmonic, 673
 alternating, 688
 defining, 674
 Fibonacci sequence and, 682e
 identifying, 746
 lower bounds of, 681e, 706e
 Maclaurin, 738, 746
 mystery, 746–747
 with positive terms, 667, 679
 rearranging, 695e
 sequences and, 705e
 of squares, 688e
 Taylor, 731–740
 convergence of, 737–740, 740e
 for functions, 731–735
 l’Hôpital’s Rule by, 750e
 limits by, 742–743, 749e
 telescoping, 665–667
 upper bounds of, 681e, 706e
 value of, 70e

Sets

bounded, 988
 closed, 933, 988
 closed bounded
 absolute maxima on, 988
 absolute minima on, 988
 in cylindrical coordinates, 1049, 1059e
 identification of, 824e, 865e
 of individual objects, 1063–1064
 open, 933
 in polar coordinates, 801e
 of real numbers, B-1, B-2
 in spherical coordinates, 1060e

Shear transformations, 1083e

Sheets

conical, 1154
 hyperboloid of one, 859
 hyperboloid of two, 861
 Shell method, 448e
 washer method and, 449e
 about x -axis, 446
 about y -axis, 446

Shells

circumference of, 440
 cylindrical, 439–447
 formulas for, 442
 height of, 440
 volume by, 439–447

Shifting

phase, 43
 power series, 730e
 p -series, 682e
 sine functions, 425

transformations, 22
 vertical, 43
 Sierpinski triangle, 707e
 Sigma notation, 344, 350e
 for Riemann sums, 345–346, 351e
 Sign, of natural logarithm, 484
 Signed area, 354
 Simple irreducible quadratic factors, 554
 Simple linear factors, 554
 partial fractions with, 548–550
 Simple nonlinear equations, direction field
 for, 608
 Simple planes, equations of, 818–819
 Simpson’s Rule, GP32, 580e
 application of, 576
 approximation, 578e
 defining, 576
 errors in, 576–577
 exact, 581e
 formulas, 582e
 in numerical integration, 574–577
 shortcut for, 582e
 Sine, C-6
 graphing of, 53e
 inverse, GP30, 49e, 54e, 227e, 730e
 defining, 44–45
 derivatives of, 218–219
 working with, 45
 Law of, 50e
 limit, 81e
 powers of, 532–533, 538e
 products of, 533
 substitution, 539–540
 Sine bowls, 442
 Sine function
 average value of, 397e
 continuity of, 115e
 curvature of, 915e
 hyperbolic, 102, 455
 defining, 503
 linear approximation for, 294
 quadratic functions and, 386e
 shifting, 425
 Sine integral function, 375, 581
 asymptote of, 381e
 Sine integrals, Riemann sums and, 387e
 Sine reduction formula, 537e
 Skew lines, 849, 853e
 Slant asymptote, 95–96, 98, 129e, 793
 Slice-and-sum method, 357, 403, 416
 Slicing
 general method, 435e, 1010
 volume by, 425–434
 Slinky curves, 871–872
 Slope
 analyzing, 150e
 on circles, 780
 of curves, 203–204
 function, 18, 25e
 of lines, 150e
 locations, 161e
 of secant lines, 7

of tangent line, 59–60, 62e, 76–77, 82e, 133, 157–158, 760, 765
 implicit differentiation and, 203–204
 polar coordinates and, 779–781
 Slope-intercept form, B-6
 Small-angle approximation, 294
 Smooth functions, 877
 Snell’s Law, 289e
 Solids
 bounded by hyperboloids, 1033e
 bounded by paraboloids, 1033e
 from integrals, 438e
 paraboloid, 1052
 from piecewise function, 438e
 of revolution, 428, 435, 516e, 864e
 infinite intervals and, 585–586
 integration by parts and, 528–529
 volume of, 480e, 524e, 555, 1008–1010, 1044e, 1085e
 Source free, 1127
 fields, 1131t
 Space
 curves in, 869–872, 912
 properties of, 918e
 lines in, 844–852, 866e
 motion in, 883–892
 planes in, 844–852
 Special series, 701t, 702
 Speed, 179, 883
 angular, 1145e
 on circles, 902e
 defining, 180, 884
 of Earth, 901e
 on ellipses, 894e
 errors in, 299e
 estimating, 299e
 of Jupiter, 901e
 Spheres, 1160e
 balls and, 820, 865e
 concentric, 1161e
 curves on, 875e, 886–887
 descriptions of, 1155t
 distance to, 1003e
 equations of, 819–820, 826e
 flux across, 1180e
 hexagonal packing of, 836e
 midpoints and, 824e
 motion on, 883e
 parameterized surfaces of, 1148
 points on, 1003e
 radial field and, 1161e
 radius of, 820
 regions between, 1005e
 in spherical coordinates, 1055t
 surface area of, 1151–1152
 temperatures on, 1153
 trajectories on, 887, 893e
 volume of, 443–444, 449e, 1035e
 cylinders in, 1003e
 Spherical cap, 450e, 950e, 1047e
 volume of, 206e

