# Our Game Theory Example: The Opener vs the Four Flush

Although traditional wisdom is that the four flush is not worth trying to improve, here Prof. Ankeny considers the case where betting was opened by the last player to have the opportunity - the player on the immediate right of the dealer. In this case all players know that none of those before him had a hand which was worth opening *on*. This in turn makes a lowly pair of jacks with crappy side cards worth trying to improve by opening, so that may be the case.

Game theory claims that for every situation there is a best and worst possible play, something we can probably all relate to from certain Tic-Tac-Toe situations. Prof. Ankeny then uses it to show us that in the above case, the four-flush can be made to be a winner; see Figure 1/Table 3.1.

POKER STRATEGY

the time? Player B catches A twice when B actually has the flush and wins $24 each time. This puts B ahead $48. But B will bet $12 the other eight times and lose it every time A calls. As a result, B loses $96. B's net for the ten hands is minus $48—he loses an average of $4.80 per hand. This is a likely outcome because if B bluffs all the time, the other players will soon catch on and call him.

What will happen to B if he bluffs all the time and A calls him half the time? On the two flushes he completes, B will win a total of $36—$24 when A calls and loses, $12 when A folds. Of the other eight times when B bluff bets $12, A will call half the time and B will lose his $12 bet, and A will fold half the time and B will win the $12 pot. On the bluffs, then, B will make nothing at all—zero profit, and zero loss. This means that he will again make $36 over the ten hands —an average of $3.60 per hand.

By this same sort of analysis we find that if B bluffs, say, twice for every strength bet (of the ten four-flush hands, he bets twice, bluffs four times, and passes four times) he will profit $7.20 per hand if A never calls, $3.60 if A calls half the time, and nothing if A always calls.

Some interesting patterns emerge from these figures. Probability tables are probably more interesting to the professional mathematician than to the professional poker player, but table 3.1 contains information that should intrigue any serious poker player. It summarizes what happens to B's profits as he varies the frequency of his bluffing and A varies the frequency of his calling. For example, table 3.1 shows that if A has a policy of never calling, Player B's profits rise the more he bluffs. Also, if A has a policy of always calling, Player B's profits decrease the more he bluffs.

This illustration of B squaring off against A can now lead

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*Bluffing: The Optimal Strategy in Draw Poker*

us to a guaranteed maximum profit strategy for Player B. Table 3.1 reveals that when B always bluffs, his winnings depend on how often A calls. The profits are high when A never calls and negative when A always calls. Player B's profits when he never bluffs are also shown. Notice that the profit is low when A never calls and high when A always calls. Player B's profits are still dependent on A's calling policy.

**Table 3.1**
***Player B's Profit: When Players A and B Vary Their Strategies***

| | Player A Calls | | |
|---|---|---|---|
| Player B Bluffs | Never | 50 Percent | Always |
| Always | $12.00 | $3.60 | –$4.80 |
| 2:1* | 7.20 | 3.60 | 0 |
| 1:1 | 4.80 | 3.60 | 2.40 |
| 1:2 | 3.60 | 3.60 | 3.60 |
| Never | 2.40 | 3.60 | 4.80 |

*2:1 means that Player B makes two bluffs for every strength bet.

We see that B's profits fluctuate wildly as each player varies his strategy. Does Player B have a strategy that will both be unaffected by Player A's unpredictable strategy and earn the most money possible against a tough opponent? Such a strategy brings Player B the *largest guaranteeable profit*—that is exactly what constitutes an optimal strategy.

Remember in what sense B's profit is optimal, but not necessarily maximal. Player B could make $12 per hand if he always bluffed and A never called. This possible profit is far larger than the optimal profit, but here the word "optimal" includes the idea of "guaranteeable." In this simple example the optimal strategy yields a profit considerably less

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Figure 1. (Table 3.1)
The optimal strategy in player B's four-flush situation is in offensive strategy, in which B adopts a bluff ratio of 1:2 - one bluff bet for every two strength bets. If B does this, A can do nothing to prevent him from winning an average profit of $3.60 per hand. Conversely, if A adopts the optimal calling ratio of 1:1, he prevents B from winning any *more* than $3.60 per hand.

# Limitations

The Theory of Slow Turns applies only to cars whose designs subscribe to the Theory of Slow Turns.  In other words:
- In theory, the Theory of Slow Turns applies only to cars which are points or bricks or otherwise enjoy a perfect distribution of mass which places their CG at their centroid, or directly below it.
- In practice, the Theory of Slow Turns applies only to mid-engine cars with front wheel steering.