- Spherical coordinates, GP78, 1083e
 - cones in, 1055*t*
 - cylinders in, 1055*t*
 - horizontal plane in, 1055*t*
 - integrals in, 1087e
 - integration in, 1056–1058
 - rectangular coordinates and, 1054
 - sets in, 1060e
 - spheres in, 1055*t*
 - in triple integrals, 1053–1058
 - vertical half-plane in, 1055*t*
 - volumes in, 1060e, 1087e
 - Spherical tanks, 475e
 - Spherical zone, 464e
 - surface area of, 460–461
 - Spiral
 - through domains, 960e
 - Euler, 914e
 - graphs of, 870
 - Spiral arc length, 787e
 - Spirals, 778e
 - regions bounded by, 788e
 - Splitting, vector fields, 1146e
 - Springs
 - compressing, 467, 474e
 - nonlinear, 476e
 - stretching, 474e
 - vertical, 476e
 - work and, 474e, 481e
 - Squared integers, 54e
 - Square region, 1026e
 - Square root
 - finding, 649e
 - repeated, 661e
 - Squares
 - completing, 523, 543, 544e
 - difference of, 538
 - expanding, 231e
 - odd, 682e
 - series of, 688e
 - Squeeze Theorem, 77–78, 127e
 - sequences and, 654–655
 - Stable equilibrium, 608
 - Standard parabolas, equations of, 790
 - Standard position, 40
 - Steady states, 101e
 - infinite series and, 705e
 - sequences and, 705e
 - solutions, 618
 - Steiner's problem, 995e
 - Step function, 70e
 - Stirling's formula, GP51, 700
 - Stirred tank reactions, 634e, 637e
 - differential equations and, 629–631
 - solutions to, 625e
 - Stokes, George Gabriel, 1172
 - Stokes' Theorem, 1162–1169, 1170e, 1179*t*
 - for line integrals, 1164–1165
 - proof of, 1168
 - for surface integral evaluation, 1165–1166
 - verification of, 1163–1164
 - Straight-line motion
 - circular motion and, 886–887
 - uniform, 886
 - Stream functions, 1130, 1146e
 - Streamlines, 1095
 - Subintervals, integrals over, 360
 - Substitution, 524e
 - integrand and, 557–558
 - for integration, 521, 557
 - secant, 540, 543, 545e
 - sine, 539–540
 - subtle, 522
 - tangent, 540–542
 - trigonometric, 532, 541
 - useful, 730e
 - Substitution Rule
 - for definite integrals, 393–395
 - geometry of, 395
 - for indefinite integrals, 388–391
 - perfect, 389–390
 - procedure, 389
 - variations on, 390–391, 397e
 - Subtraction, of vectors, 806–807
 - Summand, 344
 - Summation notation, 344–345
 - Sum Rule, 878
 - antiderivatives and, 324
 - defining, 155
 - generalized, 155
 - proof of, 883e
 - Sums. *See also* Riemann sums
 - alternating, errors in, 706e
 - geometric, 663–665, 668e
 - harmonic, 492
 - of integers, 54e
 - integrals of, 359
 - limits of, GP18, 72, 357–358
 - partial, 705e
 - errors in, 695e
 - sequence of, 644–647
 - of powers of integers, 345
 - Riemann, Midpoint Rule and, 569
 - sequence of, 682e
 - Superexponential functions, 312e, 337e
 - Supply and demand, GP5
 - Surface
 - compound, 1170e
 - constraint
 - absolute maxima on, 1000
 - absolute minima on, 1000
 - curves on, 875e
 - distance to, 1003e
 - equipotential, 1095
 - graphs of, 856
 - identifying, 867e
 - matching, 867e, 928e
 - orientable, 1149
 - parameterized
 - of cones, 1148
 - of cylinders, 1147
 - of spheres, 1148
 - parametric, 1149
 - quadric, 855, 857–862
 - identifying, 863
 - intercepts of, 858
 - traces of, 858
 - regions bounded by, 1029
 - of revolution, 766e, 801e, 1161e
 - two-sided, 1155
 - volume between, 1033e, 1053
 - walking on, 959e
 - Surface area, 525e, 1159e
 - of catenoid, 516e
 - computation of, 463e
 - of cones, 982e
 - formula for, 458
 - of cubes, 285e
 - of cylinders, 1007e, 1151–1152
 - of ellipsoid, 464
 - formula, 459–462
 - of frustum, 458, 464e
 - of funnels, 461–462
 - minimum, 249e
 - of partial cylinder, 1152–1153
 - preliminary calculations, 457–459
 - of revolution, 460
 - scaling, 464e
 - of spheres, 1151–1152
 - of spherical zone, 460–461
 - of torus, 464, 981e, 1161e
 - volume and, 481e
 - Surface-area-to-volume ratio (SAV), 464e
 - Surface integrals, 1146
 - on explicitly defined surfaces, 1153–1154
 - of scalar-valued functions, 1149–1155
 - Stokes' Theorem for evaluation of, 1165–1166
 - of vector fields, 1155–1159, 1160e
 - Suspended load, 815e
 - Symbolic methods of integration, 562, 564–565
 - Symmetric curves, 457e
 - Symmetric functions, integrating, 382–383
 - Symmetric intervals, 119–120, 125e
 - Symmetry, 12e, 53e, 272, 274
 - of composite functions, 387e
 - in functions
 - defining, 8–9
 - identifying, 9
 - in graphs, 271
 - defining, 8
 - identifying, 772
 - in integrals, GP20, 385e
 - in polar equations, 772
 - in polar graphs, 772–773
 - of powers, 387e
 - properties, 399e
 - x-axis, 8, 772
 - y-axis, 8, 772
 - Symmetry about, origin, 772
- T**
- Tables, 18
 - for derivatives, 197e
 - derivatives from, 169e
 - finding limits from, 64
 - of integrals, 562–563
 - Riemann sums from, 344

- Tabular integration, 531e
- Tail, of infinite series, 666
- Tangent function, 46
 - hyperbolic, 564
 - defining, 503
 - inverse, derivatives of, 219–222
 - powers of, 535
 - products of, 535–536
- Tangential components, 908, 1102
- Tangential form, 1124
- Tangent identities, inverse, 255e
- Tangent line, 132, 161e, 170e, 199e, 216e, 721
 - concavity and, 270e
 - defining, 59
 - derivatives and, 136
 - for ellipses, 207e, 798e
 - equation of, 134, 138e, 168e
 - horizontal, 177e, 213, 780
 - for hyperbolas, 798e
 - with implicit functions, 203–204
 - instantaneous velocity and, 60
 - locations of, 177e
 - means and, GP21
 - multiple, 206e
 - at origin, 786e
 - parametric curve and, 801e
 - rate of change and, 133–134
 - slope of, 59–60, 62e, 76–77, 82e, 133, 157–158, 760, 765
 - implicit differentiation and, 203–204
 - polar coordinates and, 779–781
 - vertical, 152e, 207e, 780
 - visualizing, 207e
- Tangent planes, 972e, 980e
 - equations of, 975–976
 - horizontal, 982e
 - linear approximation and, 973–980
- Tangents
 - horizontal, 786
 - normals and, 788e
 - vertical, 786
- Tangent substitution, 540–542
- Tangent vector, 1092–1093
 - defining, 877
 - unit, 877–878, 881e
 - curvature and, 903
 - of vector-valued functions, 875–879
- Tanks
 - conical, 475e
 - force on, 476e
 - spherical, 475e
- Tautochrone property, GP54
- Taylor, Brook, 710
- Taylor coefficients, 720e
- Taylor polynomials, 710
 - approximations with, 712–713
 - coefficients in, 711
 - graphing, 719e
 - n th-order, 711
 - real numbers and, 713–714
 - remainders and, 713
- Taylor series, GP49, GP50, GP52, 731–740
 - convergence of, 737–740, 740e
 - for functions, 731–735
 - l'Hôpital's Rule by, 750e
 - limits by, 742–743, 749e
- Taylor's Theorem, 714
 - proof of, 720e
- Technology, 15
 - analytical methods and, 279e
 - direction fields with, 612e
 - graphs and, 278e
 - surface area and, 463e
 - for volume, 1026e
- Telescoping series, 665–667, 669e
- Temperature
 - on circle, 1100
 - data, 579e
 - on spheres, 1153
- Terminal velocity, GP38, 516e
- Terms
 - defining, 639
 - leading, 666
 - positive, series with, 667
 - rational, 702–703
- Tesla, 842
- Test values, inequalities and, 259
- Tetrahedrons, 1047e
 - flux on, 1160e
 - limits of integration, 1086e
 - volume of, 1021
- Thermal constants, 624
- Three-dimensional flow, circulation of, 1107
- Three-dimensional motion, 894e
- Three-dimensional objects, 1067–1068
- Three-sigma quality control, GP52
- Tilted circles, 901e
- Tilted ellipse, 896e
- Time, estimating, 299e
- Time steps
 - errors and, 610
 - in Euler's method, 609
- TNB frame, 910
- Topographic map, 922
- Toroidal magnetic field, 902e
- Torque, 837
 - arm, 843e
 - cross products and, 840–841
- Torricelli, Evangelista, 603
- Torricelli's law, 620e
- Torricelli's trumpet, 585
- Torsion, 909–912
 - computing, 918e
 - defining, 910
 - formulas for, 912, 915e
 - of helix, 911
 - of plane curves, 918e
- Torus
 - surface area of, 464, 981e, 1161e
 - volume of, 206e, 438e, 959e
- Total moment, 1065
- Tower functions, derivatives of, 215e, 490
- Traces
 - cylinders and, 856–857
 - defining, 856
 - of quadric surfaces, 858
- Trajectories, GP61
 - on circles, 893e
 - circular, 896e
 - comparing, 885–886
 - on ellipses, 887
 - helical, 896e
 - parabolic, 895e, 915e
 - on spheres, 887, 893e
 - velocity and, 917e
- Transcendental functions, 14, 101e
 - continuity of, 109–110, 113e
 - end behavior of, 99–100, 102e
 - limits involving, 110
- Transformations, 25e
 - of functions, 20–22
 - of graphs, 20–22, 43–44
 - image of, 1073–1074
 - linear, 1083e
 - one-to-one, 1074
 - of parabolas, 21
 - in plane, 1073–1078
 - scaling, 22
 - shear, 1083e
 - shifting, 22
 - of three variables, 1079
 - of two variables, 1075
- Trapezoid Rule, 580e
 - approximation, 571
 - concavity and, 581e
 - defining, 571
 - derivation of, 570
 - errors in, 571, 576–577
 - exact, 581e
 - in numerical integration, 570–574
 - shortcut for, 581e
- Trapezoids, area of, 358, 369
- Traveling wave, GP66
- Triangles
 - angles of, 833e
 - area of, 839–840, 843e, 844e, 995e
 - circles in, 291e
 - disappearing, 235e
 - inequality, 122, 816e, 836e
 - median of, 826e, 1072e
 - in rectangles, 290e
 - Sierpinski, 707e
 - Triangle Rule, 806, 820
- Triangular plates, 476e
- Trigonometric functions, 39–40, 109
 - defining, 14, 39
 - derivatives for, 171–175
 - evaluating, 41, 49e
 - graphing, 42–43
 - hyperbolic functions and, 502
 - indefinite integrals of, 324–326
 - inverse, 14, 44–48, 49e
 - defining, 47
 - derivatives of, 218–224
 - working with, 48
 - limits of, 88
 - period of, 42
 - solving, 49e

Trigonometric identities, C-4, 41–42
 deriving, 200e
 Trigonometric inequalities, 82e
 Trigonometric integrals, 532–536
 Trigonometric limits, 78–79, 176e
 Trigonometric substitutions, 532, 541
 Triple integrals, 1036–1043, 1088e
 change of variables for, 1050, 1078–1080
 in cylindrical coordinates, 1048–1055
 in rectangular coordinates, 1036–1041
 in spherical coordinates, 1053–1058
 Tripling time, 500e
 Two-dimensional curl, 1125
 Two-dimensional divergence, 1127
 Two-dimensional flows, flux of, 1109–1110
 Two-dimensional motion, 894e
 in gravitational field, 887–892
 Two-dimensional objects, 1066–1067
 Two-Path Test, 935
 Two-sided limits, one-sided limits and, 66
 Two-sided surfaces, 1155
 Tyrolean traverse, 512

U

UFO interception, GP62
 Unbounded integrands, 586–590
 improper integrals with, 587
 Unbounded intervals, B-1
 Unbounded regions, 992–993
 Undetermined coefficients, 547e, 548
 Uniform straight-line motion, 886
 Unit area cubic, 402e
 Unit binormal vector
 defining, 910
 formulas for, 912
 Unit cost, 183
 Unit tangent vector, 877–878, 881e
 curvature and, 903
 Unit vectors, 811
 coordinate, 809, 842
 cross product of, 838–839
 magnitude and, 810, 822–823, 825e
 in polar coordinates, 1098e
 Unknown functions, 401e
 Upper bounds, 362
 of series, 681e, 706e
 Utility functions, 959e
uv-plane, 1080

V

Vallée Poussin, Charles de la, 307
 Variable cost, 183
 Variable density, 960e, 1067
 center of mass with, 1069
 mass from, 465
 Variable flow rate, 478e
 Variable forces, 466
 Variable gravity, 415e
 Variable of integration, 356
 Variables. *See also* Independent variables
 in Bernoulli equations, 626e
 Chain Rule with one, 958e
 change of, 450e, 615, 902e
 double integral with, 1076
 integrand and, 1077–1078
 in multiple integrals, 1072–1081
 region and, 1078–1079
 for triple integrals, 1050, 1057
 with triple integrals, 1078–1080
 defining, 1
 dependent, 2, 647t
 derivatives with two, 940–943
 domain of two, 920
 dummy, 344
 functions of three, 937, 945–946
 average value of, 1043
 functions of two, 920
 applications of, 924–925
 continuity of, 935–937
 graphs of, 921–924
 limit laws for, 932
 range of, 920
 of integration, 323
 intermediate, 953
 strategies for selection of, 1080–1081
 transformations
 of three, 1079
 of two, 1075
 Vector calculus, GP78
 Vector fields, GP75, GP76, GP77, GP78
 circulation of, 1106–1110
 conservative, 1114–1121, 1170e, 1183e
 curl of, 1141
 properties of, 1143
 curl of, 1140
 defining, 1089, 1093
 designing, 1096e
 divergence of, 1137–1138, 1143e
 drawing, 1091
 flux of, 1107–1108
 general rotation, 1140
 linear, 1123e
 line integrals of, 1102–1106
 matching, 1096e
 in polar coordinates, 1098e
 quadratic, 1123e
 radial
 defining, 1090–1091
 divergence of, 1138–1139, 1144e
 in three dimensions, 1093
 sketching, 1096e
 splitting, 1146e
 surface integrals of, 1155–1159, 1160e
 in three dimensions, 1093–1095
 in two dimensions, 1090–1092
 Vector functions, arc length for, 897
 Vectors, GP62, GP63, GP65. *See also*
 Unit vectors
 addition of, 806–807
 applications of, 811–813

binormal, 909–912
 computing, 913e, 918e
 unit, 910, 912
 clock, 816e
 components of, 807–809
 decomposing, 835e
 equal, 805
 with equal projections, 835e
 equations, 815e
 of lines, 845
 force, 812–813
 gradient, 964–965
 magnitude of, 808, 822–823
 normal, 849, 1092–1093
 curvature and, 902–912
 principal unit, 906–907
 operations, 804–805, 809, 814e, 821
 properties of, 811
 orthogonal, 840
 defining, 828
 unit, 835e
 parallel, 805, 816e, 825e
 in planes, 804–813
 from polar coordinates, 815e
 position, 807
 subtraction of, 806–807
 tangent, 1092–1093
 defining, 877
 unit, 877–878, 881e, 903, 912
 of vector-valued functions, 875–879
 in three dimensions, 817–823
 velocity, 805, 811–812, 865e
 zero, 805
 multiplication by, 811
 Vector-valued functions, 916e
 antiderivatives of, 880
 calculus of, 875–881
 continuity for, 872–873
 defining, 868
 derivatives of, 875, 881e
 integrals of, 880–881
 limits for, 872–873, 875e
 lines as, 869
 tangent vector of, 875–879
 Velocity, 178, 501e, 883
 acceleration and, 180, 408, 412e
 antiderivatives of, 329
 average, 56–59, 61e, 386e, 396e
 defining, 179
 instantaneous velocity and, 60
 secant lines and, 58, 60
 for circular motion, 884–885
 comparing, 188e
 constant, 415e
 decreasing, 478e
 defining, 180, 404, 884
 displacement from, 350e, 401, 403–405, 410e, 411e, 478e
 graphs, 411e
 initial, 188e, 895e
 initial value problems for, 330
 instantaneous, 58–59, 61e, 132–133
 defining, 179