# Disclaimer

Do not try this in your Porsche.

# Prerequisite Information

There are two pieces of background knowledge the reader must have at this point: the definition of slow turns, and the braking and turning force fundamentals.

## 1. Definitions

**Fast turn**: A turn for which you as the driver would not lift.  A clean example of a fast turn can be seen at 1:27 here
Hakkinen Battles Schumacher At Spa | 2000 Belgian Grand Prix
**Medium turn**: One for which you would lift and perhaps brake lightly to moderately
**Slow turn**: One for which you would brake hard and deep into; perhaps ⅓ - ½ way.  A clean example of a slow turn can be seen at 0:24 here
fernando alonso onboard pole lap at magny cours 2005
**Apex:** A.K.A. the clipping point, the middle of or tightest point in a turn

## 2. Force Vectors

This theory accepts as correct the assertions regarding the forces acting upon the car made in "The Technique of Motor Racing" by Piero Taruffi[1].

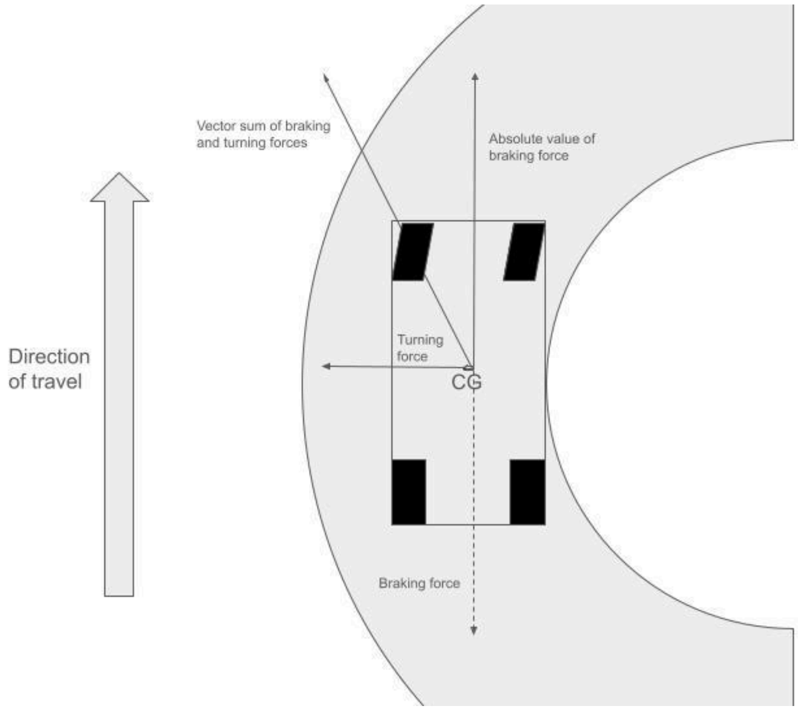


Figure 1. Forces acting on a car in a turn.

**Figure 1** shows the forces. The braking force is towards the rear of the car; we are more interested in its absolute value. Taruffi claims that when the magnitude of the vector sum of the braking and turning force vectors exceeds the available traction, the car will lose adhesion and leave the road in the direction of the vector sum. This diagram presently has this error: the car is shown in the middle of the turn, when the brakes should be fully off, but the braking force vector is not drawn with zero length.

# Theory

### *1(a) Postulate 1*

At the end of the straightaway the driver must be braking as hard as is possible without locking the wheels. In other words using, by braking, as much of the available traction as is possible without using it all.

### *1(b) Postulate 2*

At the apex of the turn the car should be going around the turn as fast as possible without losing adhesion and leaving the road. In other words using - by turning - as much of the available traction as is possible without using it all.

### *1(c) Assertion*

The optimal driver behavior between the entrance of the turn and the apex is to release the brakes in such a way that the magnitude of the vector sum of the braking and turning forces is kept constant at that value which requires as close as is possible to but without equalling 100% of the available traction.

*1(d) Corollary Hypothesis*

For any given car, for every recognizable slow turn we can identify, there is an optimal de-braking curve.

## Comments

**"Thus, for Poker players, Game Theory bears fruit. Game Theory tells us that for every recognizable poker situation we can name, there is an optimal strategy. The strategy may be too complex for us to discern, but in theory there is one."**
**- Ankeny, Nesmith: Poker Strategy: Winning with Game Theory**

Game Theory also tells us that for every slow turn that we can identify, there exists an optimal de-braking strategy.