Velocity (*continued*)

net change and, 402–410
 piecewise, 412e
 to position, 333e
 position and, 403–405, 411e
 terminal, GP38, 516e
 trajectories and, 917e
 vectors, 805, 811–812, 865e
 of waves, 512–513, 515e
 zero, 62e

Velocity curve
 area under, 338–340
 displacement and, 339–340

Velocity functions, 138e, 149e

Verhulst, Pierre François, 631

Vertex
 of ellipses, 791
 of hyperbolas, 792
 major-axis, 792
 minor-axis, 792
 of parabolas, 790

Vertical asymptotes, 18, 98, 101e
 defining, 85
 functions with, 91e
 location of, 87, 90e

Vertical coordinates, 779

Vertical half-plane, 1048
 in spherical coordinates, 1055

Vertical lines, B-6

Vertical motion, 337e

Vertical shift, 43

Vertical springs, 476e

Vertical tangent lines, 152e, 207e

Visual approximation, 514e

Volterra, Vito, 631

Volume, 537e, 591
 approximation of, 1017e
 area and, 479e, 545e
 of boxes, 950e
 without calculus, 450e
 comparing, 437, 480e, 530e, 595e
 of cones, 286e, 438e, 463e
 changes in, 981e
 of cubes, 285e
 of cylinders, 287e, 438e, 463e, 1007e
 in spheres, 1003e
 in cylindrical coordinates, 1060e, 1086e

as definite integral, 440
 of ellipsoid, 799e, 1007e
 equal, 450e, 1035e
 estimating, 438e
 Fermat's calculation of, 437
 formulas, 1047e, 1063e
 height and, 279e
 of hemispheres, 233e, 438e
 of hyperbolic cap, 799e
 integrals, 1040
 miscellaneous, 1062e
 of parabolic cube, 426
 of paraboloids, 799e, 981e, 1029
 of prisms, 1039
 of pyramids, 958e
 by shells, 439–447
 by slicing, 425–434
 of solids, 480e, 524e, 555, 1008–1010, 1044e, 1085e
 of spheres, 443–444, 449e, 1035e
 in spherical coordinates, 1060e, 1087e
 surface area and, 481e
 between surfaces, 1033e, 1053
 technology for, 1026e
 of tetrahedrons, 1021
 of torus, 206e, 438e, 959e
 by washer method, 430

Vorticity, 1146e

W

Wallis products, 596e

Washer method, 443
 defining, 429
 shell method and, 449e
 volume by, 430
 about x -axis, 429, 446

Water, depth, 513

Water clock, designing, GP25

Watts, 414e

Wave equation, GP66, 951e

Waves, GP6
 height of, 386e
 traveling, GP66, 972e, 973e
 velocity of, 512–513, 515e

Weierstrass, Karl, 116

Work
 calculating, 832
 defining, 466, 832
 dot products and, 832
 from force, 474e

in force field, 1105, 1122e
 function, 474e
 in gravitational fields, 477e
 integrals, 1104–1105, 1112e
 physical applications, 466–467
 springs and, 474e, 481e
 sum of, 466

World population, 494–495

X

x -axis
 disk method about, 428, 446
 shell method about, 446
 symmetry, 8, 772
 washer method about, 429, 446

xyz -coordinate system, 817
 distances in, 819
 origin in, 818
 points in, 818

xz -plane, 817

Y

y -axis
 disk method about, 446
 revolution about, 430–431
 shell method about, 446
 symmetry, 8, 772
 washer method about, 446

y -coordinate, average, 1032

yz -plane, 817

Z

Zeno's paradox, 670e

Zero average value, 1016e

Zero curl, 1171e

Zero curvature, 915e

Zero derivatives, constant functions and, 253

Zero log integral, 595e

Zero net area, 367e, 381e

Zeros, 13

Zero scalar, multiplication by, 811

Zero vector, 805
 multiplication by, 811

Zero velocity, 62e

Zeta function, 681e

Index of Applications

Note:

- Italics indicate figures or margin notes.
- “t” indicates a table.
- “e” indicates an exercise.
- GP indicates a Guided Project (located online in MyLab Math).
- AP-, B-, and C- indicate Appendices A, B, and C respectively (Appendices B and C are online at bit.ly/2y3Nck3 and bit.ly/2Rvf08V).

Economics and Business

Average and marginal cost, 183–185, 188, 240
Average and marginal profit, 189
Average marginal production, 189
Bad loans, 626
Bank accounts, 198
Car loans, 670
Compounding, 495, 501
Consumer Price Index, 649
Demand functions, 270
Depreciation of equipment, 500
Economic models, 1001–1002
Economics, GP5, GP13, GP19, GP44, GP68
Elasticity, 185–186, 188, 270
Endowment models, 626
Finance, GP42, GP44, GP45
Financial models, 495
Fuel economy, 190
Gini index, GP19
House loans, 670
Inflation, 501
Interest rates, 114
Investment problems, 114
Loan payments, 623, 625, 670
Mortgage payments, 114
Oil production, 413, 572, 580
Periodic savings, 668
Perpetual annuities, 591
Production costs, 410
Profit maximization, 249
Retirement accounts, 239, 320
Rising costs, 499
Savings plans, 217, 499, 660, 705, 929
Shipping regulations, 993, 1003
Supply and demand, GP5
Utility functions, 959, 1001–1002
Wealth distribution, 707

Engineering and Physical Sciences

Acceleration, 186, 408, 501, 516
Air drops, 764, 765
Airline travel, 187
Airplane pressure, 1016
Air resistance, 501
Arm torque, 843