This theory claims that that strategy is to let no amount of available traction go unused. If traction goes unused then either the driver failed to either brake or turn as hard as was possible, or the entry speed was too low. Where it was the former, the brakes could have been used harder, there also existed less resultant downforce on the front wheels at that time, so there was less available traction, compounding the penalty for this error.

We have, then, that regarding Corollary 1(d), the single fastest de-braking curve simultaneously permits and requires the single fastest entry speed.

It is not a game of pushing down. It is a game of lifting up.

*Release the brakes as you would approach God.*

## Exercises

1. Watch the slow turn example of Fernando Alonso many times as is necessary to see that he spends the entire second from 0:27 to 0:28 simultaneously releasing the brakes and turning the wheel in. One can hear the RPMs continue to drop after he downshifts into 1st - between 0:26.0 and 0:26.5 - Until the apex right around 0:27.9
   fernando alonso onboard pole lap at magny cours 2005
2. Sit in a chair with no arms and scoot forward so your sitbones are near the front edge of the seat. Hold your right foot on an already depressed imaginary brake pedal, with your heel 1" above the floor. Hold your hands at the 3 and 9 o'clock positions on an imaginary steering wheel.
3. When you are ready, suddenly but smoothly begin to lift your foot up a few inches and turn the wheel 90 degrees at the same time. Do it at a rate that both finish in exactly one full second.
   When lifting your foot, do not use your foot or ankle - use the muscles in your thigh or hip, letting everything from your kneecap down hang.

   Lifting your foot and turning the wheel at the same time is like rubbing your stomach and patting your head at the same time. It is not difficult but nobody gets it right the first time. You must practice it if you wish to use it.

4. Now try making it take two seconds, and try one-half of a second.
5. Now do it over 1 second but starting slowly and ending quickly, then do it starting quickly and ending slowly.
6. Do them again but turn the wheel 180 degrees.
7. Before you get good at it, come to San Jose, CA, and race me at indoor go-karts for $100/lap. We will do 20 laps and switch cars after 10. Bring $2K USD + track fees to lose to me. I assure you it will be worth every penny.
8. Now watch this video of Lewis Hamilton in turn 11 at Suzuka at 0:13 here
   Lewis Hamilton Smashes Suzuka Track Record | 2017 Japanese Grand Prix
   Whether that is a video game or too much camera firmware is making it look like one; either way, what is going on with that de-braking curve?
9. Now go back to Alonso and watch his steering between 0:26.5 and 0:27. He turns the wheel 90 degrees first, before 0:27, then 90-100 more after, but is the rate constant? It first looks to me like there is a pause, but if I focus on his hands not the steering wheel center, they appear to move at a constant rate.
10. Now forward to 0:57 as he approaches the Chateau d'Oh. A few moments later the announcer will say he has never seen a car turn in so well. How might it be possible to reword "so well"?

## Known Bugs

This theory contains at least one error, but this may be it.

## Purpose

This paper is about racing, and specifically not about driving on the street. It is, for the most part, not transferable on the street.

For example, suppose you are cruising down main street, Anytown, USA, and approaching a 4-way intersection where the speed limit is 30 in all directions. For your cruising speed to be 100-110 would be neither safe nor legal nor practical. For you to then brake quite hard down to 60 at the white line, compressing the front springs by a goodly amount, and then meter out that stored energy by staying on the brakes and exiting the turn at a crawl - 15 - because you refused to let the front bob up and 60 was too slow - this would place your street driving squarely in the realm of totally unacceptable to all other human beings.

There is one item in this paper which can be applied to street driving, and that is the comment "Release the brakes as you would approach God". We were all taught to release the clutch smoothly, but not the brakes.

Also, many of us were taught not to brake in turns, and the reader now understands the mechanics of and good reason for this. It is not clear to me that this is always possible, given the realities of rush hour traffic behind you. But that is a different subject.

In any case, there is no conflict here. Whether you choose to brake before the turn or into it, release them with the very utmost of care and fine attention to detail.

## Authenticity

If you have any questions or concerns regarding the authenticity of the information presented here, I suggest you direct your inquiries to Elf/Renault Winfield Ecole du Pilotage, or to Mika Hakkinen, Fernando Alonso, Alain Prost, Nelson Piquet, Gerhard Berger, Nigel Mansell, Riccardo Patrese, Rene Arnoux, or Jean Alesi, or the McLaren, Williams, Ferrari, Brabham, Lotus, or Lola Racing teams.

## References

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