Atmospheric CO₂, GP7
Atmospheric pressure, 500
Avalanches, 256
Bike race, 415
Bike rides, 478
Black holes, 593
Blocks on springs, 51
Boat rates, 240
Boats in currents, 815
Boats in wind, 815
Boiling points, 52
Bouncing balls, 643, 649, 707
Box design, 286, 335
Box volume, 27
Bubbles, 671
Building tunnels, 706
Buoyancy, 477, GP26
Canoes in currents, 866
Carbon dating, 500
Carbon uptake, 414
Cardboard boxes, 993
Cars, 914
Catenary, 515
Center of mass, GP72
Change in energy, 336
Channel flow, 778
Chaos, GP41
Charging capacitors, 38
Chemical rate equations, 605, 613, 620
Coiling ropes, 474
Comparing vehicles, 188
Comparing volumes, 437
Compressing a spring, 467, 474
Computing torque, 843
Conservation of energy, 959, 1123
Converging airplanes, 228
Cooling a bowl, 624
Cooling times, 626
Crankshafts, 290
Crosswinds, 815, 823, 825
Crystal lattice, GP48
Cycling distances, 412
Damped oscillators, 199, 320
Deceleration, 412, 478
Deep-water velocity equation, 513
Depletion of natural resources, 413
Descent times, 546
Diagnostic scanning, 216
Displacement, 348, 404–405, 410, 478
Distance traveled, 411
Dogs running, 287
Double glass windows, 670
Drag racer acceleration, 255
Draining pools, 591
Draining tanks, 11, 474, 475, 605
Draining troughs, 235, 475
Draining water heaters, 232
Drinking juice, 476

Driving speeds, 299
Earth-Mars system, 778
Earthquake magnitudes, 216
Eiffel Tower, 592, GP72
Electric fields, 81, 519, 1097, 1180
Electric field integrals, GP70
Electric potential, 973, 1006
Electronic chips, 591
Electron speed, 844
Electrostatic force, 169
Elevation, 927
Emptying pools, 474
Energy consumption, 499
Engines, 813
Enzyme kinetics, GP12
Equilibrium, 825
Escape velocity, 593
Falling bodies, 516
Feather dropped on moon, 187
Ferris wheels, 235, 290
Filling bowls, 1035
Filling reservoirs, 413
Filling swimming pools, 187, 232
Filling tanks, 413, 475
Firing angles, 917
Flow from tanks, 191, 603
Flow rate, 333, 351, 413, 478, 578
Fluid forces, 481
Flying into headwind, 412, 1112
Flying kites, 231
Folding boxes, 286, 290
Force and pressure, 471–472
Force on buildings, 475
Force-on-dam problems, 472, 475, 482
Force on inclined plane, 834
Force on protons, 841
Force on tanks, 476
Force on windows, 476
Four-cable loads, 826
Free falls, 349, 605, 613, 619, 620
Fuel consumption, 478
Gas law calculations, 949
Gas tanks, 1063
Gateway Arch, 386, 456
Glass design, 865
Golden Gate cables, 456, 798
Gravitational fields, GP74
Gravitational force, 169, 337, 408, 833, 1063, 1123
Gravitational potential, 1145
Hand tracking, 864
Harvesting model, 602
Head start problem, 333
Heat equation, 950
Heat flux, 1145, 1161
Heat transfer, 1181
Hot air balloons, 230, 232, 240
Hydrostatic pressure, 472

Ideal fluid flow, GP75
 Ideal Gas Law, 299, 930, 960
 Jet altitude, 234
 Kiln design, 516
 Knee torque, 866
 Ladders, 51, 233, 283, 286
 Lapse rate, 198, 252, 255
 Leaky buckets, 474, 476
 LED lighting, 500
 Lifting a chain and bucket, 468
 Lifting problems, 481
 Light cones, 864
 Lighthouse light beams, 234
 Light transmission, 287
 Magnetic fields, 519
 Magnetic force, 841
 Mass from density, 352
 Mass of one-dimensional objects, 472
 Mass on springs, 333
 Maximum height, 889
 Minimizing sound intensity, 335
 Mixing tanks, 200
 Moments of inertia, GP73
 Motion with gravity, 330, 333, 408, 601, 605
 Motorboats, 811
 Motorcycles, 232
 Mountains, 930
 Newton's Law of Cooling, 623–625, 637
 Nonlinear springs, 476
 Oil consumption, 499
 Opening laptops, 843
 Optimal boxes, 994
 Orientation and force, 477
 Oscillations, 279
 Oscillators, GP15
 Paddle wheel, 1145
 Painting funnels, 461
 Painting surfaces, 336, 463
 Parabolic dam, 475
 Parachutes, 815
 Paths of moons, 766, 767
 Patrol car velocity, 179
 Peaks and valleys, 930
 Pendulums, 477, 567, 580
 Period of pendulum, GP37
 Periodic motion, 396
 Piston compression, 232
 Piston position, 240
 Planetary orbits, 386, 897–898
 Pole in corner, 51
 Power and energy, 150, 200, 414
 Power lines, 515
 Pressure and altitude, 198
 Probe speed, 412
 Projectiles, 48, 160, 567, 765, 891, 902, 917
 Projectiles on Mars, 238
 Protons, 593
 Pulling wagons, 813
 Pumping gasoline, 471
 Pumping problems, 469
 Pumping water, 470, 481
 Radar, 776
 Radioactive decay, 518, 649

Radiometric dating, 497
 Rain gutters, 289
 Rain on roofs, 1158, 1161
 RC circuit equations, 635
 Relativity theory, 81
 Resistors in parallel, 929, 950
 Resource consumption, 496
 Rolling wheels, 759
 Rope, 232
 Running models, 501
 Sag angle, 515
 Sand piles, 232
 Searchlights, 236
 Shadow length, 240
 Shallow-water velocity equation, 515
 Shipping crates, 285
 Shock absorption, 474
 Silos, 289
 Snowplow problems, 415
 Soda can design, 286, 1072
 Speed of Earth, 901
 Speed of Jupiter, 901
 Speed of sound, 186
 Spreading oil, 227
 Spring oscillations, 190
 Spring vibrations, 199
 Stacking dominoes, 683
 Stereographic projections, 54
 Stirred tanks, 629–631, 634, 637
 Stone dropped on Earth, 187
 Stone dropped on Mars, 187
 Stone trajectory, 10, 187
 Stretching a spring, 474
 Submarines, 825
 Suspension systems, 288
 Temperature, 26, 191, 198, 579, 1153
 Terminal velocity, 407, 516
 Three-cable loads, 826
 Three-way tug-of-war, 816
 Tightening bolts, 843
 Time-lagged flights, 231
 Towing boats, 226, 815
 Tracking dives, 226
 Tsunamis, 515
 Uranium dating, 500
 Velocity, 58–59, 61–62, 132–133, 180, 189,
 347, 396, 404–405, 408, 410–411,
 501, 512–513
 Vertical motion, 337
 Vertical springs, 476
 Viewing angles, 288
 Voltage on capacitors, 151
 Volume of wooden object, 437
 Walking and rowing, 285
 Walking and swimming, 285
 Water bottle design, 867
 Water-level changes, 982, 1007
 Water towers, 10, 284
 Water volume, 1016
 Water waves, 930
 Wave velocity, 515
 Wind energy, 234, 248, 249
 Winding chains, 474

Windows, 287
 Wine barrels, 287
 Work done by springs, 474, 481

General

Anchored sailboats, 50
 Ballparks, 222
 Baseball, 38, 128, 232
 Baseball pitches, 895
 Basketball, 288
 Batting averages, 982
 Billiards shot, 847, 853
 Boats, 48
 Bungee jumpers, 191
 Caffeine levels, 139, 500, 518
 Carnival rides, 893
 Cheese, 1047
 Coffee, 229
 Dancing, 249
 Day hike, 412
 Daylight, 199
 Drinking soda, 232
 DVD grooves, 918
 Earned run averages, 929
 Field goal attempts, 50
 Footballs, 449, 864
 Fundraisers, 23
 Gardening, 286
 Golf ball flight, 890–891
 Golf slices, 875, 895
 GPS data, 7, 12, 194
 Hockey, 335
 Ice cream cones, 1058–1059
 Mercator map projection,
 596, GP35
 Monk and the mountain, 115
 Parking fees, 24
 Pens, 285, 291
 Postage rates, 69
 Problems of antiquity, 386
 Publishing costs, 52
 Pursuit problem, GP40
 Quarterback ratings, 929
 Races, 190, 234, 333, 501
 Random events, 1047
 Rugby, 449
 Scuba diving, 398
 Sierpinski triangle, 707
 Ski jumps, 895
 Slicing a cake, 1035
 Slinky curves, 871
 Smartphones, 198
 Snowboard rental, 129
 Snowflake island fractal, 670
 Taxicab fees, 23
 Tennis probabilities, 26
 Traffic flow, 596
 Velocity of skydivers, 38,
 239, 407
 Video game calculations, 847
 Walking and rowing, 26
 Walk in the park, 115

Witch of Agnesi, 206
 World record free fall, 10
 Zeno's paradox, 670

Life Sciences

Air flow in lungs, 414
 Antibiotic decay, 239
 Antibiotic dosing, 130
 Ants, 816
 Aspirin, 705
 Atlantic salmon, 148, 160
 Average lifetime, GP34
 Bacteria growth rate, 239
 Bald eagle population, 23
 Bamboo growth, 336
 Beak length, 238
 Bioavailability, 588, 591
 Blood flow, 289, 413
 Blood testing, 287
 Body mass index, 949, 979
 Body surface area, 960
 Cancer treatment, 413
 Cell growth, 160, 409
 Cell populations, 198

Chemotherapy, 501
 Desert tortoises, 37, 198
 Drug dosing, 622, 625, 660
 Drug elimination, 649
 Drug infusion, 605, 613
 Drug sequencing, 655
 Ecological diversity, GP67
 Epidemics, 619
 Fish harvesting, 626, 660
 Fishing, 217, 232, 234
 Fish length, 189
 Gliding mammals, 289
 Grazing goats, 788, GP57
 Indian spotted owlets, 137, 169, 270
 Owlet talons, 139
 Pectin extraction, 994
 Periodic dosing, 660, 668
 Pharmacokinetics, 498, GP14, GP43
 Physiological models, 635
 Population models, 38
 Predator-prey models, 631–633,
 635–637, GP36
 Radioiodine treatment, 500
 Sleep models, 660
 Spring runoff, 191

Tortoise growth, 499
 Tree growth, 189
 Tree notches, 291, 451
 Tumor size, 319, 500, 605, 620
 Water-level changes, 982
 Yeast growth, 23

Social and Behavioral Sciences

Carrying capacity, 627
 City urbanization, 160
 Crime rates, 500
 Geographic vs. population centers,
 1071
 Police, 256, 404
 Population growth, 169, 188, 270, 412, 499,
 518, 605
 Population models, 38, 396, 627,
 628, 1016
 Population of China, 500
 Population of Las Vegas, 138
 Population of Texas, 499
 Population of West Virginia, 500
 U.S. Population growth, 189, 239, 635
 World population, 217, 495

TABLE OF INTEGRALS

Substitution Rule	Integration by Parts
$\int f(g(x))g'(x) dx = \int f(u) du \quad (u = g(x))$	$\int u dv = uv - \int v du$
$\int_a^b f(g(x))g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$	$\int_a^b uv' dx = uv \Big _a^b - \int_a^b vu' dx$

Basic Integrals

- $\int x^n dx = \frac{1}{n+1} x^{n+1} + C; n \neq -1$
- $\int \frac{dx}{x} = \ln |x| + C$
- $\int \cos ax dx = \frac{1}{a} \sin ax + C$
- $\int \sin ax dx = -\frac{1}{a} \cos ax + C$
- $\int \tan x dx = \ln |\sec x| + C$
- $\int \cot x dx = \ln |\sin x| + C$
- $\int \sec x dx = \ln |\sec x + \tan x| + C$
- $\int \csc x dx = -\ln |\csc x + \cot x| + C$
- $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$
- $\int b^{ax} dx = \frac{1}{a \ln b} b^{ax} + C; b > 0, b \neq 1$
- $\int \ln x dx = x \ln x - x + C$
- $\int \log_b x dx = \frac{1}{\ln b} (x \ln x - x) + C$
- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C, a > 0$
- $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C, a > 0$
- $\int \sin^{-1} x dx = x \sin^{-1} x + \sqrt{1 - x^2} + C$
- $\int \cos^{-1} x dx = x \cos^{-1} x - \sqrt{1 - x^2} + C$
- $\int \tan^{-1} x dx = x \tan^{-1} x - \frac{1}{2} \ln (1 + x^2) + C$
- $\int \sec^{-1} x dx = x \sec^{-1} x - \ln (x + \sqrt{x^2 - 1}) + C$
- $\int \sinh x dx = \cosh x + C$
- $\int \cosh x dx = \sinh x + C$
- $\int \operatorname{sech}^2 x dx = \tanh x + C$
- $\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$
- $\int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + C$
- $\int \tanh x dx = \ln \cosh x + C$
- $\int \coth x dx = \ln |\sinh x| + C$
- $\int \operatorname{sech} x dx = \tan^{-1} \sinh x + C = \sin^{-1} \tanh x + C$
- $\int \operatorname{csch} x dx = \ln |\tanh (x/2)| + C$

Trigonometric Integrals

- $\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$
- $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$
- $\int \sec^2 ax dx = \frac{1}{a} \tan ax + C$
- $\int \csc^2 ax dx = -\frac{1}{a} \cot ax + C$
- $\int \tan^2 x dx = \tan x - x + C$
- $\int \cot^2 x dx = -\cot x - x + C$
- $\int \cos^3 x dx = -\frac{1}{3} \sin^3 x + \sin x + C$
- $\int \sin^3 x dx = \frac{1}{3} \cos^3 x - \cos x + C$

38. $\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C$
39. $\int \csc^3 x \, dx = -\frac{1}{2} \csc x \cot x - \frac{1}{2} \ln |\csc x + \cot x| + C$
40. $\int \tan^3 x \, dx = \frac{1}{2} \tan^2 x - \ln |\sec x| + C$
41. $\int \cot^3 x \, dx = -\frac{1}{2} \cot^2 x - \ln |\sin x| + C$
42. $\int \sec^n ax \tan ax \, dx = \frac{1}{na} \sec^n ax + C; n \neq 0$
43. $\int \csc^n ax \cot ax \, dx = -\frac{1}{na} \csc^n ax + C; n \neq 0$
44. $\int \frac{dx}{1 + \sin ax} = -\frac{1}{a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) + C$
45. $\int \frac{dx}{1 - \sin ax} = \frac{1}{a} \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right) + C$
46. $\int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan \frac{ax}{2} + C$
47. $\int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \cot \frac{ax}{2} + C$
48. $\int \sin mx \cos nx \, dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C; m^2 \neq n^2$
49. $\int \sin mx \sin nx \, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C; m^2 \neq n^2$
50. $\int \cos mx \cos nx \, dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C; m^2 \neq n^2$

Reduction Formulas for Trigonometric Functions

51. $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$
52. $\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$
53. $\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx; n \neq 1$
54. $\int \cot^n x \, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx; n \neq 1$
55. $\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx; n \neq 1$
56. $\int \csc^n x \, dx = -\frac{\csc^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} x \, dx; n \neq 1$
57. $\int \sin^m x \cos^n x \, dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x \, dx; m \neq -n$
58. $\int \sin^m x \cos^n x \, dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x \, dx; m \neq -n$
59. $\int x^n \sin ax \, dx = -\frac{x^n \cos ax}{a} + \frac{n}{a} \int x^{n-1} \cos ax \, dx; a \neq 0$
60. $\int x^n \cos ax \, dx = \frac{x^n \sin ax}{a} - \frac{n}{a} \int x^{n-1} \sin ax \, dx; a \neq 0$

Integrals Involving $a^2 - x^2$; $a > 0$

61. $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$
62. $\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$
63. $\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$
64. $\int x^2 \sqrt{a^2 - x^2} \, dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \frac{x}{a} + C$
65. $\int \frac{\sqrt{a^2 - x^2}}{x^2} \, dx = -\frac{1}{x} \sqrt{a^2 - x^2} - \sin^{-1} \frac{x}{a} + C$
66. $\int \frac{x^2}{\sqrt{a^2 - x^2}} \, dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$
67. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C$

Integrals Involving $x^2 - a^2$; $a > 0$

68. $\int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln |x + \sqrt{x^2 - a^2}| + C$
69. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln |x + \sqrt{x^2 - a^2}| + C$
70. $\int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$
71. $\int x^2 \sqrt{x^2 - a^2} \, dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln |x + \sqrt{x^2 - a^2}| + C$
72. $\int \frac{\sqrt{x^2 - a^2}}{x^2} \, dx = \ln |x + \sqrt{x^2 - a^2}| - \frac{\sqrt{x^2 - a^2}}{x} + C$
73. $\int \frac{x^2}{\sqrt{x^2 - a^2}} \, dx = \frac{a^2}{2} \ln |x + \sqrt{x^2 - a^2}| + \frac{x}{2} \sqrt{x^2 - a^2} + C$
74. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$
75. $\int \frac{dx}{x(x^2 - a^2)} = \frac{1}{2a^2} \ln \left| \frac{x^2 - a^2}{x^2} \right| + C$

Integrals Involving $a^2 + x^2$; $a > 0$

76. $\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + C$
77. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln(x + \sqrt{a^2 + x^2}) + C$
78. $\int \frac{dx}{x\sqrt{a^2 + x^2}} = \frac{1}{a} \ln \left| \frac{a - \sqrt{a^2 + x^2}}{x} \right| + C$
79. $\int \frac{dx}{x^2 \sqrt{a^2 + x^2}} = -\frac{\sqrt{a^2 + x^2}}{a^2 x} + C$
80. $\int x^2 \sqrt{a^2 + x^2} dx = \frac{x}{8} (a^2 + 2x^2) \sqrt{a^2 + x^2} - \frac{a^4}{8} \ln(x + \sqrt{a^2 + x^2}) + C$
81. $\int \frac{\sqrt{a^2 + x^2}}{x^2} dx = \ln|x + \sqrt{a^2 + x^2}| - \frac{\sqrt{a^2 + x^2}}{x} + C$
82. $\int \frac{x^2}{\sqrt{a^2 + x^2}} dx = -\frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + \frac{x\sqrt{a^2 + x^2}}{2} + C$
83. $\int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$
84. $\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 + x^2}} + C$
85. $\int \frac{dx}{x(a^2 + x^2)} = \frac{1}{2a^2} \ln \left(\frac{x^2}{a^2 + x^2} \right) + C$

Integrals Involving $ax \pm b$; $a \neq 0, b > 0$

86. $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C; n \neq -1$
87. $\int (\sqrt{ax + b})^n dx = \frac{2}{a} \frac{(\sqrt{ax + b})^{n+2}}{n+2} + C; n \neq -2$
88. $\int \frac{dx}{x\sqrt{ax - b}} = \frac{2}{\sqrt{b}} \tan^{-1} \sqrt{\frac{ax - b}{b}} + C$
89. $\int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}} \right| + C$
90. $\int \frac{x}{ax + b} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax + b| + C$
91. $\int \frac{x^2}{ax + b} dx = \frac{1}{2a^3} ((ax + b)^2 - 4b(ax + b) + 2b^2 \ln|ax + b|) + C$
92. $\int \frac{dx}{x^2(ax + b)} = -\frac{1}{bx} + \frac{a}{b^2} \ln \left| \frac{ax + b}{x} \right| + C$
93. $\int x\sqrt{ax + b} dx = \frac{2}{15a^2} (3ax - 2b)(ax + b)^{3/2} + C$
94. $\int \frac{x}{\sqrt{ax + b}} dx = \frac{2}{3a^2} (ax - 2b)\sqrt{ax + b} + C$
95. $\int x(ax + b)^n dx = \frac{(ax + b)^{n+1}}{a^2} \left(\frac{ax + b}{n+2} - \frac{b}{n+1} \right) + C; n \neq -1, -2$
96. $\int \frac{dx}{x(ax + b)} = \frac{1}{b} \ln \left| \frac{x}{ax + b} \right| + C$

Integrals with Exponential and Trigonometric Functions

97. $\int e^{ax} \sin bx dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + C$
98. $\int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + C$

Integrals with Exponential and Logarithmic Functions

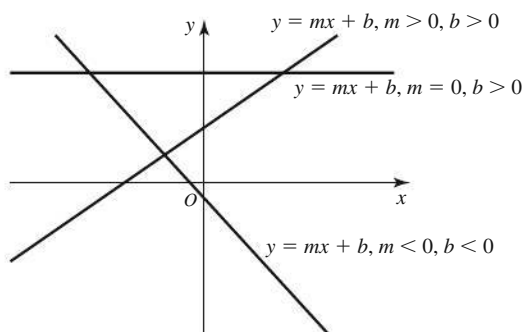
99. $\int \frac{dx}{x \ln x} = \ln|\ln x| + C$
100. $\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \left(\ln x - \frac{1}{n+1} \right) + C; n \neq -1$
101. $\int x e^x dx = x e^x - e^x + C$
102. $\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx; a \neq 0$
103. $\int \ln^n x dx = x \ln^n x - n \int \ln^{n-1} x dx$

Miscellaneous Formulas

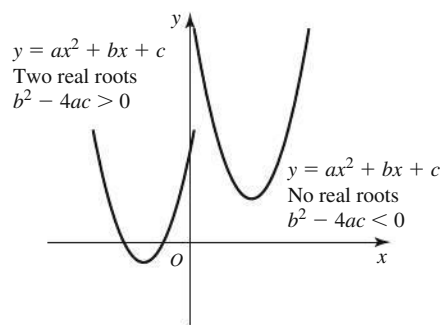
104. $\int x^n \cos^{-1} x dx = \frac{1}{n+1} \left(x^{n+1} \cos^{-1} x + \int \frac{x^{n+1} dx}{\sqrt{1-x^2}} \right); n \neq -1$
105. $\int x^n \sin^{-1} x dx = \frac{1}{n+1} \left(x^{n+1} \sin^{-1} x - \int \frac{x^{n+1} dx}{\sqrt{1-x^2}} \right); n \neq -1$
106. $\int x^n \tan^{-1} x dx = \frac{1}{n+1} \left(x^{n+1} \tan^{-1} x - \int \frac{x^{n+1} dx}{x^2 + 1} \right); n \neq -1$
107. $\int \sqrt{2ax - x^2} dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + C; a > 0$
108. $\int \frac{dx}{\sqrt{2ax - x^2}} = \sin^{-1} \left(\frac{x-a}{a} \right) + C; a > 0$

GRAPHS OF ELEMENTARY FUNCTIONS

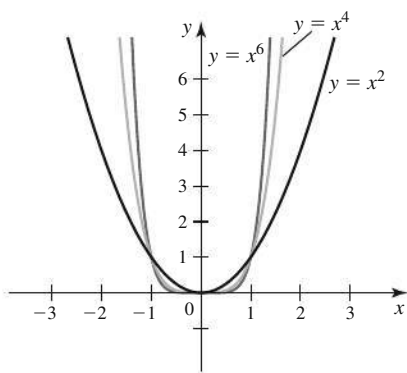
Linear functions



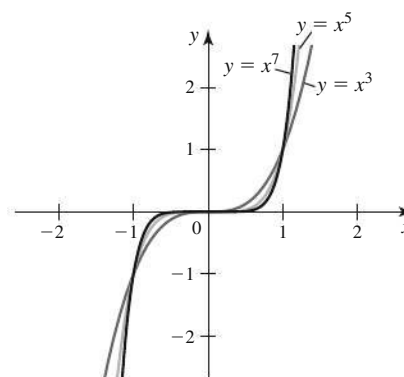
Quadratic functions



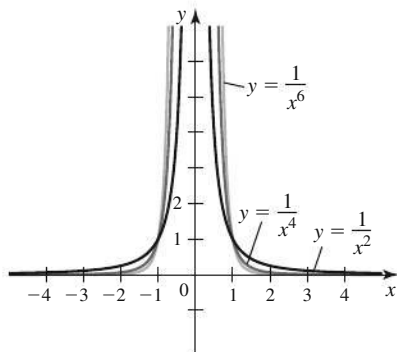
Positive even powers



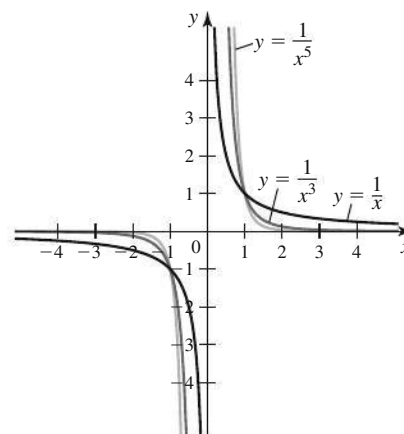
Positive odd powers



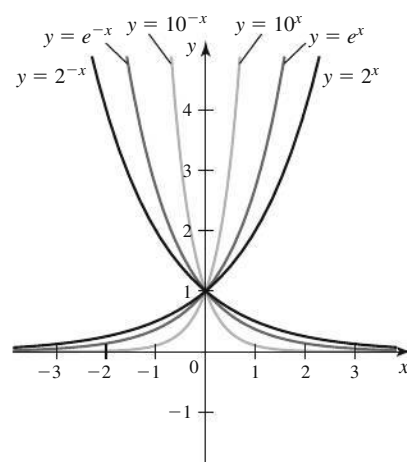
Negative even powers



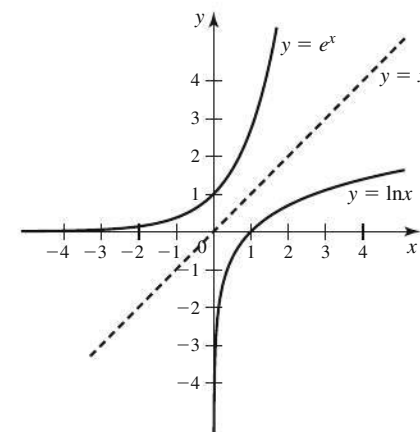
Negative odd powers



Exponential functions



Natural logarithmic and exponential functions



DERIVATIVES

General Formulas

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}(x^n) = nx^{n-1}, \text{ for real numbers } n$$

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

$$\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

Exponential and Logarithmic Functions

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln |x|) = \frac{1}{x}$$

$$\frac{d}{dx}(b^x) = b^x \ln b$$

$$\frac{d}{dx}(\log_b |x|) = \frac{1}{x \ln b}$$

Hyperbolic Functions

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2} \quad (|x| < 1)$$

$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}} \quad (0 < x < 1)$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}} \quad (x > 1)$$

$$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2} \quad (|x| > 1)$$

$$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{1+x^2}} \quad (x \neq 0)$$

FORMS OF THE FUNDAMENTAL THEOREM OF CALCULUS

Fundamental Theorem of Calculus	$\int_a^b f'(x) dx = f(b) - f(a)$
Fundamental Theorem for Line Integrals	$\int_C \nabla f \cdot d\mathbf{r} = f(B) - f(A)$ (A and B are the initial and final points of C .)
Green's Theorem	$\iint_R (g_x - f_y) dA = \oint_C f dx + g dy$ $\iint_R (f_x + g_y) dA = \oint_C f dy - g dx$
Stokes' Theorem	$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS = \oint_C \mathbf{F} \cdot d\mathbf{r}$
Divergence Theorem	$\iiint_D \nabla \cdot \mathbf{F} dV = \iint_S \mathbf{F} \cdot \mathbf{n} dS$

FORMULAS FROM VECTOR CALCULUS

Assume $\mathbf{F}(x, y, z) = f(x, y, z)\mathbf{i} + g(x, y, z)\mathbf{j} + h(x, y, z)\mathbf{k}$, where f, g , and h are differentiable on a region D of $\mathbb{R}^3$.

Gradient: $\nabla f(x, y, z) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

Divergence: $\nabla \cdot \mathbf{F}(x, y, z) = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$

Curl: $\nabla \times \mathbf{F}(x, y, z) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f & g & h \end{vmatrix}$

$\nabla \times (\nabla f) = \mathbf{0} \quad \nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\mathbf{F}$ conservative on $D \Leftrightarrow \mathbf{F} = \nabla \varphi$ for some potential function φ

$\Leftrightarrow \oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ over closed paths C in D

$\Leftrightarrow \int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path for C in D

$\Leftrightarrow \nabla \times \mathbf{F} = \mathbf{0}$ on D